NAVPERS 93400 A-1b

FUNDAMENTALS OF ELECTRONICS

VOLUME 1b

BASIC ELECTRICITY

Alternating Current

Bureau of Naval Personnel

PREFACE

This book is part of a nine-volume set entitled "Fundamentals of Electronics". The nine volumes include:

Volume 1a - NavPers 93400A-1a, Basic Electricity, Direct Current
Volume 1b - NavPers 93400A-1b, Basic Electricity, Alternating Current
Volume 2 - NavPers 93400-2, Power Supplies and Amplifiers
Volume 3 - NavPers 93400-3, Transmitter Circuit Applications
Volume 4 - NavPers 93400-4, Receiver Circuit Applications
Volume 5 - NavPers 93400-5, Oscilloscope Circuit Applications
Volume 6 - NavPers 93400-6, Microwave Circuit Applications
Volume 7 - NavPers 93400-7, Electromagnetic Circuits and Devices
Volume 8 - NavPers 93400-8, Tables and Master Index

If you are becoming acquainted with electricity or electronics for the first time, study volumes one through seven in their numerical sequence. If you have a background equivalent to the information contained in volumes one and two, you are prepared to study the material contained in any of the remaining volumes. A master index for all volumes is included in volume eight. Volume eight also contains technical and mathematical tables that are useful in the study of the other volumes.

A question (or questions) follows each group of paragraphs. The questions are designed to determine if you understand the immediately preceding information. As you study, write out your answers to each question on a sheet of paper. If you have difficulty in phrasing an answer, restudy the applicable paragraphs. Do not advance to the next block of paragraphs until you are satisfied that you have written a correct answer.

When you have completed study of the text matter and written satisfactory answers to all questions on two facing pages of the book, compare your answers with those at the top of the next even-numbered page. If the answers match, you may continue your study with reasonable assurance that you have understood and can apply the material you have studied. Whenever your answers are incorrect, restudy the applicable material to determine why the book answer is correct and yours is not. If you make an honest effort to follow these instructions, you will have achieved the maximum learning benefits from each study assignment.

Follow the directions of your instructor in answering the review questions included at the end of each chapter.

CHAPTER 8

ALTERNATING CURRENT

The first faltering steps of scientific achievement in the field of electricity were performed with crude, and for the most part, homemade apparatus. Great men, such as Georg Simon Ohm, had to fabricate nearly all the laboratory equipment used in their experiments. The only convenient source of electrical energy available to these early scientists was the voltaic cell, invented some years earlier. Due to the fact that cells and batteries were the only sources of power available, some of the early electrical devices were designed to operate from DIRECT CURRENT (dc).

When the use of electricity became widespread, certain disadvantages in the use of direct current became apparent. In a direct current system the supply voltage must be generated at the level required by the load. To operate a 220 volt lamp for example, the generator must deliver 220 volts. A 110 volt lamp could not be operated from this generator by any convenient means. A resistor could be placed in series with the 110 volt lamp to drop the extra 110 volts, but the resistor would waste an amount of power equal to that consumed by the lamp.

Another disadvantage of direct current systems is the large amount of power lost due to the resistance of the transmission wires used to carry current from the generating station to the consumer. This loss could be greatly reduced by operating the transmission line at very high voltage and low current. This is not a practical solution in a dc system, however, since the load would also have to operate at high voltage. As a result of the difficulties encountered with direct current, practically all modern power distribution systems use a type of current known as ALTERNATING CURRENT (ac). In an alternating current system, the current flows first in one direction then reverses and flows in the opposite direction.

Unlike dc voltage, ac voltage can be stepped up or down by a device called a TRANSFORMER. This permits the transmission lines to be operated at high voltage and low current for maximum efficiency. Then at the consumer end the voltage is stepped down to whatever value the load requires by using a transformer. Due to its inherent advantages and versatility, alternating current has replaced direct current in all but a few commercial power distribution systems.

In the theory to follow, a knowledge of the use of the natural trigonometric functions is required. This material is found in Volume 8.

8-1. Types of Voltage

Many other types of current and voltage exist in addition to direct current and voltage. If a graph is constructed showing the magnitude of dc voltage across the terminals of a battery with respect to time it would appear as in Figure 8-1A. The dc voltage is shown to have a constant amplitude. Some voltages go through periodic changes in amplitude like those shown in Figure 8-1B. The pattern which results when these changes in amplitude with respect to time are plotted on graph paper is known as a WAVEFORM. Figure 8-1B shows some of the common electrical waveforms. Of those illustrated, the sine wave will be dealt with most often.

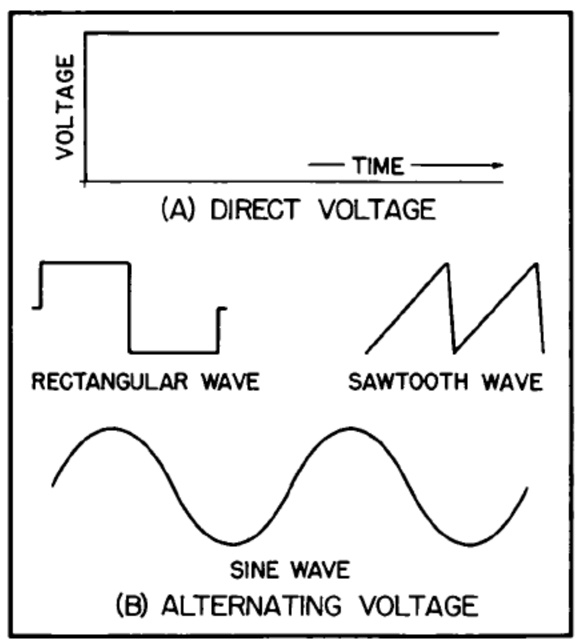

Figure 8-1 - Voltage waveforms.

8-2. Review of Induction

As stated in Chapter 4, electromagnetic induction occurs any time a conductor is passed through a magnetic field in such a way as to cut the lines of force. In the diagram shown in Figure 8-2, a conductor is passed upward through a magnetic field in a direction perpendicular to the direction of the flux lines. As the conductor cuts through the flux lines, the magnetic field will exert a force on the free electrons within the conductor. This force causes the free electrons to leave one end of the conductor and pile up on the other, creating a difference of potential across the conductor. The direction of this electron displacement and the resulting polarity of induced voltage can be determined by the use of the LEFT-HAND RULE FOR GENERATORS stated as: With the thumb, forefinger, and middle finger on the left hand held perpendicular to each other, point the forefinger in the direction of the magnetic field, and the thumb in the direction of motion of the conductor; the middle finger will then point in the direction of electron displacement (negative end of conductor).

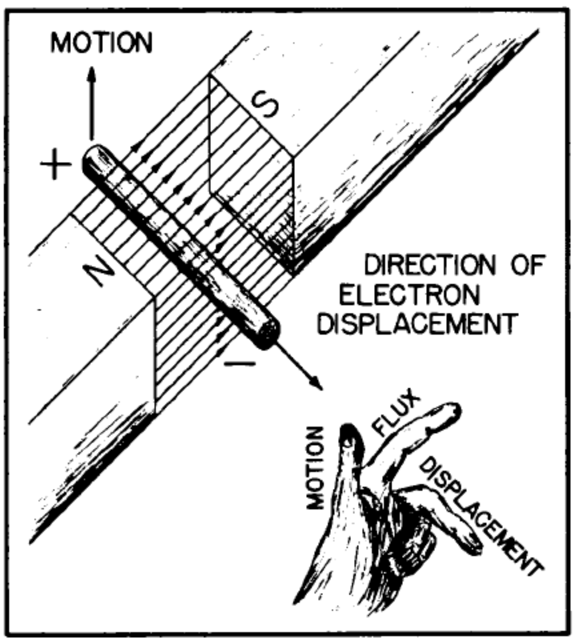

Figure 8-2 - Left-hand rule for induced EMF

The application of this rule is demonstrated in Figure 8-2. Notice that the middle finger points to the end of the conductor which assumes a negative charge as a result of the electron displacement.

Q1. What would happen if the direction of motion of the conductor were reversed?

8-3. Magnitude of Induced Voltage

The amount of voltage induced into a conductor cutting a magnetic field is dependent on the number of lines cut per unit of time. This is determined by four basic factors which are:

1. The speed of relative motion between the field and the conductor.
2. The strength of the magnetic field.
3. The length of the conductor within the field.
4. The angle at which the conductor cuts the field.

If the speed at which the conductor cuts the lines of force is increased, the force on the free electrons within the conductor is greater and the generated voltage is increased.

Increasing the strength of the magnetic field also increases the force on the electrons, thereby increasing the induced voltage. The induced voltage is directly proportional to the strength of the field.

The length of the conductor within the field affects the amount of induced voltage. A longer conductor permits the magnetic field to perform more work on the free electrons. A long conductor can be considered to consist of a number of short conductors connected in series. The individual induced voltages in each of the short sections add to produce a large total voltage across the long conductor. For this reason practical generators use long conductors which are formed into coils to conserve space.

The angle at which the conductor cuts the lines of force affects the number of lines cut per unit of time and, therefore, the amount of induced voltage. The effects of the cutting angle on induced voltage can be illustrated by the drawings in Figures 8-3 and 8-4. In Figure 8-3, a conductor is shown moving parallel to the lines of force between the poles of a magnet. Since the conductor cuts no lines of force, no voltage is induced in the conductor.

In Figure 8-4 a conductor is shown cutting the magnetic field at an angle of 30° with respect to the direction of the lines of flux. In the drawing the conductor is traveling at a fixed velocity and requires one second to travel from point (A) to point (B). The conductor could have cut the same number of flux lines by moving from point (A) to point (C). Since ABC forms a 30°, 60°, 90° right triangle, side AC is one-half the length of side AB. The conductor traveling at a constant velocity thus takes twice as long to travel from A to B as it would to travel from A to C. Since the conductor cuts only half as many lines of force per unit of time when cutting at 30° as it does when cutting at 90°, only one-half as much voltage will be induced into the conductor when cutting at an

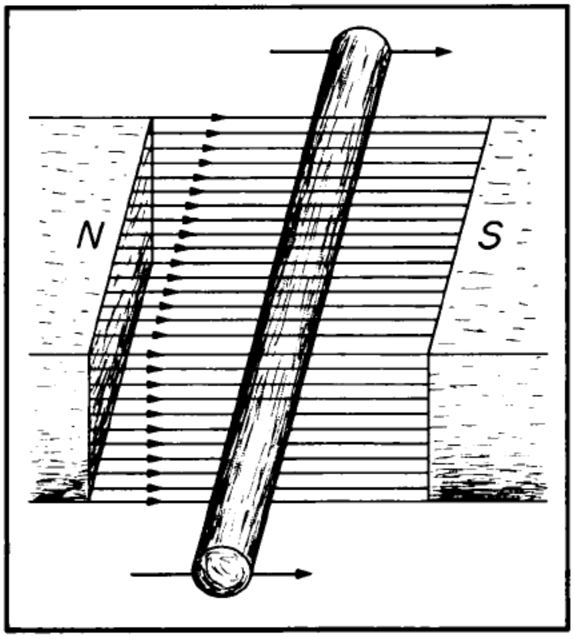

Figure 8-3 - Conductor moving parallel to lines of force.

angle of 30°. This principle can be developed into a formula which can be used to determine the instantaneous voltage induced into a conductor as it cuts a magnetic field.

Since the maximum voltage will be induced in the conductor when it cuts the field at 90°,

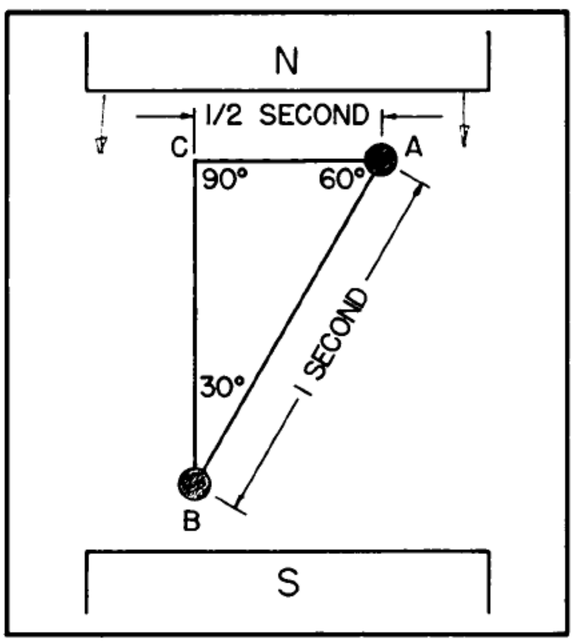

Figure 8-4 - Conductor cutting field at 30°.

any angle less than 90° will produce some fraction of the maximum voltage.

As the conductor moves through the field at an angle other than 90°, its motion can be resolved into two components as shown in Figure 8-5. One of these components of motion (line AC) is parallel to the lines of flux and thus produces no induced voltage. The other component

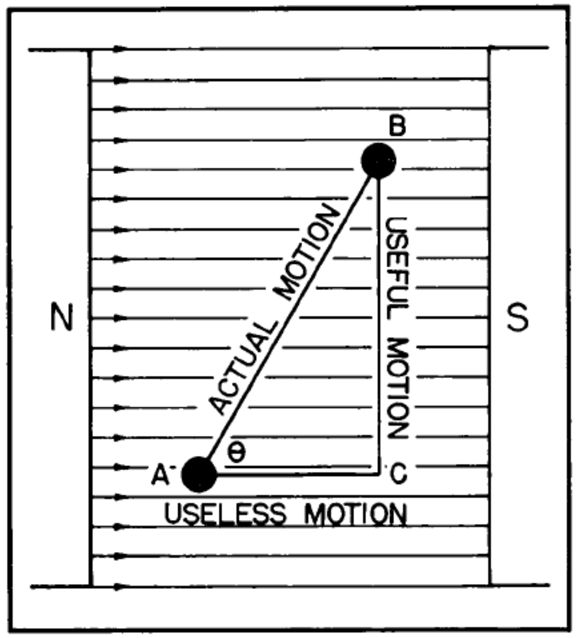

Figure 8-5 - Components of motion.

of motion (line CB) is at right angles to the lines of flux and is the movement which actually produces the induced voltage. By the use of the sine function, the magnitude CB can be determined if the angle THETA (θ) and the magnitude AB are known. As an equation:

$$CB = AB \times \text{sine } \theta \qquad (8\text{-}1)$$

To see how this equation is utilized to determine the induced voltage in a conductor, let us examine the operation of a simple alternator.

8-4. Simple Alternator

The construction of a simple two pole alternator is shown in Figure 8-6. Notice that the conductor has been formed into a loop and placed between the poles of a magnet in such a way as to permit rotation of the loop within the magnetic field. The rotating loop and its supporting structure is called an ARMATURE. The free ends of the loop are attached to cylindrical

A1. The polarity of the induced voltage would reverse.

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pieces of metal called SLIP RINGS, which rotate along with the loop. As the loop rotates the induced voltage will appear at the slip rings.

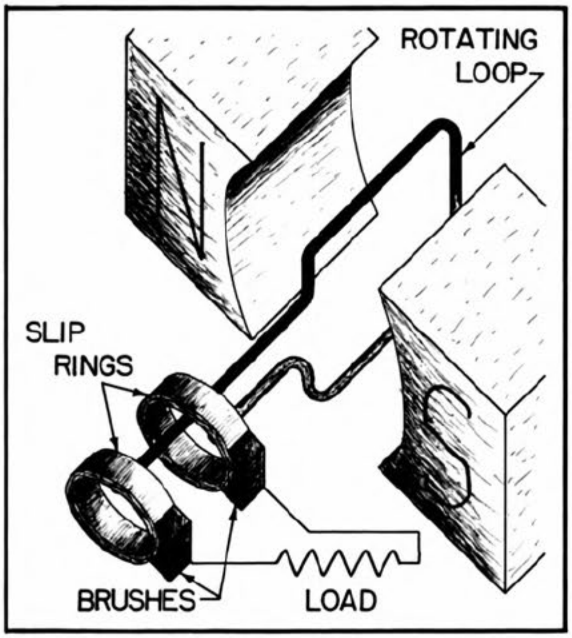

Figure 8-6 - Simple alternator.

In order to connect the stationary load to the rotating loop without the possibility of the leads twisting as the loop rotates, sliding contacts, called BRUSHES, are used. These are generally of carbon and press against the slip rings to form a rotating contact. Any voltage present on the rings is transferred to the stationary brushes and thence to the load.

8-5. Induced Voltage

To analyze the operation of the simple alternator, refer to Figure 8-7. In this drawing the loop of wire is shown as it rotates counterclockwise within the magnetic field. At the instant of time illustrated, the section of the loop from A to B is passing downward through the magnetic field. Application of the left-hand rule for generators to this side of the loop shows the electrons to be forced from A toward B as indicated by the arrow on the left-hand side of the loops. Since the electrons are leaving A and being concentrated at B, end A assumes a positive charge while end B takes on a negative charge.

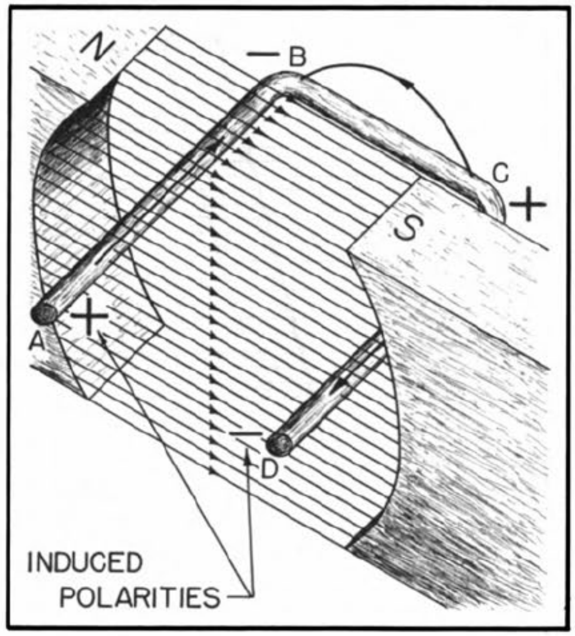

Figure 8-7 - Polarity of induced voltage.

While the left-hand side of the loop is passing downward through the magnetic field, the right-hand side of the loop is traveling upward through the field. Since the direction of motion of side CD is opposite to the direction of motion of side AB, the voltages induced into these two sides will be of opposite polarity. Application of the left-hand rule to side CD shows the electrons to be displaced from C to D. This displacement current, shown by the arrow, leaves C positive and makes D negative. Notice that in tracing around the wire loop the voltages induced in the two sides are SERIES-AIDING. The voltage which appears between the ends of the loop (A-D) is thus equal to the SUM of the voltages in each side. This total voltage is present at the brushes and can be applied to an external load.

Q2. Would any voltage be induced into the loop if it were to open between points B and C?

8-6. A Complete Cycle

If the loop of wire in the simple alternator is rotated at a constant speed, a voltage will be generated which varies in amplitude and periodically reverses polarity. To show how this waveform of voltage is generated, the amplitude of the voltage induced into the loop will be plotted for each 45° of rotation. (See Figure 8-8).

As in the previous explanations, the loop of wire will be assumed to rotate in a counter-

clockwise direction. The illustration represents a cross-sectional view in which the end view of one side of the loop is shown, the other side of the loop being omitted for simplicity.

At the start of the revolution (time 0) the loop is traveling parallel to the lines of force. Since no lines of force are cut by the conductor, no voltage is induced in the loop.

At time ONE the conductor has advanced 45° to the position shown. Since the angle at which the conductor cuts the lines of force is known, equation 8-2 can be used to compute the voltage induced into the conductor in terms of the maximum possible induced voltage. Assuming that the maximum voltage (E_m) that can be induced in the loop is one volt (when the conductor cuts the lines at right angles), the instantaneous voltage (e) induced at an angle of 45° will be:

$$e = E_m \times \sin \theta \quad (8\text{-}2)$$

$$e = E_m \times \sin 45^\circ = 1.0 \times 0.707$$

$$e = 0.707 \text{ volts}$$

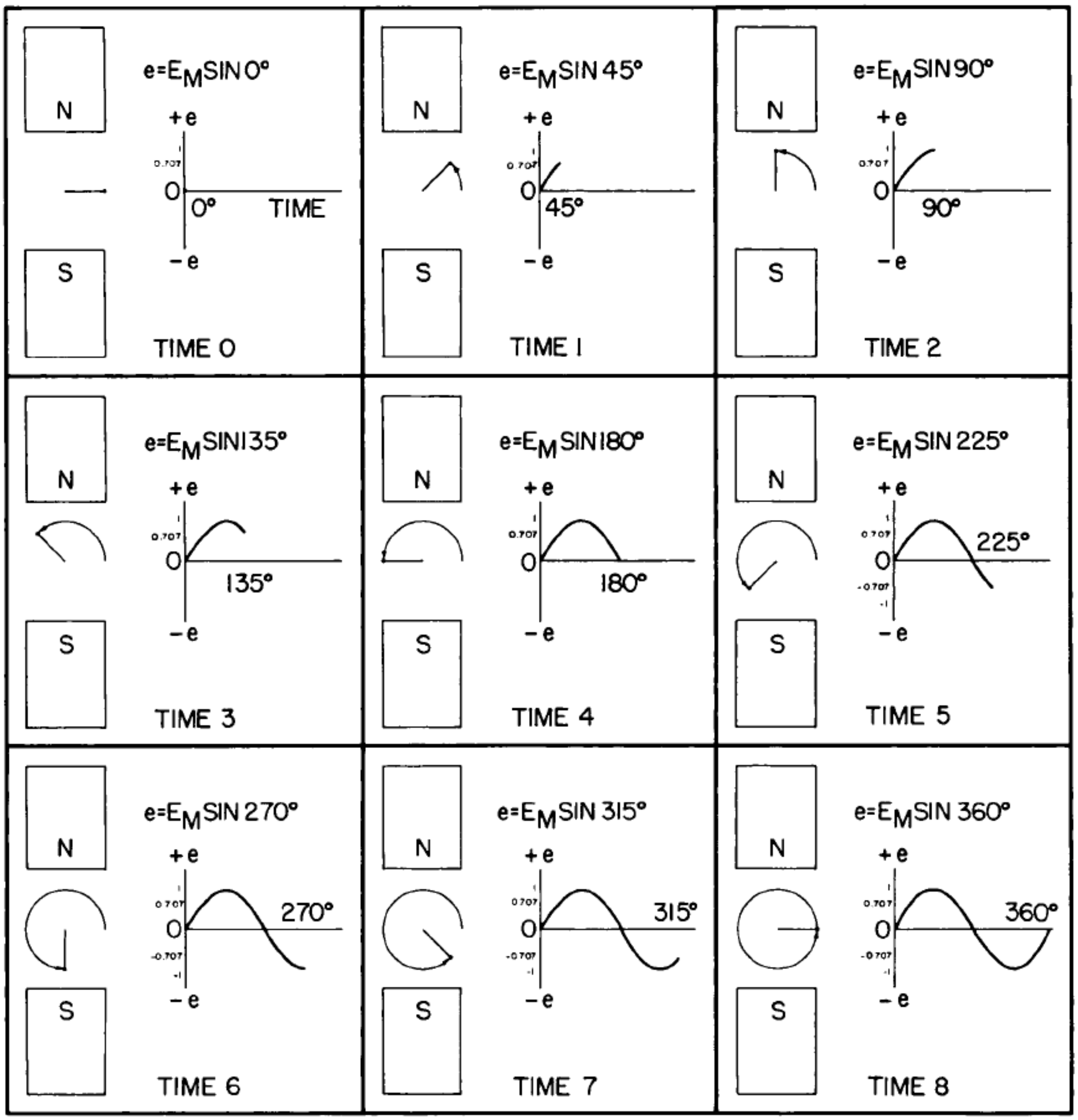

Figure 8-8 - Generation of a sine wave.

A2. Yes. The induced voltage is not dependent upon the existance of a complete current path.

Thus, the instantaneous induced voltage at 45° of rotation is 0.707 volts.

By the time the conductor reaches 90° at time TWO, it is cutting perpendicular to the lines of force and the instantaneous voltage is:

$$e = E_m \times \sin\theta \qquad (8\text{-}2)$$

$$e = 1.0 \times \sin 90^\circ$$

$$e = 1.0 \times 1.0$$

$$e = 1.0 \text{ volt}$$

The induced voltage at 90°, is, therefore, one volt.

As the conductor rotates another 45° to 135° at time THREE, the induced voltage is reduced from its maximum value to the same value that was developed at 45° or 0.707 volts.

At 180° of rotation (time FOUR), the conductor is again traveling parallel to the lines of force and no voltage is induced. At this instant, the conductor has rotated 180 mechanical degrees and the induced voltage has varied from zero to its maximum positive value and back to zero.

As the conductor passes 180° it once more begins to cut lines of force. Notice, however, that the conductor is cutting the lines of force in a direction OPPOSITE to the direction of travel during the first 180°. Since the direction of cutting has reversed, the voltage induced into the conductor will be of the OPPOSITE POLARITY. Thus, at time FIVE the voltage induced into the conductor will be a NEGATIVE 0.707 volts.

At 270° of rotation (time SIX), the conductor is once again cutting the lines of force at right angles, and the induced voltage is at its maximum negative value (-1.0 volt).

In traveling from 270°, through 315°, to 360° the amount of induced voltage will decrease, becoming zero at 360° when the conductor is again traveling parallel to the lines of force. At this point the conductor is back at the starting position and one complete CYCLE of events has occurred. Should the conductor continue rotating, additional cycles will be generated identical to the one just completed. Since the instantaneous amplitude of the generated waveform is proportional to the sine of the angle of cutting, this waveform is called a SINE WAVE.

Q3. What would happen to the output sine wave of voltage if the strength of the magnetic field were increased?

SINE WAVE ANALYSIS

8-7. Frequency

In examining the operation of a simple alternator, one complete rotation of the loop was seen to produce one cycle of ac voltage. If the loop had taken one second for a complete rotation, one cycle of voltage would be generated each second. If the speed of rotation is increased so that one rotation requires only one-tenth of a second, ten cycles of voltage would be generated each second. The number of cycles of voltage that occur each second is called the FREQUENCY of the voltage, and is symbolized by the **letter (f).**

The frequency of the voltage generated by an alternator depends on both the speed at which the armature rotates, and the number of magnetic poles the alternator contains. Regardless of the number of pairs of poles, a complete cycle of voltage will be generated each time an armature coil passes under a north and a south pole in succession. Figure 8-9 shows a four pole alternator. In this device one complete mechanical rotation of the armature will produce two cycles of voltage. Since one cycle is

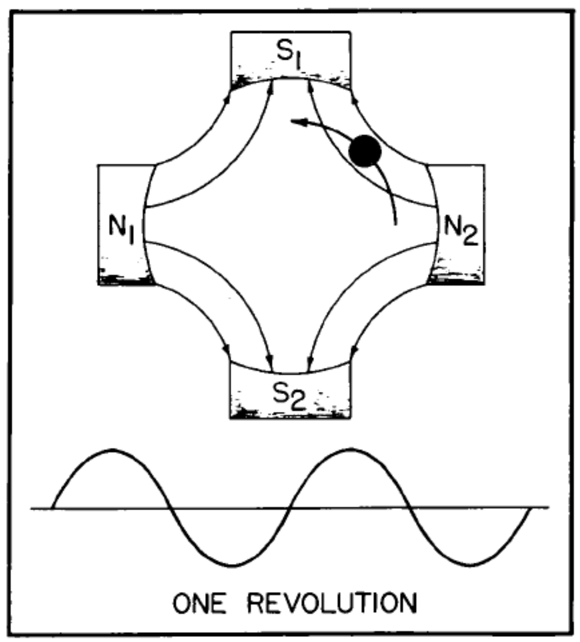

Figure 8-9 - Basic four pole alternator.

generated for each TWO POLES, the number of cycles produced is proportional to one-half the number of poles used, or P/2. Normally, the

speed (S) of rotation of the armature is given in revolutions per minute (RPM). Since the frequency of the voltage is desired in cycles per second, the speed of the armature must be converted from RPM to revolutions per second (RPS). To do this, the RPM are divided by 60 or:

$$RPS = \frac{S}{60} \qquad (8\text{-}3)$$

Since frequency is proportional to speed and one-half the number of poles:

$$f = \frac{P}{2} \times \frac{S}{60}$$

$$f = \frac{PS}{120} \qquad (8\text{-}4)$$

where: f = the frequency in cycles per second (cps)

P = the number of magnetic poles

S = the speed in RPM

Example. An alternator has four poles and is driven at a speed of 1800 RPM. What is the frequency of the generated voltage?

Given: P = 4

S = 1800 RPM

Find: f = ?

Solution:

$$f = \frac{PS}{120} \qquad (8\text{-}4)$$

$$f = \frac{4 \times 1800}{120}$$

$$f = 60 \text{ cps}$$

Mechanical alternators can be constructed to generate a wide range of frequencies. However, due to their mechanical limitations they cannot be constructed to generate frequencies above about 200 kilocycles per second. Fortunately, with the advent of the vacuum tube, circuits were invented which permitted the generation of frequencies of less than one cycle per second to thousands of megacycles (gigacycles) per second.

MEASUREMENT OF PERIOD

An individual cycle of any sine wave represents a finite amount of TIME. Figure 8-10 shows two cycles of a sine wave which has a frequency of two cycles per second. Since two cycles occur each second, one cycle must require one-half second of time. The time required to complete one cycle of a waveform is called the PERIOD of the wave. In this example the period is one-half second.

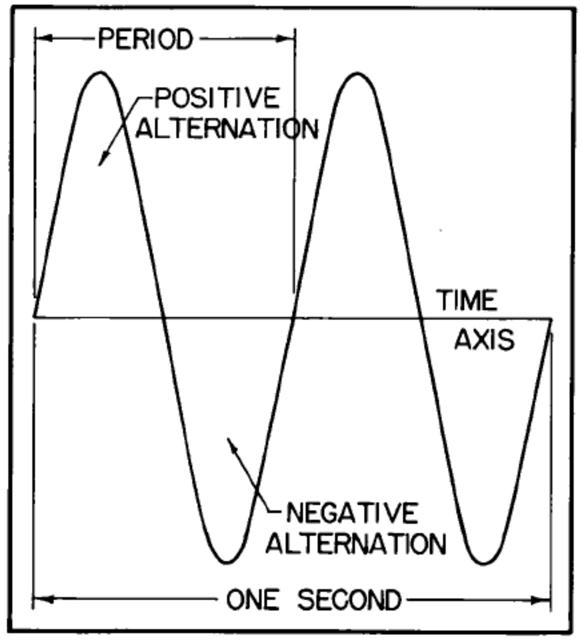

Figure 8-10 - Period of a sine wave

Each cycle of the waveform in Figure 8-10 is seen to consist of two pulse shaped variations in voltage. The pulse which occurs during the time the voltage is positive is called the POSITIVE ALTERNATION. The pulse which occurs during the time the voltage is negative is called the NEGATIVE ALTERNATION. For a sine wave these two alternations will be identical in size and shape, and opposite in polarity.

The period of a wave is inversely proportional to its frequency. Thus, the higher the frequency (greater number of cps), the shorter the period. In terms of an equation:

$$t = \frac{1}{f} \qquad (8\text{-}5)$$

where: t = period in seconds

f = frequency in cycles per second

A3. The amplitude of the sine wave would increase since the induced voltage would be a greater value.

Example. What is the period of a sine wave which has a frequency of 1000 cycles per second?

Given: f = 1000 cps

t = ?

Solution:

$$t = \frac{1}{f} \tag{8-5}$$

$$t = \frac{1}{1000}$$

t = 0.001 second or 1 millisecond

Q4. A waveform has a period of 2 microseconds. What is its frequency?

Q5. How much time is required to complete a single alternation if the sine wave has a frequency of 1 kc?

8-8. Degree Measure

Upon occasion it is necessary to locate a specific point on a sine wave. This could be done by stating the amount of time lapse between the beginning of the sine wave and the point in question. Although this method is accurate, it is more convenient to let the horizontal axis represent degrees rather than time.

In the two pole alternator, one full cycle of voltage is generated as the loop makes one complete rotation of 360 mechanical degrees. This relationship holds true regardless of the output frequency of the generator. Since 360 mechanical degrees of rotation produce one cycle of voltage, it is convenient to divide the cycle into 360 ELECTRICAL DEGREES.

Notice in Figure 8-11 that the sine wave has been divided into 360 degrees. Since the positive and negative alternations have the same time duration each alternation contains 180 degrees. The positive alternation reaches its maximum value at 90 degrees and the negative alternation reaches its maximum value at 270 degrees.

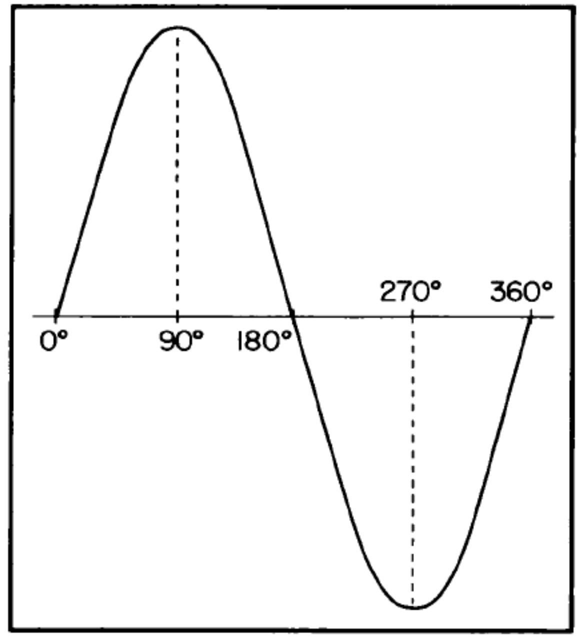

Figure 8-11 - Sine wave showing degree measure.

Q6. Given a 60 cps sine wave and a 100 cps sine wave, which one would contain the greatest number of degrees per cycle?

8-9. Radian Measure

Another system of measurement that is quite frequently applied to sine waves is RADIAN MEASURE or π MEASURE. An angle of one RADIAN is defined as "that angle subtended by a section of a circle equal in length to the radius of that circle." An angle of one radian is shown in Figure 8-12. In this drawing an arc (A-B) equal in length to the radius of the circle has been constructed on the circumference of the circle. The angle AOB that this arc forms with the center of the circle is equal to one radian. Since the circumference of a circle is equal to two pi times the radius, there must be two pi radians in a circle. Therefore:

$$2\,\pi \text{ radians} = 360^\circ \tag{8-6}$$

or:

$$6.28 \text{ radians} = 360^\circ \tag{8-7}$$

and:

$$\pi \text{ radians} = 180^\circ \tag{8-8}$$

or:

$$3.14 \text{ radians} = 180^\circ \tag{8-9}$$

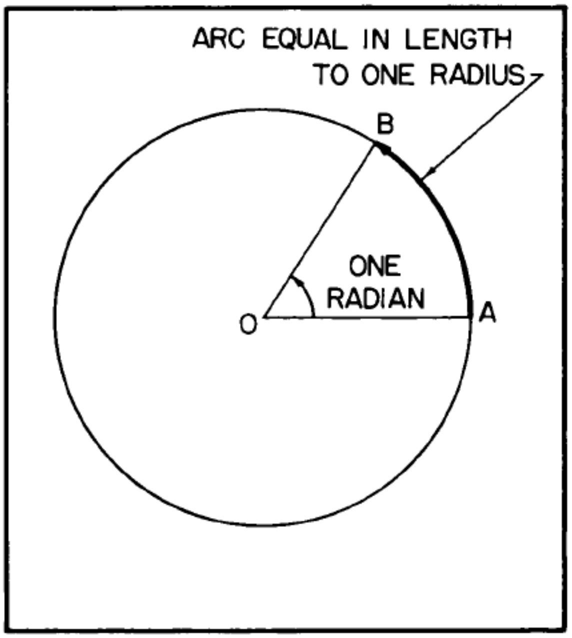

Figure 8-12 - Angle of one radian.

Since 3.14 radians are equal to 180° one radian is:

$$1 \text{ radian} = \frac{180^\circ}{3.14} \qquad (8\text{-}10)$$

$$1 \text{ radian} = 57.32 \text{ degrees} \qquad (8\text{-}11)$$

Quite frequently it is necessary to convert degrees to radians. To do this the following equations is used:

$$\text{radians} = \text{degrees} \times \frac{\pi}{180^\circ} \qquad (8\text{-}12)$$

Example. How many radians are contained in two cycles of a sine wave?

Solution: Since two cycles of a sine wave contain 720°:

$$\text{radians} = \text{degrees} \times \frac{\pi}{180^\circ} \qquad (8\text{-}12)$$

$$\text{radians} = \frac{720^\circ}{1} \times \frac{\pi}{180^\circ}$$

$$\text{radians} = 4\pi$$

$$\text{radians} = 12.56$$

Thus, two cycles of a sine wave contain 12.56 radians.

To convert radians to degrees the following equation is used:

$$\text{degrees} = \text{radians} \times \frac{180^\circ}{\pi} \qquad (8\text{-}13)$$

Example. How many degrees are contained in two-thirds of a radian?

Solution: $\text{degrees} = \text{radians} \times \frac{180^\circ}{\pi} \qquad (8\text{-}13)$

$$\text{degrees} = \frac{2}{3} \times \frac{180}{\pi}$$

$$\text{degrees} = \frac{120^\circ}{\pi}$$

$$\text{degrees} = 38.2$$

Thus, two-thirds of a radian is equal to 38.2 degrees.

Since thedifferent points on a sine wave can be expressed in degrees, it also follows that they can be expressed in radians. Thus, one alternation of a sine wave (180°) could also be expressed as π radians in duration. A complete cycle would be 2π radians, or 6.28 radians in length.

Q7. How many radians are there between the positive and negative maximum points of a single cycle?

8-10. Angular Velocity

The VELOCITY of an object moving uniformly in a straight line is defined as "the distance traveled per unit of time." Thus, velocity is measured in feet per second, miles per hour, etc.

In many electronics problems, the velocity of an object having circular motion must be determined. Circular velocity such as that of the armature of an alternator is called ANGULAR VELOCITY and is symbolized by the lower case Greek letter OMEGA (ω). Angular velocity is measured by determining the size of the angle the moving object generates per unit of time. Thus, angular velocity is measured in degrees per second, or radians per second, the latter being most common.

To analyze many electronic circuits the angular velocity of a sine wave must be known.

A4. 500 kc.

A5. 500 us.

A6. Neither. Each contains 360° per cycle.

A7. π radians.

One cycle of a sine wave contains 360 electrical degrees or 2π radians. The angular velocity of a sine wave is therefore, "the number of radians the sine wave completes per second."

A sine wave having a frequency of one cycle per second would complete an angle of 2π radians, or 6.28 radians per second. A sine wave having a frequency of 10 cycles per second would complete 10 times 2π radians per second and would have an angular velocity of 20π radians per second (62.82 radians per second). The angular velocity is seen to be dependent upon the number of radians in a cycle (2π) and the number of cycles per second (frequency) of the sine wave. In the form of an equation, the angular velocity of a sine wave is:

$$\omega = 2\pi f \tag{8-14}$$

where: ω = angular velocity in radians per second

f = frequency in cycles per second

2π = number of radians in one cycle (6.28)

Notice that the higher the frequency becomes the greater will be the angular velocity.

Example. What is the angular velocity of a sine wave having a frequency of 2 kc?

Solution:

$$\omega = 2\pi f \tag{8-14}$$

$$\omega = 6.28 \times 2 \times 10^3$$

$$\omega = 12{,}560 \text{ radians per second}$$

In certain circumstances it is necessary to obtain the total angle a sine wave completes in a given amount of time. This total angle (θ) is found by multiplying the angular velocity in radians per second by the time in seconds or:

$$\theta = 2\pi f t \tag{8-15}$$

or:

$$\theta = \omega t \tag{8-16}$$

where: θ = total angle in radians

ω = angular velocity in radians per second

t = time in seconds

Example. What is the angle in radians completed by a 100 cycle sine wave in 2.5 seconds?

Given: f = 100 cyclers per second

t = 2.5 seconds

Solution:

$$\theta = \omega t$$

$$\theta = 2\pi \times 100 \times 2.5$$

$$\theta = 6.28 \times 2.5 \times 10^2$$

$$\theta = 1570 \text{ radians}$$

MEASURING SINE WAVE AMPLITUDE

8-11. Peak Amplitude

One of the most frequently measured characteristics of a sine wave is its amplitude. Unlike dc measurement, the amount of alternating current or voltage present in a circuit can be measured in various ways. In one method of measurement, the maximum amplitude of either the positive or the negative alternation is measured. The value of current or voltage obtained is called the PEAK VOLTAGE or the PEAK CURRENT. To measure the peak value of current or voltage, an oscilloscope or a special meter (peak reading meter) must be used. The peak value of a sine wave is illustrated in Figure 8-13.

8-12. Peak-to-Peak Amplitude

A second method of indicating the amplitude of a sine wave consists of determining the total voltage or current between the positive and negative peaks. This value of current or voltage is called the PEAK-TO-PEAK VALUE (see Figure 8-13). Since both alternations of a pure sine wave are identical, the peak-to-peak value is twice the peak value. Peak-to-peak voltage is usually measured with an oscilloscope,

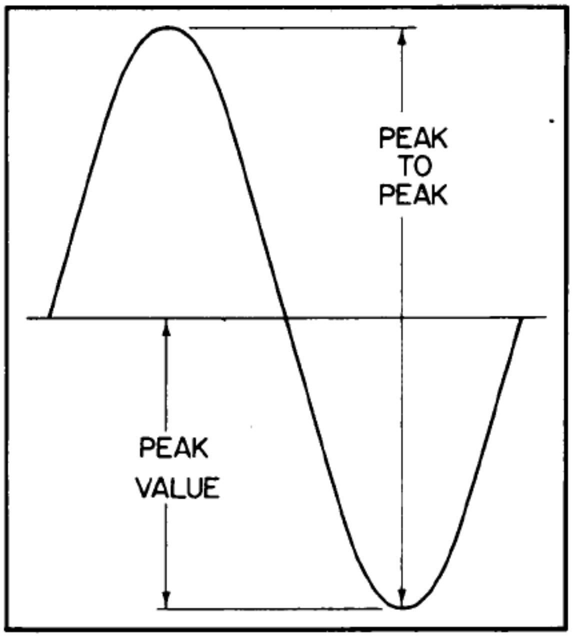

Figure 8-13 - Peak and Peak-to-Peak values.

although some voltmeters have a special scale calibrated in peak-to-peak volts.

8-13. Instantaneous Amplitude

The equation for the instantaneous value of a sine wave of voltage was given in Section 8-6 as:

$$e = E_m \times \sin \theta \qquad (8\text{-}2)$$

where: e = the instantaneous voltage

E_m = the maximum or peak voltage

$\sin \theta$ = the sine of the angle at which e is desired

Similarly the equation for the instantaneous value of a sine wave of current would be:

$$i = I_m \times \sin \theta \qquad (8\text{-}17)$$

where: i = the instantaneous current

I_m = the maximum or peak current

$\sin \theta$ = the sine of the angle at which i is desired

Example. What is the instantaneous voltage at point (P), 30° after the start of the sine wave in Figure 8-14?

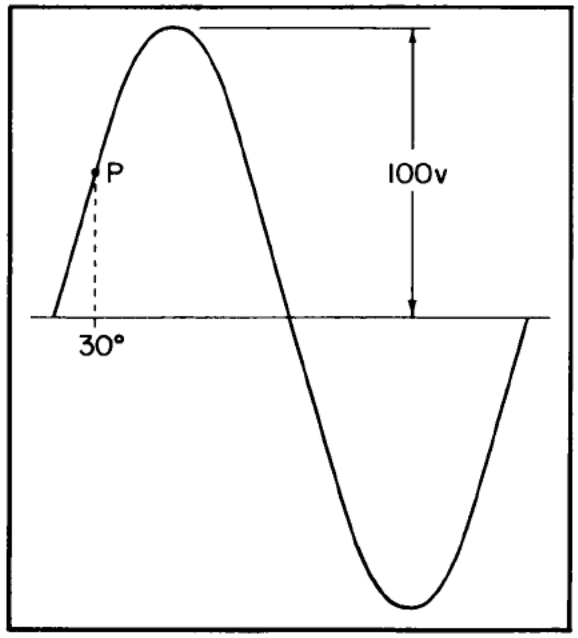

Figure 8-14 - Instantaneous value.

Given: $E_m = 100$ volts

$\theta = 30^\circ$

Find: $e = ?$

Solution:

$e = E_m \times \sin \theta$

$e = 100 \times \sin 30^\circ$

$e = 100 \times 0.5$

$e = 0.5 \times 10^2$

$e = 50$ volts

Q8. If the instantaneous current (i) at 30° is 5 ma, what is the peak current?

8-14. Effective or RMS Value

As the use of alternating current gained popularity, it became increasingly apparent that some common basis was needed on which ac and dc could be compared. A 100 watt light bulb for example, should work just as well on 120 volts ac as it does on 120 volts dc. It can be seen, however, that a sine wave of voltage having a peak value of 120 volts would not supply the lamp with as much power as a steady value of 120 volts dc.

Since the power dissipated by the lamp is a

A8. $I_m = \dfrac{i}{\sin \theta} = 10$ ma.

---

result of current flow through the lamp, the problem resolves to one of finding a MEAN alternating current ampere which is equivalent to a steady ampere of direct current.

Figure 8-15 shows a circuit in which the peak alternating current through the 10 ohm resistor is 1.414 amperes. Since the current through the resistor is changing continously the power dissipated by the resistor will also vary. It will be maximum when the current is maximum and zero when the current is zero.

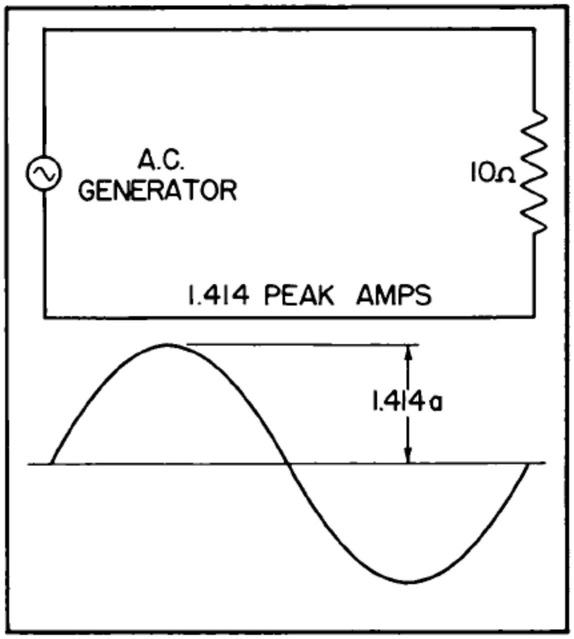

Figure 8-15 - Basic ac circuit.

The variations in power throughout the cycle can best be analyzed by plotting a curve showing the instantaneous power at each point in the cycle. In the procedure to follow, the instantaneous current, the square of the instantaneous current, and the instantaneous power will be calculated in 10 degree steps for the first quarter of the cycle. These values are shown in Table 1.

Notice that at 0° the instantaneous current (i) is zero causing the power dissipated by the resistor to be zero. At 10° the instantaneous current is 0.245 amperes, the current squared is 0.060 and the power is 0.60 watt. At 90° the current has reached its maximum value of 1.414 amperes, the square of the current is 2.000 and the power dissipated is 20.00 watts.

During the part of the sine wave of current from 90° to 180° the same values could be used as before but in a reverse order. Thus, at 100° the values of current and power would be identical to those at 80°.

Degrees	i	i^2	P
0°	.000	.000	.00
10°	.245	.060	.60
20°	.486	.236	2.36
30°	.707	.500	5.00
40°	.909	.826	8.26
50°	1.083	1.173	11.73
60°	1.225	1.500	15.00
70°	1.329	1.766	17.66
80°	1.393	1.940	19.40
90°	1.414	2.000	20.00

TABLE 1

Using the values of i and P from Table 1, a graph can be constructed showing the way in which power varies throughout the cycle. This graph is shown in Figure 8-16.

In this graph a sine wave of current is plotted first, using the instantaneous values from Table 1. Next the curve representing i^2 and power is constructed.

Notice that the power curve has twice the frequency of the current curve, and that ALL POWER IS POSITIVE. This is due to the fact that heat is dissipated regardless of which way

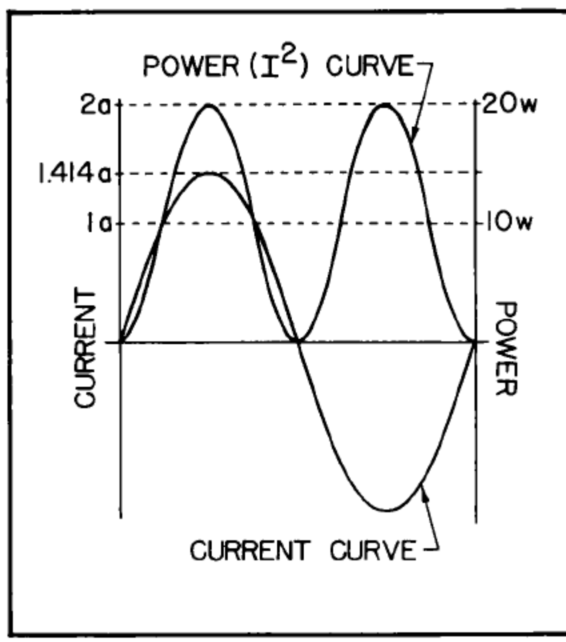

Figure 8-16 - Current and power curves

the current flows through the resistor.

Since all the alternations of the power curve are identical, the MEAN or AVERAGE POWER is the value HALF-WAY between the maximum and minimum values of power. Thus, the average power dissipated by the 10 ohm resistor is 10 watts, one-half the peak power. Since the curve representing power also represents current squared (i^2), the average or mean of the curve also lies half-way between the maximum and minimum values of i^2. As power is proportional to i^2, a dc current having a value equal to the square root of the mean of the i^2 values would produce the same average power as the original sine wave of current. This mean current is called the ROOT MEAN SQUARE (RMS) current. One RMS ampere of alternating current is as effective in producing heat as one steady ampere of direct current. For this reason an RMS ampere is also called an EFFECTIVE ampere. In Figure 8-16 the peak current of 1.414 amperes produces the same amount of average power as one ampere of effective (RMS) current.

ANYTIME AN ALTERNATING VOLTAGE OR CURRENT IS STATED WITHOUT ANY QUALIFICATIONS, IT IS ASSUMED TO BE AN EFFECTIVE VALUE. Since effective values of ac are the ones generally used, most meters are calibrated to indicate effective values of voltage and current.

In many instances it is necessary to convert from effective to peak or vice-versa. Figure 8-16 shows that the peak value of a sine wave is 1.414 times the effective value and therefore:

$$E_m = E \times 1.414 \qquad (8\text{-}18)$$

where: E_m = maximum or peak voltage

E = effective or RMS voltage

and:

$$I_m = I \times 1.414 \qquad (8\text{-}19)$$

where: I_m = maximum or peak current

I = effective or RMS current

Example. What is the peak voltage between the terminals of an ac outlet, if the voltage is 120 volts ac?

Given: E = 120 volts ac

Find: E_m = ?

Solution: $E_m = E \times 1.414$ (8-18)

$$E_m = 120 \times 1.414$$

$$E_m = 169.7 \text{ volts}$$

Upon occasion it is necessary to convert a peak value of current or voltage to an effective value. The conversion factor may be derived as follows:

$$E_m = E \times 1.414 \qquad (8\text{-}18)$$

Multiplying both sides of (8-18) by 1/1.414

$$E_m \times \frac{1}{1.414} = E \times 1.414 \times \frac{1}{1.414}$$

$$E_m \times \frac{1}{1.414} = E$$

Dividing 1 by 1.414:

$$E = E_m \times 0.707 \qquad (8\text{-}20)$$

where: E = the effective voltage

E_m = the maximum or peak voltage

Similarly for current:

$$I = I_m \times 0.707 \qquad (8\text{-}21)$$

where: I = the effective current

I_m = the maximum or peak current

Example. What is the effective voltage across a 20 ohm resistor, if the peak current through it is 7 amperes?

Solution. Using Ohm's Law the peak voltage across the resistor is:

$$E_m = I_m R$$

$$E_m = 7 \times 20$$

$$E_m = 140 \text{ volts peak}$$

Since the peak voltage is 140 volts, the effective voltage is:

$$E = E_m \times 0.707 \quad (8\text{-}20)$$

$$E = 140 \times 0.707$$

$$E = 98.98 \text{ volts effective}$$

Q9. Derive a formula that could be used to convert effective values to peak-to-peak values.

8-15. Average Value

The average value of a complete cycle of a sine wave is zero, since the positive alternation is idential to the negative alternation. In certain types of circuits however, it is necessary to compute the average value of one alternation. This could be accomplished by adding together a series of instantaneous values of the wave between 0 degrees and 180 degrees, and then dividing the sum by the number of instantaneous values used. Such a computation would show one alternation of a sine wave to have an average value equal to 0.637 of the peak value. In terms of an equation:

$$E_{avg} = E_m \times 0.637 \quad (8\text{-}22)$$

where: E_{avg} = the average voltage of one alternation

E_m = the maximum or peak voltage

similarly: $$I_{avg} = I_m \times 0.637 \quad (8\text{-}23)$$

where: I_{avg} = the average current in one alternation

I_m = the maximum or peak current

Example. What is the average value of a sine wave of current if the peak value of the wave is 3 amps?

Solution: $$I_{avg} = I_m \times 0.637 \quad (8\text{-}23)$$

$$I_{avg} = 3 \times 0.637$$

$$I_{avg} = 1.911 \text{ amps}$$

Figure 8-17 shows a comparison between the various values that are used to indicate the amplitude of a sine wave.

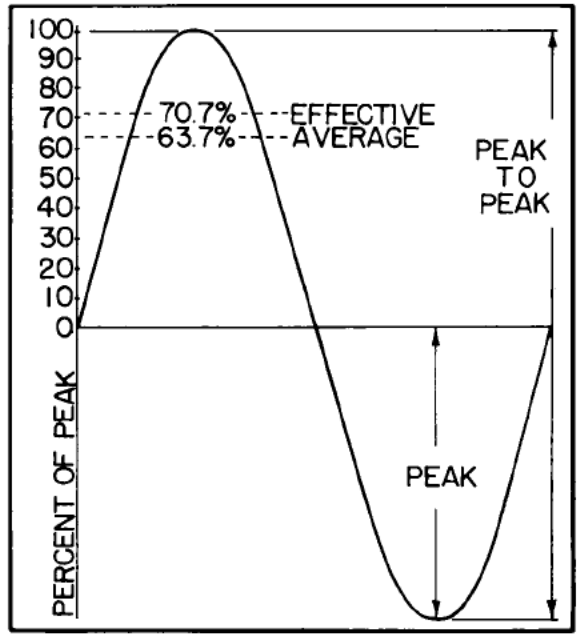

Figure 8-17 - Various values used to indicate sine wave amplitude.

PHASE MEASUREMENT

8-16. Sine Waves in Phase

If a sine wave of voltage is applied to a resistance, the resulting current will also be a sine wave. This follows Ohm's law which states that the current is directly proportional to the applied voltage. Figure 8-18 shows a sine wave of voltage and the resulting sine wave of current superimposed on the same time axis. Notice that as the voltage increases in a positive direction the current increases along with it. When the voltage reverses direction, the current reverses direction. At all times the voltage and current pass through the same relative parts of their respective cycles at the same time. When two waves, such as those in Figure 8-18, are precisely in step with one another they are said to be IN PHASE. To be in phase, the two waves must go through their maximum and minimum points at the same time and in the same direction.

In some circuits, several sine waves can be in phase with each other. Thus, it is possible to have two or more voltage drops in phase with each other and also in phase with the circuit current.

8-17. Sine Waves Out of Phase

Figure 8-19 shows a voltage wave E_1 con-

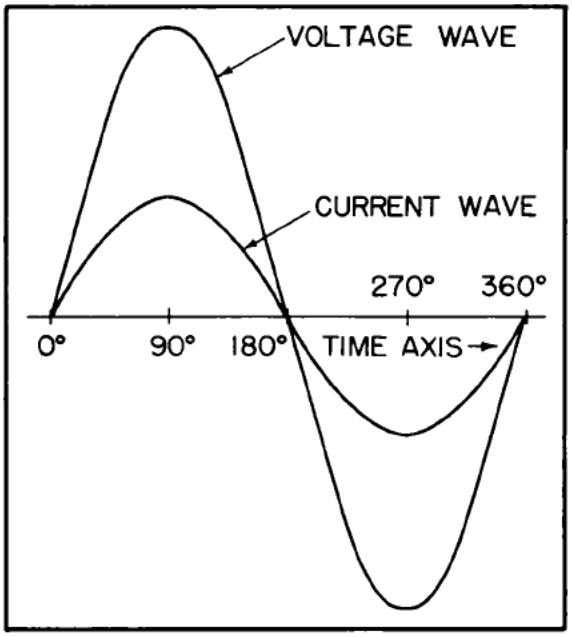

Figure 8-18 - Voltage and current waves in phase.

sidered to start at 0 degrees (time zero). As voltage wave E_1 reaches its positive peak, a second voltage wave E_2 starts its rise (time 1). Since these waves do not go through their maximum and minimum points at the same instant of time, a PHASE DIFFERENCE exists between

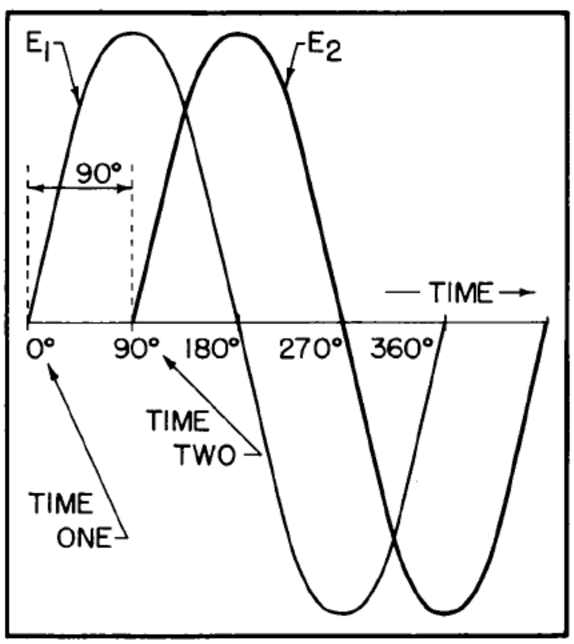

Figure 8-19 - Voltage waves 90° out of phase.

the two waves. The two waves are said to be out of phase. For the two waves in Figure 8-19 this phase difference is 90 degrees.

To further describe the phase relationship between two waves the terms LEAD and LAG are used. The amount by which one wave leads or lags another is measured in degrees. Referring again to Figure 8-19, wave E_2 is seen to start 90 degrees later in time than wave E_1, thus wave E_2 lags wave E_1 by 90 degrees. This relationship could also be described by stating that wave E_1 leads wave E_2 by 90 degrees.

It is possible for one wave to lead or lag another by any number of degrees, except 0 or 360 degrees, in which condition the two waves are in phase. Thus, two waves may differ in phase by 45 degrees, but two waves differing by 360 degrees would be considered as in phase.

A phase relationship that is quite common is the one shown in Figure 8-20. The two waves illustrated have a phase difference of 180 degrees.

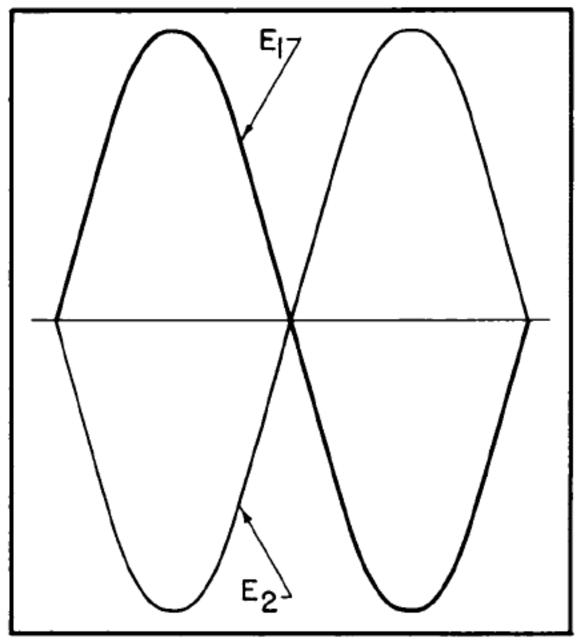

Figure 8-20 - Two waves 180° out of phase.

Notice that although the waves pass through their maximum and minimum values at the same time, their instantaneous voltages are always of opposite polarity. If two such waves existed across the same component they would have a cancelling effect on each other. If the two waves are equal in amplitude the resultant wave would be zero. However, if they have different amplitudes the resultant wave would have the polarity of the larger and be the difference of the two.

To determine the phase difference between two sine waves, locate the points on the time

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A9. Since the maximum value is 1.414 times the effective value and peak-to-peak is 2 times the maximum value, the peak-to-peak value is 2.828 times the effective value. P-P = effective x 1.414 x 2
P-P = effective x 2.828

axis where the two waves cross the time axis traveling in the same direction. The number of degrees between the crossing points is the phase difference. The wave that crosses the axis at the later time (to the right on the time axis) lags the other.

Q10. Does a phase difference always indicate a time difference?

VECTORS

In many theoretical descriptions, drawings are used to give a simple pictorial explanation of a complex process. In the preceding section, drawings of sine waves were used as an aid to the development of the concept of phase. However, should a circuit contain three or four sine waves of different phases, the diagram would become a maze of lines too confusing to be useful. Fortunately, voltage, current, force, and many other mathematical quantities can be represented by a simple graphic symbol called a VECTOR.

8-18. Nature of a Vector

To suitably represent quantities such as voltage, current, and force, a vector must indicate both the amount of the quantity and its direction. A vector is therefore composed of a line, the length of which denotes the amount of the quantity, and an arrow head which shows the direction of the quantity. Figure 8-21 shows a vector used to represent the attracting force of gravity. The arrow head shows that the force is directed toward the earth and the length of the vector shows that the force magnitude is 50 pounds. The vector thus indicates the DIRECTION and MAGNITUDE of the force.

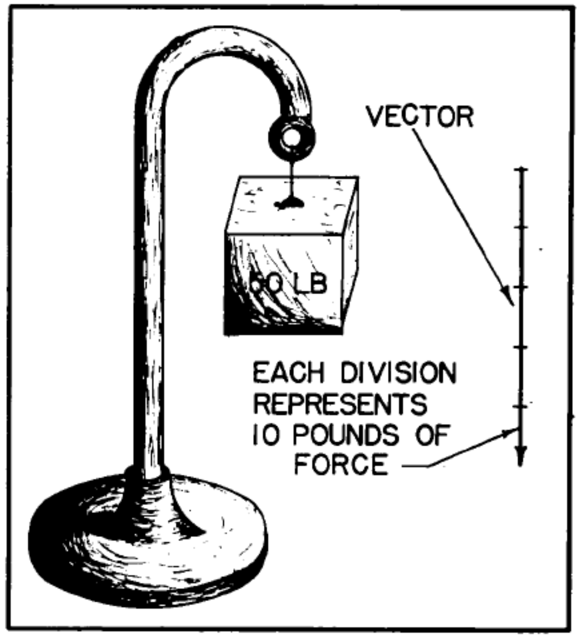

Figure 8-21 - Vector representing force of gravity.

8-19. Electrical Quantities as Vectors

Although a voltage does not have a true direction in terms of three dimensional space, it does have a phase which in a sense can be considered as direction. A vector can thus be used to represent electrical quantities such as voltage and current.

Figure 8-22 shows how a vector can be used to represent a sine wave of voltage. In part A of the diagram the sine wave is at zero degrees, just beginning the positive alternation. To illustrate this condition, a vector is constructed whose length corresponds to the magnitude of the voltage under consideration. The vector

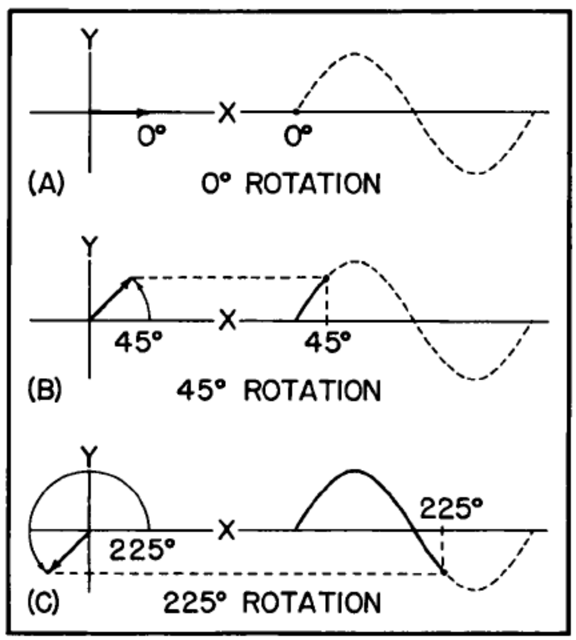

Figure 8-22 - Sine wave represented by rotating vector.

can be constructed to represent any of the voltage values previously mentioned such as peak, effective, etc. To show that the sine wave is at zero degrees of phase, the vector is placed on the X-axis pointing to the right. When in this position the vector is said to be in STANDARD POSITION.

In part B of Figure 8-22 the sine wave has completed 45 degrees of the cycle. During this period of time the vector has also completed an angle of 45 degrees. This is shown by advancing the vector through an angle of 45 degrees in a COUNTER-CLOCKWISE direction. (In mathematics counter-clockwise rotation is used to indicate positive angles). Vectors representing electrical quantities will always be assumed to rotate counter-clockwise unless otherwise specified.

In part C of Figure 8-22 the sine wave has completed 225 degrees of rotation. The vector has likewise rotated through an angle of 225 degrees. After 360 degrees the sine wave completes one full cycle and the vector will be back to its original position having completed 360 degrees of rotation.

Q11. How many revolutions per second would a vector complete if it represented a 60 cps sine wave?

Q12. Can vectors representing different frequencies be plotted on the same diagram?

8-20. Vectors and Phase Difference

Since a single vector can be used to represent a sine wave, two vectors can be used to represent two sine waves. Thus, a vector diagram is a convenient method of showing the phase difference between two sine waves.

To illustrate this application of vectors, two sine waves E_1 and E_2 are shown in Figure 8-23. Sine wave E_2 is phased so as to lag sine wave E_1 by 90 degrees.

In constructing the vector diagram, one of the sine waves is chosen as a reference to which the second wave may be compared. In Figure 8-23 wave E_1 has been chosen as the reference and a vector representing this wave is constructed in standard position on the X-axis.

Once the vector representing wave E_1 has been placed on the diagram, a second vector can be drawn to represent E_2. This vector must be placed on the diagram so as to show E_2 lagging 90 degrees behind E_1. Since the vectors rotate in a counter-clockwise direction, vector E_2 is drawn pointing downward along the Y-axis. The complete vector diagram appears as though a high speed photograph were taken just as reference vector E_1 arrived at the X-axis. Vector E_2 then appears to be trailing along 90 degrees behind E_1.

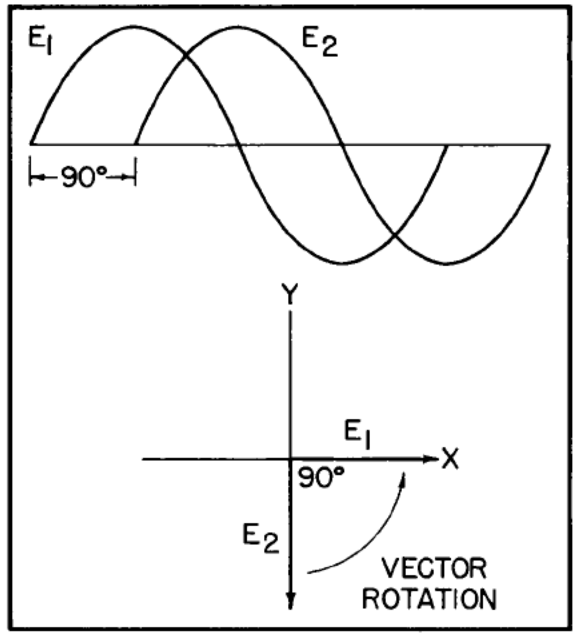

Figure 8-23 - Vectors showing phase relationships.

Q13. How would the vector diagram appear if wave E_2 were used as the reference wave?

8-21. Addition of Vectors

Quite frequently it is necessary to add out of phase voltages or currents in the process of solving ac circuit problems. The vector lends itself well to such solutions and is a valuable, time saving tool.

In Figure 8-24 two vectors E_a and E_b are to be added together. These vectors are shown in Figure 8-24A as they would normally appear on a vector diagram.

To obtain the sum of these two vectors, the tail of the second vector E_b is placed at the head of the first vector E_a, as shown in Figure 8-24B. This must be done without changing the direction or magnitude of either vector. A third vector is then drawn from the tail of the first vector E_a to the arrow head of the second vector E_b. This third vector is called the RESULTANT and is the SUM of the two original vectors. Using this procedure any number of vectors can be added. Each vector is placed in turn with its tail at the arrowhead of the previous vector. Again the sum is found by drawing a vector from the tail of the first vector to the head of the last vector. Figure 8-25 shows the addition of three vectors of various directions and magnitudes.

A10. Yes.

A11. 60 rps.

A12. Yes. The vector representing the higher frequency would rotate at a faster rate.

A13. Vector E_2 would be drawn to the right along the X-axis and vector E_1 would point vertically (upward) along the Y-axis.

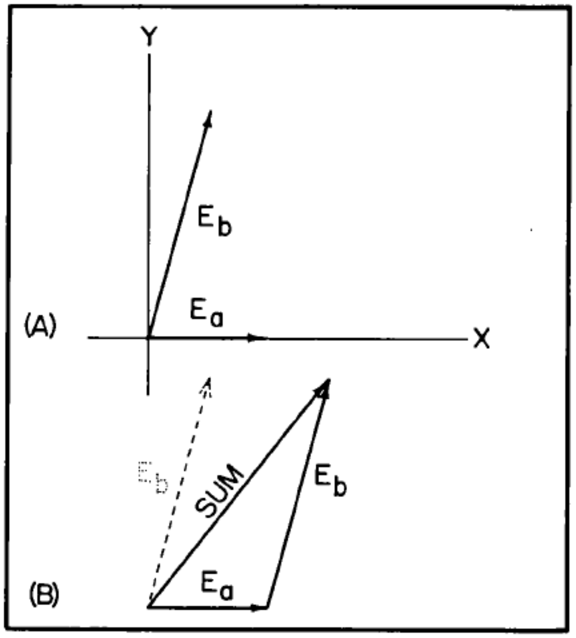

Figure 8-24 - Obtaining a vector sum.

A simplified procedure for adding two vectors consists of constructing a parallelogram in which the vectors form two adjacent sides. In Figure 8-26A the resultant is to be obtained for the two vectors shown. A parallelogram is developed by constructing line CD parallel to vector OE, and a line ED parallel to vector OC (Figure 8-26B). A vector is now drawn from O to D. This third vector is then the sum of the two original vectors.

If this method is compared to that used in Figure 8-24, the end result of the two methods is seen to be identical. In using the parallelogram method, however, the vectors must be added in pairs. Three vectors could not be added simultaneously. In adding more than two vectors by the parallelogram method, two of the vectors are added and then their resultant is added to a third vector. This process is continued until all the vectors have been added and a final resultant obtained.

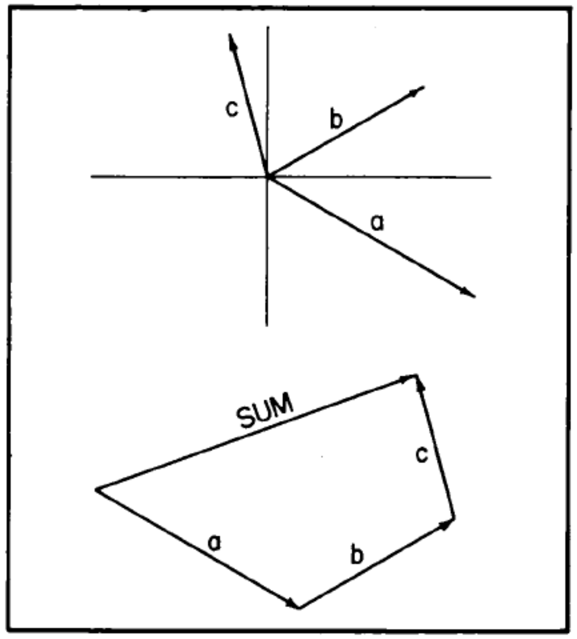

Figure 8-25 - Addition of three vectors.

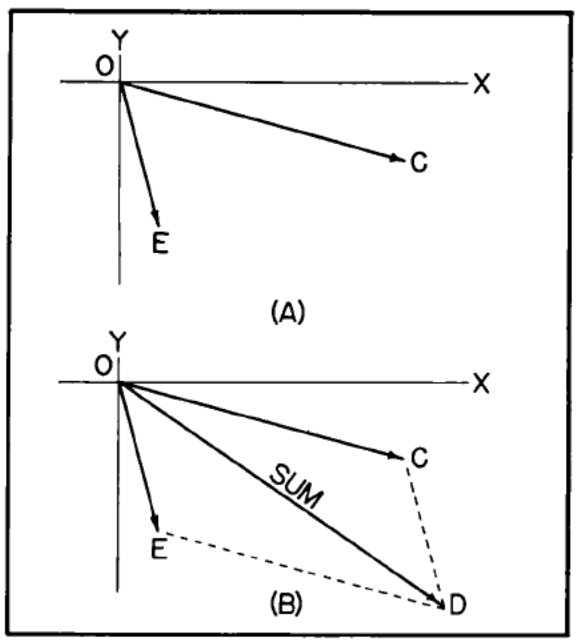

Figure 8-26 - Addition by parallelogram method.

RATE OF CHANGE

In many instances the behavior of a circuit depends more on the way in which the voltage or

current changes with respect to time, than it does on the amount of the voltage or current. If the voltage or current has the form of a sine wave the rate of change will vary continuously throughout the cycle.

8-22. Sine Wave Rate of Change

Figure 8-27 shows a sine wave of voltage on which two 30 degree segments of time have been chosen for comparison. The first of these segments is from 0 degrees to 30 degrees, and the second extends from 60 degrees to 90 degrees.

Notice that during the first 30 degrees of the cycle the sine wave rises from 0 volts to 50 volts, a change of 50 volts. During the second segment the voltage changes from 86.6 volts at 60 degrees to 100 volts at 90 degrees, a change of only 13.4 volts. Thus, the rate of change of voltage is much greater near 0 degrees than it is near 90 degrees. The concept of sine wave rate of change will be utilized in the following chapter to explain the characteristics of an inductor.

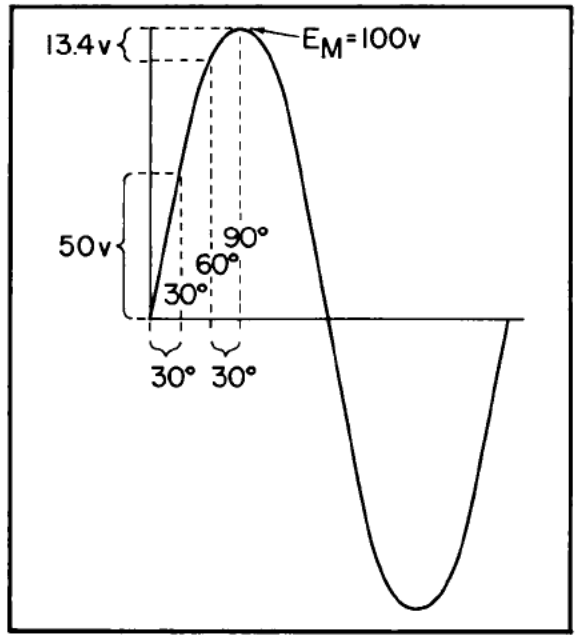

Figure 8-27 - Rate of change of a sine wave.

APPLICATIONS FOR SINE WAVES

The alternating current used commercially for light and power is a prime example of the use of ac. Electronic equipment makes extensive use of ac in the form of signal voltages. From the tiny ac signal generated by a microphone to the thousands of watts radiated into space by radio transmitters, ac plays a role of primary importance.

EXERCISE 8

1. Define alteranting current.
2. What is meant by the term waveform?
3. List four factors that determine the amount of voltage induced into a conductor.
4. What is a cycle of voltage?
5. What is an alternation?
6. What is meant by the term frequency?
7. What is meant by the phrase "period of a sine wave?"
8. What is the frequency of a sine wave if the positive alternation lasts for 50 microseconds?
9. Convert 720 degrees to radians.
10. A sine wave has an angular velocity of 3140 radians per second. What is its frequency?
11. What percentage of its maximum amplitude has a sine wave attained, when it has completed the first 30 degrees of a cycle?
12. Find the instantaneous amplitude of a 1 kc sine wave 200 microseconds after the start of the wave. Assume the sine wave to have a peak value of 100 volts.
13. What is the effective value of current through a circuit if the peak current is 20 amperes?
14. What is the peak voltage across a resistor if an ac voltmeter connected across the resistor indicates 220 volts?
15. How does the mathematical average value of a sine wave of voltage compare to the electrical average value?
16. How may the effective value of a sine wave be converted to the RMS value?
17. Draw two sine waves of equal amplitude displaced 45° in phase. Label the leading wave E_1 and the trailing wave E_2.
18. Draw two sine waves displaced by an angle of pi radians.
19. What two things does a vector represent?
20. In which direction are vectors assumed to rotate?
21. Two vectors A and B are placed in standard position on a vector diagram. If vector A represents a 100 cps sine wave and vector B represents a 200 cps sine wave, where will vector B appear when vector A has rotated 90 degrees?
22. At what point in the cycle is the rate of change of a sine wave maximum?
23. At what point in the cycle is the rate of change of a sine wave minimum?

CHAPTER 9

INDUCTANCE

The study of inductance presents a very challenging but rewarding segment of your electronic career. It is challenging in the sense that, at first, it will seem that new concepts are being introduced. The student will realize as this chapter progresses that these "new concepts" are merely extensions and enlargements of fundamental principles that have been acquired previously in the study of magnetism and electron physics. The study of inductance is rewarding in the sense that a thorough understanding of it will enable the student to acquire a working knowledge of electronic circuits more rapidly and with more surety of purpose than would otherwise be possible.

Inductance is the characteristic of an electrical circuit that makes itself evident by opposing the starting, stopping, or changing of current flow. The above statement is of such importance to the study of inductance that it bears repeating in a simplified form. Inductance is the characteristic of an electrical conductor which opposes a CHANGE in current flow.

One does not have to look far to find a physical anology of inductance. Anyone who has ever had to push a heavy load (wheelbarrow, car, etc.) is aware that it takes more work to start the load moving than it does to keep it moving. This is because the load possesses the property of inertia. Inertia is the characteristic of mass which opposes a CHANGE in velocity. Therefore, inertia can hinder us in some ways and help us in others. Inductance exhibits the same effect on current in an electric circuit as inertia does on velocity of a mechanical object. The effects of inductance are sometimes desirable—sometimes undesirable.

The study and comprehension of the action of current and voltage on inductive components require that the student be familiar with the use of logarithms and exponents. These subjects are discussed in detail in Volume 8.

HISTORY OF INDUCTANCE

9-1. History

On September 22, 1791 in Newington Butts, London, a man was born who was destined to play a great part in the laying of the foundation of the growing science of electricity. This man, Michael Faraday, started to experiment with electricity around the year 1805 while working as an apprentice bookbinder. It was in 1831 that Faraday performed experiments on magnetically coupled coils. A voltage was induced in one of the coils due to a magnetic field created by current flow in the other coil. From this experiment came the induction coil, the theory of which eventually made possible many of our modern conveniences such as the automobile, doorbell, auto radio, etc. In performing this experiment Faraday also invented the first transformer, but since alternating current had not yet been discovered the transformer had few practical applications. Two months later, based on these experiments, Faraday constructed the first direct current generator. At the same time Faraday was doing his work in England, Joseph Henry was working independently along the same lines in New York. The discovery of the property of self-induction of a coil was actually made by Henry a little in advance of Faraday and it is in honor of Joseph Henry that the unit of inductance is called the HENRY.

It was from the experiments performed by these, and many other men that the laws and theories of inductance grew.

ELECTROMAGNETISM

9-2. Relation of Current and Magnetic Field in a Conductor

Scientists and physicists have performed many and varied experiments involving current and magnetism. You are already familiar with one of the basic discoveries arising from these experiments, namely that an electric current is always associated with a magnetic field and vice versa. Prior to 1820, it was thought by many that a magnetic field was unrelated to current flow. However, Hans Oersted believed that there was a definite relationship between current and magnetism. He spent many hours seeking this relationship. Unfortunately, in trying to disprove this popular theory, Oersted and others were laboring under the misconception that the associated electromagnetic field would be exerted in the same direction as

the current flow in a conductor. In trying to prove his idea he positioned a magnetic needle so that it lay at RIGHT ANGLES to the wire. Oersted assumed the magnetic force would cause the needle to SWING PARALLEL to the wire. He was puzzled, when upon forcing a current through the wire, no deflection of the needle was observed. This fact was taken by many as proof that no relationship between magnetism and current existed.

Oersted's revolutionary discovery, that the magnetic field exists at right angles to the current flow, came about one day while he was teaching a group of students. He inadvertently placed the magnetic needle PARALLEL to the wire and noticed a deflection as current was started in the wire. Further investigation after class confirmed his findings and led him to formulate his theory of electromagnetism.

9-3. Left Hand Rule for a Conductor

A simple rule has been established which allows us to determine the direction of magnetic flux around a conductor if we know the direction of current flow through it.

LEFT HAND RULE FOR CONDUCTORS: If the left hand is placed so that the thumb points in the direction of ELECTRON flow, the curled fingers will point in the direction of the flux lines encircling the conductor. (Chapter 4 Section 16)

The left hand rule for conductors and the manner in which the intensity of the flux lines diminish with distance from the conductor, are illustrated in Figure 9-1.

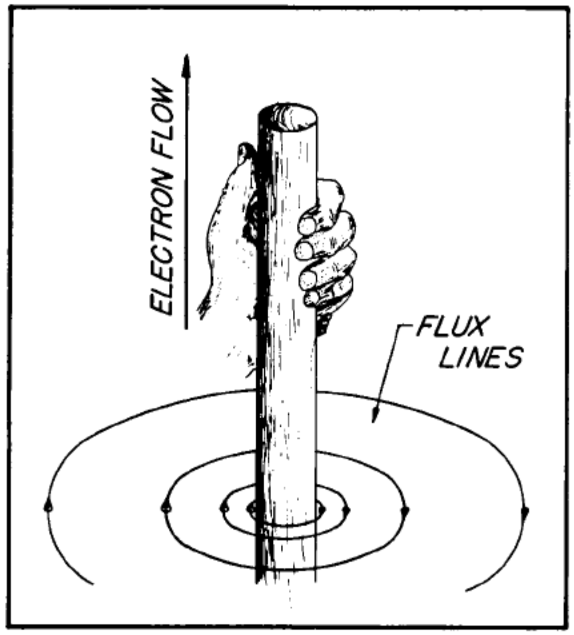

Figure 9-1 - Left Hand Rule for Conductors.

If the electron flow in Figure 9-1 is reversed and the left hand rule again applied, the thumb will point downward, indicating that the direction of the lines of flux is reversed. The left hand rule may also be used to advantage to determine the direction of electron flow if the direction of the flux lines is known. Point the curled fingers in the same direction as the arrow heads of the circular flux lines and the thumb will point in the direction of electron flow.

Q1. What would the reversal of the direction of circular flux lines around a conductor indicate concerning the electron flow?

9-4. Field Around A Loop Carrying Current

The magnetic field about a single conductor is very weak. Furthermore, as the distance from the conductor increases the intensity of the field rapidly decreases.

If the straight conductor in Figure 9-1 were formed into a loop such as in Figure 9-2, and the left hand rule for conductors is applied at various points around the loop, it will be noted that all the circular flux lines point in the same direction through the center of the loop. Looping the wire causes a concentration of flux lines in the enclosed area, thereby allowing the weak magnetic field of the single wire to produce a comparatively intensified field. Since it has been established that magnetic lines of force enter at the south pole of a magnet and leave at the north pole, magnetic poles may be assigned to the field associated with a current carrying loop. Figure 9-2 illustrates the concentration of flux lines in the area enclosed by the loop and also the position of the magnetic poles established by the lines of flux.

NOTE: All electromagnetic lines of force are closed loops identical to the lines of flux produced by a permanent magnet. Some of the circular flux lines of Figure 9-2 are shown incomplete merely for clarity. All lines leaving the north pole must circle around and join with the same line entering at the south pole.

The single loop of wire carrying current in Figure 9-2 possesses the properties of a very weak magnet. In fact, if the loop is mounted in such a manner that it is free to rotate, it would align itself with the earth's magnetic field. As with the magnetic field of the single wire, the magnetic field of a single loop is too weak to be of much practical use.

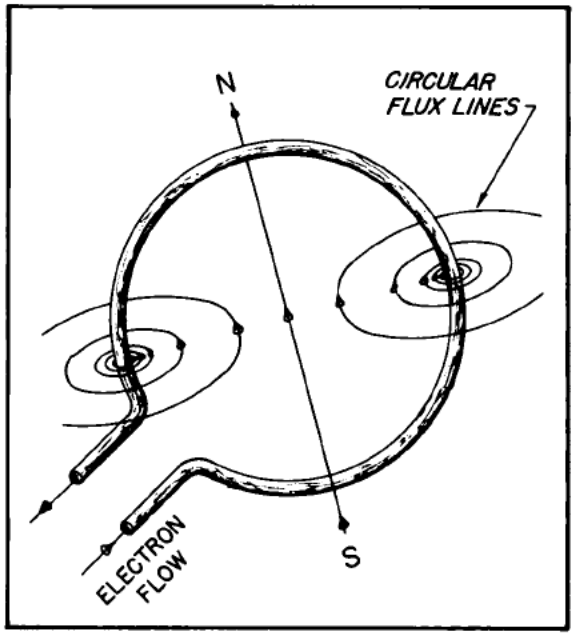

Figure 9-2 - Current carrying loop.

Q2. What is the difference in the effect of electromagnetic lines of force compared to magnetic lines of force?

9-5. Field Around a Coil

A coil is constructed of many individual current carrying loops placed close enough together so that the flux (Figure 9-3B) lines from many of the loops interact. This concentrates a large number of flux lines in the center of the coil. Figure 9-3A illustrates the interaction of flux lines in a coil by taking two segments of wire out of adjacent loops. Note first that the current is traveling in the same direction in both conductors. Secondly, when the coils are not tight against each other, the flux lines very close to the wire are still closed loops around their own individual wire. The significance of this will be shown at a later time.

The important fact to be noted is that the outer flux lines interact and extend themselves to encompass both conductors. As additional loops carrying current in the same direction are added, the magnetic lines of force will merely extend themselves to provide a single field around the resulting coil. This is the internal action in a coil which allows lines of force to maintain closed loops around the entire coil. Just as there is a rule to indicate the relationship between current and flux in the single conductor and loop, there is also a rule to indicate the relationship between the current and flux in a coil.

LEFT HAND RULE FOR COILS: If the four fingers are held (or wrapped) around the conductor of the coil so as to point in the direction of electron flow, then the thumb held at right angles to the fingers will indicate the direction of magnetic flux lines through the coil.

The north pole is considered to be the end of the coil from which the flux leaves. Flux enters the coil at the south pole. Using the left hand rule the direction of current flow can be found if the poles of the coil are known. If the left hand is placed around the coil with the thumb pointing in the direction of the north pole, the fingers will point in the direction of current flow.

Figure 9-3B illustrates the magnetic field distribution around the coil and the left hand rule for coils. Notice that the fingers point along the conductor in the direction of the electron flow and the thumb points in the direction of the flux lines through the center of the coil. Notice also that the flux lines enter the south pole and leave the north pole of the coil.

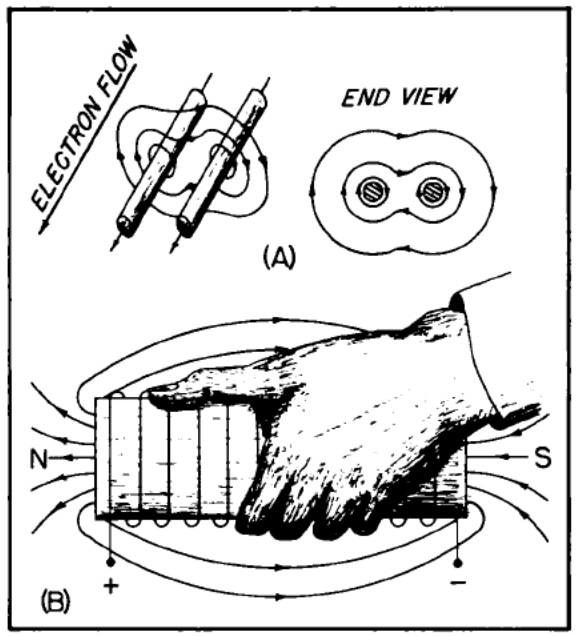

Figure 9-3 - Left Hand Rule for Coils

9-6. Factors Affecting Flux Density and Direction

Before entering into the factors affecting the flux density, it might be well to re-define the term. FLUX DENSITY is defined as the number of flux lines passing through a given area. In other words, given a specific cross-sectional area such as one square centimeter, the more lines of flux that can be crowded into this area the higher will be the flux density. The student

A1. This would indicate the electron flow had reversed direction.

A2. None. The two follow basically the same laws and the same effect would be produced in a conductor being cut by either line of force.

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desiring to refresh his memory is directed to review Chapter 3, Section 11.

CURRENT: It has been shown by the left hand rule for coils how reversing the direction of electron flow will reverse the direction of the flux lines and thereby the polarity of the coil. In addition to determining the direction of flux, the current also determines the amount of flux. All other factors remaining constant, INCREASING the current through the coil will INCREASE the flux density, and DECREASING the current will DECREASE the flux density. Flux density is also a direct function of the number of coil turns.

CORE MATERIAL: The core of a coil is defined as "that material enclosed by the turns of the coil, and through which the lines of flux pass." (In certain applications the core may be extended, to surround the outside of the coil as well.) Core materials may be either magnetic or non-magnetic. In order to better understand the effect of core materials on flux density we must first expand a bit on knowledge previously acquired in Chapter 3.

RELUCTANCE: Reluctance has been defined as "the opposition a material offers to the magnetic lines of force". A mathematical expression of reluctance is offered here merely to aid the student in picturing the factors involved in the reluctance of a material used as a core.

$$R = \frac{L}{\mu A} \tag{9-1}$$

Where: R = reluctance

L = length of core in cm.

A = area in sq/cm.

μ = permeability of core material

It can be seen from equation (9-1) that reluctance is a direct function of the length of the core and an inverse function of the cross-sectional area and permeability of the material used. For example, if a core with a LOW reluctance is desired, it should be physically large in cross-sectional area and made of material with a high permeability.

PERMEABILITY: Permeability has been defined as "the ease with which magnetic lines of force distribute themselves throughout the material." It should be noted that permeability is a term indicating the ease with which a material will conduct lines of force IN RELATION TO AIR AS THE REFERENCE MEDIUM. In other words, permeability is a ratio of the flux density in a given area of core material to the flux density in the same given area of air (with the same current in the coil). Air has been assigned a permeability of unity or one. Therefore, any core material having a permeability with a value greater than one will produce a greater flux density than an air core coil.

Summing up the effect of core materials on flux density, the flux density of a coil can be increased without changing the current or the number of coil turns, by inserting a core with a high permeability into the coil.

Q3. What is the main advantage of a coil compared to a single loop?

9-7. Electromagnetic Induction

The process of producing an electrical effect by magnetic means, or a magnetic effect by electrical means without physical contact is called ELECTROMAGNETIC INDUCTION.

It was stated previously that a current carrying coil possesses the properties of a magnet. The fact that this current carrying coil can produce a magnetic effect without physical contact is shown by the common door chime. Pushing a button will send a current through a coil. The current causes a magnetic field, which in turn attracts a metal plunger, causing the plunger to strike the chime. Therefore, a magnetic effect is produced in the plunger without physical contact.

This chapter will deal primarily with the production of an electrical effect. Michael Faraday discovered that he could produce an electrical effect by moving a conductor rapidly through a magnetic field, thus inducing a voltage in the conductor. Faraday then formulated a law which is used as a foundation for the study of electromagnetic induction.

FARADAY'S LAW: The EMF induced (or generated) in a conductor by electromagnetic action is proportional to the rate at which the conductor cuts magnetic lines of force.

9-8. Requirements for Induced EMF

There are three basic requirements to be

fulfilled to obtain an induced electromotive force. These are:

1. The existance of a conductor.
2. The existance of a magnetic field.
3. Relative motion between the conductor and field in such a manner as to have the conductor cut the lines of force.

Before proceeding with an analysis of these three basic requirements, two important fundamental statements must be well understood. These are: the left hand rule for generators and Lenz's law.

LEFT HAND RULE FOR GENERATORS: If the thumb, forefinger, and the middle finger of the left hand are held at right angles to each other with the thumb pointing in the direction of movement of the conductor and the forefinger pointing in the direction of the magnetic flux (from north to south), then the middle finger will point in the direction of the induced EMF (electron displacement).

LENZ'S LAW: The polarity of an induced EMF is such that it tends to set up a current, the magnetic field of which always opposes the change in the existing field caused by the original current.

Figure 9-4 illustrates the left hand rule for generators. As an aid to help the reader visualize the mutually perpendicular relationship between the three factors, notice the box being held in the left hand with the three vectors extending outward from the single corner in the palm. A quick glance at any square object or the corner of a book should make the arrangement clear. This vector arrangement will be used from now on when the rule is applied.

Notice in Figure 9-4 the thumb is pointing toward the top of the page along the vector marked "movement of conductor". This indicates that the conductor is being moved from the bottom of the page toward the top, cutting the flux lines in the process. The forefinger, as stated in the rule, is at right angles to motion and is pointing along the vector marked magnetic flux. The direction of the forefinger indicates that the flux direction is FROM the north pole to the south pole. The middle finger can be seen to be pointing out of the page along the vector marked induced EMF.

The terminology "direction of induced EMF" can prove to be confusing. The direction of electron flow WITHIN A SOURCE is from plus to minus. Since even the movement of an open circuited conductor through a flux field will cause a displacement of electrons (giving a

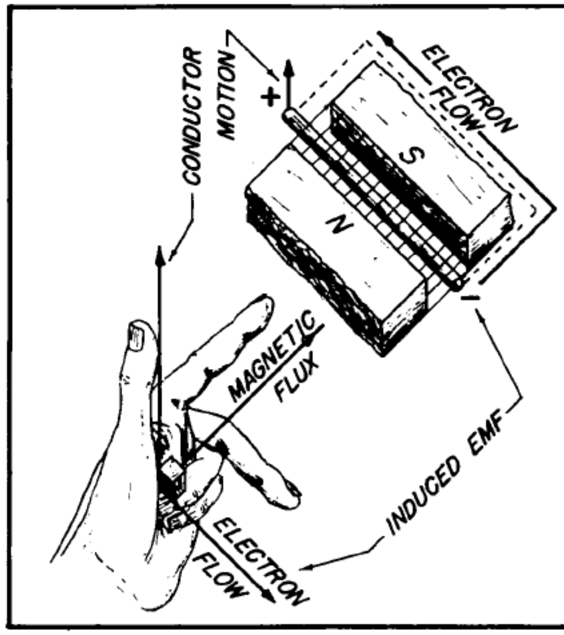

Figure 9-4 - Left hand rule for generators.

deficiency of electrons at one end of the conductor and a surplus at the other), the middle finger will point in the direction that electrons will move inside the "source". Since the section of conductor within the flux field will act as a source, a polarity is given to indicate the potential acquired from electron displacement. If a circuit were connected as indicated by the dotted line, there would be electron flow in this circuit in the same direction as indicated by the middle finger of the left hand generator rule. Thus, when a complete circuit exists, the last line of the left hand rule for generators may be changed to read: "THEN THE MIDDLE FINGER WILL POINT IN THE DIRECTION OF THE ELECTRON FLOW."

Figure 9-4 may also be used to show the effect of changing the direction of any of the factors involved. For example, point the thumb down but keep the flux lines in the same direction. This will indicate the conductor moving DOWN through the field. It will now be noted that the electron flow is opposite to its previous direction. Now keep the thumb pointed DOWN but reverse the direction of the flux lines by moving the forefinger around to point in the opposite direction (against the arrows). It will be noted that again the direction of electron flow has been changed.

From this simple illustration it can be seen that changing the direction of EITHER the motion of the conductor OR the flux lines will REVERSE electron flow, but changing the direction of

A3. The coil is capable of producing a greater flux density for a given amount of current.

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motion AND flux lines will not change the direction of current flow.

Q4. Can there be current flow in an open circuit?

Q5. What will be the effect on electron flow if the direction of motion of a conductor through a flux field were changed but the flux remained unchanged?

In order to visualize Lenz's law the simplified arrangement in Figure 9-5 will be explained step-by-step.

Using the left hand rule for a conductor (Figure 9-5A) flux lines are seen to be established around the conductor when the switch is closed. Only a few flux lines are shown for ease of explanation. As current rises in the conductor, the flux lines will expand outward in ever widening loops. If another closed loop conductor containing a meter is placed close enough to the first conductor, then some of these expanding flux lines will cut through the second conductor. When a conductor is cut by a line of flux, a voltage is induced in the conductor. It does not matter if the flux field is stationary and the conductor is moved through the field (as shown in Figure 9-4) or if the conductor is stationary and the flux field moves past the conductor (as in Figure 9-5). The important thing to remember is that as long as there is relative motion between the two then there will be an induced EMF. In Figure 9-5B there is relative motion between the conductor and the flux field, but since the left hand rule for generators specifies the motion of the CONDUCTOR the flux field must be assumed to be standing still. This merely means that if the flux field were originally expanding TOWARD the conductor, and the field is now assumed to be standing still, then the conductor can be assumed to be moving toward the flux field (right to left).

Applying the left hand rule for generators to Figure 9-5B by pointing the thumb toward the left of the page in the direction of the relative motion of the conductor, the forefinger directly out of the page in the direction of the flux lines, we then find the middle finger pointing toward the bottom of the page in the direction of the induced electron flow.

What has all this to do with Lenz's law? The relationship can be investigated by considering the law in parts. The first half of the law states "THE DIRECTION OF AN INDUCED EMF IS SUCH THAT IT TENDS TO SET UP A CURRENT

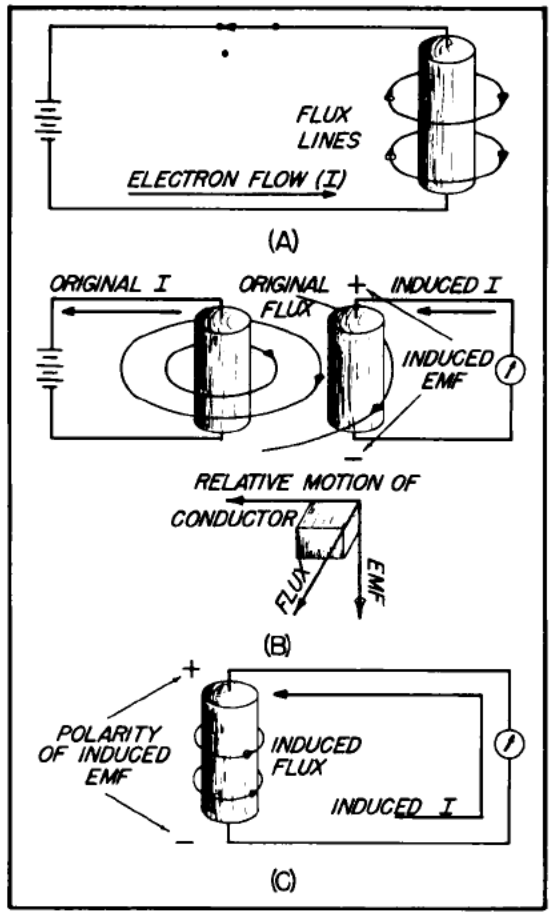

Figure 9-5 - Illustration of Lenz's law and mutual induction.

...." The induced EMF in Figure 9-5B, with a polarity of (+) at the top and (-) at the bottom, has in fact set up a current which is shown flowing along the conductor toward the bottom of the page. The second half of the law continues with ...."THE MAGNETIC FIELD OF WHICH ALWAYS OPPOSES THE CHANGE IN THE EXISTING FIELD CAUSED BY THE ORIGINAL CURRENT". The existing field set up by the original current is seen in Figure 9-5B to be in a clockwise direction as viewed from above. Applying the left-hand rule for conductors to Figure 9-5C, it is found that the induced current will set up its own magnetic field which will be in a counter-clockwise direction and oppose the existing field set up by the original current.

Although a simplified circuit has been used to demonstrate Lenz's law, the action and theory are identical to that of more complicated applications. This principle will be encountered

again for a more detailed analysis at a later time.

In cases where an EMF is induced, three requirements must be satisfied. These three requirements are: the existance of a magnetic field, the existance of a conductor, and relative motion between the field and conductor. It should be remembered that if there is no relative motion, or the relative motion is in such a manner that no lines of force are being cut, then there will be no induced EMF.

It should also be remembered that an EMF may be induced in a conductor regardless of whether or not there is a complete circuit, but for the induced EMF to cause a current flow there must be a complete circuit.

9-9. Factors Affecting Polarity and Amplitude of Induced EMF

It was shown in section 9-8 by the use of left hand rule for generators, that changing either the direction of motion of the conductor or the direction of the flux lines will reverse the polarity of the induced EMF.

According to Faraday's law the amplitude of induced EMF is affected by the rate at which lines of force are cut. This can be expressed mathematically as:

$$E_{av} = \frac{N\phi}{10^8 t} \qquad (9\text{-}2)$$

Where: N = the number of conductors

t = time in seconds, taken to cut all flux lines

ϕ = the number of lines of force (flux)

10^8 = a constant

The constant (10^8) comes from the fact that an EMF of one volt is produced if one conductor cuts 100,000,000 (10^8) lines of force in one second.

It can be seen from equation (9-2) that the amplitude of E_{av} is a direct function of the number of conductors and the number of flux lines and an inverse function of time. Doubling the number of conductors or lines of flux will double E_{av}. Decreasing the time taken to cut the flux lines will increase E_{av}.

Q6. What will be the effect on E_{av} if both the number of flux lines and the number of conductors were doubled?

9-10. Self-Induction and CEMF

The property of a conductor to induce a voltage within itself is known as SELF INDUCTION.

The induced EMF is called a COUNTER ELECTROMOTIVE FORCE (CEMF) because it will always oppose the applied EMF (voltage).

As stated previously (and illustrated in Figure 9-1) a current carrying conductor is encircled by flux lines. If it were possible to look into a cross-section of this wire, it would be seen that the circular flux lines actually start at the center of the conductor and expand outward. In expanding outward the flux lines cut the conductor itself, and any time flux lines cut a conductor there will be an EMF generated. Figure 9-6 illustrates the action of an EMF being generated in a single conductor.

Applying the left hand generator rule to Figure 9-6 will show the CEMF producing a current flow in opposition to the original electron flow. At first glance it might seem that the CEMF would prevent current flow, but it must be remembered that CEMF will be produced only so long as there is relative motion (flux expanding or collapsing). If a steady state current (no change) is reached self induction will cease to exist.

The CEMF produced due to self induction depends on the amount of flux linkage between one part of the conductor and another part of the same conductor. In other words, a single circular flux line expanding out from the center

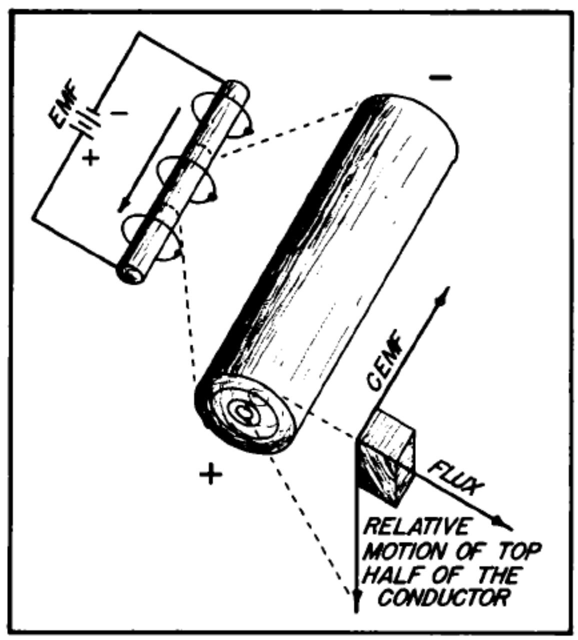

Figure 9-6 - Self-Induction of a straight conductor.

A4. No! There can be a momentary displacement of electrons toward one terminal of the open circuit but this value is too small and happens too fast to be detected by normal means.

A5. The direction of electron flow would be changed. (Providing a complete circuit existed.)

A6. E_{av} would increase to four times as much.

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of the straight wire will cut only the radius of the wire from the center to the surface. Therefore the CEMF produced will be very small. If the conductor is wound into a coil then a single circular flux line expanding out from the center of the conductor in one of these loops will cut many of the loops. Therefore, the CEMF produced will increase because the flux linkage between one part of the conductor and another will be greater.

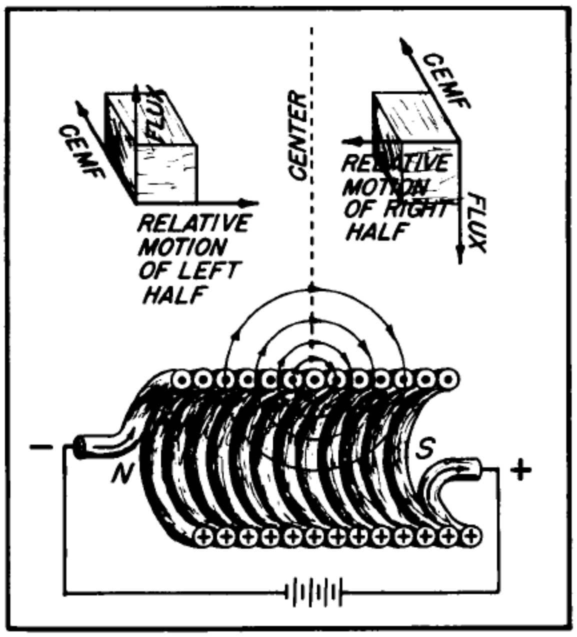

Figure 9-7 - Self-induction of a coil.

NOTE: Dot (·) in center of conductor indicates current leaving and cross (+) indicates current entering.

Figure 9-7 illustrates a cut-away view of a coil with a single expanding circular flux line. The original electron flow is out of the page on the top of the coil. Applying the left hand generator rule to the coil will indicate the induced electron flow INTO the page on the top of the coil, opposing the original current flow. The polarity of the CEMF produced by the coil, is indicated as being in opposition to the EMF (battery).

Although only one flux line is shown it must be remembered that all parts of the conductor are producing flux lines at the same time.

Q7. Why does a straight wire produce less CEMF than a coil?

9-11. Mutual Induction

MUTUAL INDUCTION is defined as the ability of a circuit or device to transfer energy to another electrically isolated circuit or device.

Faraday's and Lenz's laws express the concept involved in the process of mutual induction. Faraday discovered that induced EMF was proportional to the rate of cutting of lines of force. Faraday also found that an EMF was induced only when there was a change in the flux linkages. Lenz's law states the same ideas in a more refined manner, but one fact is present in both cases. In order for there to be an induced EMF or a transfer of energy there must be a CHANGE OF FLUX LINKAGE.

Figure 9-8 illustrates the transferring of energy from a circuit containing a source to an electrically isolated circuit. As was stated previously, when the switch is closed there will be a momentary expansion of flux lines as the field builds up around the wire of the circuit on the left. These flux lines will cut across the conductor of the electrically isolated circuit on the right and produce a momentary EMF. If a steady state of current flow is reached there will be no more expansion of the field and therefore no more flux lines will be cut. Of course, since no flux lines are being cut, there will be no CEMF produced. This does not mean the flux field has disappeared. It merely means, that as long as there is a steady current flow in the source circuit (primary) the field will exist around the conductors of the primary and the isolated circuit (secondary), BUT THERE WILL BE NO RELATIVE MOTION. In order to again induce energy in the secondary there must be a CHANGE in the field (flux linkage). This can be accomplished in one of two ways. The current in the primary could be increased, causing the field to expand further and additional flux lines would be cut as they passed through the secondary wire. All actions would be the same as the initial buildup of the field. The primary current could also be decreased (by opening the switch). Since there is no longer a current in the primary to sustain it, the field will collapse back into the primary wire. In the process of collapsing the flux lines will have to pass

through the secondary wire, again producing a CEMF. Closer examination of Figure 9-8 will show that when the flux lines collapse, they will pass through the secondary in the opposite direction to that which they took when expanding. Applying the left hand generator rule will show the relative motion of the secondary as being in the opposite direction and the polarity of the CEMF opposite to that shown when the flux was expanding.

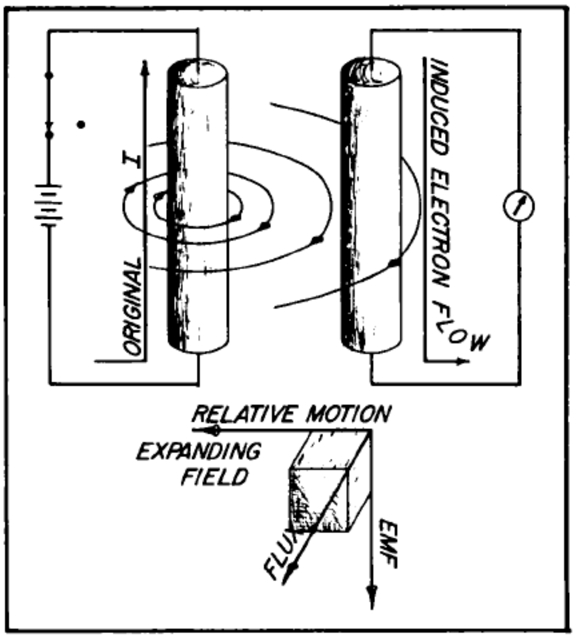

Figure 9-8 - Energy transfer between circuits.

From the above a new concept can be seen to emerge. A field does not necessarily have to increase from zero to maximum and back to zero in order to produce CEMF. It is now possible to see how a CEMF can be produced by a field which never drops to zero but simply increases and decreases in density about some average value. This is an important basic fact which will find application at a later date.

Q8. Can a flux field exist when there is no relative motion?

9-12. Definition of Inductance

INDUCTANCE is the property of an electrical circuit that opposes any change in the current through that circuit. That is, if the current increases, a self-induced voltage (CEMF) opposes this change and delays the increase. If the current decreases, a self-induced voltage tends to aid, or prolong the current flow. Thus the most noticable effect of inductance in a circuit is that the current can neither increase nor decrease as fast as it can in a circuit without inductance.

9-13. Self-Inductance

It has already been pointed out that all conductors and coils induce within themselves an EMF which will oppose any change in the internal current. Therefore, any conductor or coil, by definition, possesses the property of inductance.

In order for a coil or piece of wire to possess inductance, current need not be present. From this it can be seen that a wire hanging in free space will still possess the property of self-induction or simply inductance.

It was stated in section 9-10 how coiling a conductor would increase its ability to produce CEMF, due to increased flux linkages between various sections of the same conductor. Extending the line of reasoning introduced above, it is not too difficult to see how a coil, possessing the ability to produce a larger CEMF than a straight wire, will possess a larger inductance.

9-14. Unit of Inductance

The unit for measurement of inductance is the HENRY named in honor of Joseph Henry who discovered the property of self-induction of a coil.

A coil is said to possess an inductance of one henry if an EMF of one volt is induced in the coil when the current through the coil is CHANGING at the rate of one ampere per second. This is expressed mathematically as:

$$L = \frac{e}{\frac{\Delta i}{\Delta t}} \qquad (9\text{-}3)$$

Where: L = the electrical symbol for self-inductance in henrys

e = induced voltage in volts

(delta)Δ = "a small change in"

Δi = a small change in current

Δt = a small change in time

Equation (9-3) may be more easily grasped if expressed as follows:

$$\text{Inductance (L) of one henry} = \frac{\text{Induced EMF of one volt}}{\text{a change in current of one amp per second}}$$

A7. Less flux linkage. A line of flux can only cut a straight conductor once but a single line of flux can cut its conductor many times in a coil.

A8. Yes! The flux field can be maintained by a steady current but there will be no induced voltage without relative motion.

9-15. Factors Affecting Self-Inductance

A single piece of wire will possess a given amount of inductance. This same wire, when coiled, will possess a larger inductance. The amount of inductance possessed by a conductor is determined mainly by its physical arrangement and not the applied voltage or current.

The important factors are the number of turns, the spacing of the turns, permeability of the core material, and the physical shape of the core (length, area, etc.)

TURNS: Inductance has been defined as a measure of the ability to produce a CEMF, and section 9-10 showed that the production of CEMF depends on the flux linkage between various parts of the same conductor. Therefore, putting more turns on a coil will cause more flux linkage, thereby increasing the inductance.

SPACING OF TURNS: It was stated in section 9-5 that the flux lines very close to the conductor form loops of small diameter. The significance of coil spacing can now be seen. Wide spacing will allow only the outer flux lines of each conductor to participate in flux linkage. Close spacing will allow more flux lines to participate in linkage. The closer the spacing the higher the inductance.

CORE MATERIAL: Any increase in flux density will increase flux linkage and thereby increase inductance. It was shown in section 9-6 how the physical factors of core material affected flux density.

The effect of all the above factors on inductance can be expressed mathematically as follows:

$$L = \frac{1.26 \mu A N^2}{l_m \times 10^8} \qquad (9\text{-}4)$$

Where: L = inductance in henrys

N = number of coil turns

10^8 = a constant (explained in equation 9-2) used here to convert L to practical units

μ = permeability of core material

A = cross sectional area of core enclosed by one turn, in sq/cm.

1.26 = a constant

l_m = mean length of core in cm.

NOTE: This formula is approximately correct when used with a single layer coil whose length is at least ten times its width. The inductance formula is presented here merely to aid the reader in visualizing the effect of the physical factors of the coil on inductance. In equation 9-4 the area is given in sq/cm. If the area is measured in square inches, then the constant 1.26 must be multiplied by 2.54. The new constant, 3.20 is then substituted for the constant 1.26.

Equation (9-4) indicates L is a direct function of core permeability, area, and coil turns. Inductance is shown to be an inverse function of core length.

One other factor of note which affects inductance is the type of winding. Due to the complexity and specialization of this branch of coil design, only brief mention will be given this phase of coil construction.

It can readily be seen that in a single layer coil, flux expanding away from the coil in a perpendicular direction is wasted. A multilayer coil would provide more flux linkage than a single layer coil with the same core dimensions. The manner in which these extra layers are wound on the coil is a concern of the coil designer.

Q9. How much change will there be in the inductance of a coil when it is moved from one circuit to another?

INDUCTORS

Figure 9-9 shows examples of two types of inductors and their schematic symbols. The air-core type inductor is most frequently used in circuits that are above the audio range. The iron-core type is widely used in the audio range (below 20 KC). The iron-core type is usually made of laminated sheets of iron to reduce core losses.

9-16. Mutual Inductance

In order to understand the methods employed in computing the total inductance of a number of inductors (coils) connected in various circuit configurations, the factors affecting these computations must be understood.

In section 9-11 the process of MUTUAL INDUCTION was analyzed and later Figure 9-8

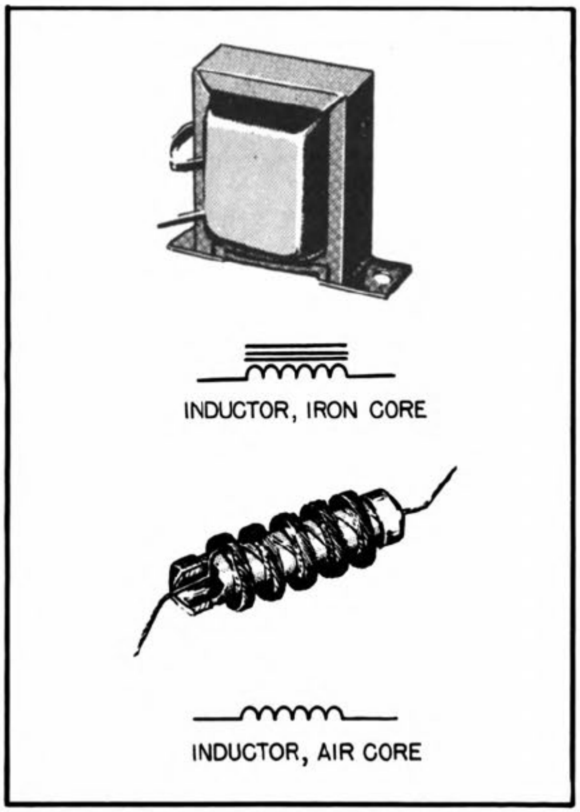

Figure 9-9 - Inductor types and schematic symbols.

was used to show a change of flux in one circuit (primary) inducing a voltage in an electrically isolated circuit (secondary). When two coils are positioned so that flux lines from one cuts the turns of the other, they are said to exhibit MUTUAL INDUCTANCE. The action of mutual inductance in producing an EMF is basically the same as that of self-inductance, the main difference being that self-inductance is the property of a single coil while mutual inductance is the property of two (or more) coils acting together.

Mutual inductance is measured in henrys and is designated by M. The equations for mutual inductance is:

$$M = \frac{e}{\Delta i/\Delta t} \qquad (9\text{-}5)$$

where: M = mutual inductance in henrys

e = induced voltage in SECONDARY in volts

$\Delta i/\Delta t$ = rate of change of primary current per unit of time

Again, the above may be more readily understood if expressed as follows:

$$\text{Mutual inductance of one henry} = \frac{\text{Induced EMF of one volt in secondary}}{\text{a CHANGE in primary current of one amp per second}}$$

Example. Two coils L_1 (primary) and L_2 (secondary) are positioned close together. Find the mutual inductance.

Given: The current in L_1 changes from 2 amp to 5 amp in one second and causes an induced voltage of 21 volts in L_2.

Solution: First determine the rate of change of the current in L_1:

$$\frac{\Delta i}{\Delta t} = \frac{5-2}{1} = \frac{3}{1} = 3 \text{ amps per sec.}$$

Substituting known values in equation (9-5):

$$M = \frac{e}{\Delta i/\Delta t}$$

$$M = \frac{21}{3}$$

$$M = 7 \text{ henrys}$$

Q10. If a rate of change of current of 500 ma per second in one coil induces a voltage of 1-1/2 volts in another coil, what is the mutual inductance?

Equation (9-5) may be transposed to arrive at another very useful equation.

$$e = \frac{M\,\Delta i}{\Delta t} \qquad (9\text{-}6)$$

This equation is useful because it allows us to predict the amount of induced voltage for two magnetically coupled circuits or coils. It is also useful because it clearly shows induced

770-696 O - 65 - 3

A9. None. Inductance is a function of the physical qualities of the coil which will not change from one circuit to another.

A10. 3 henrys.

voltage to be a direct function of the mutual inductance and the rate of change of current.

Q11. If the mutual inductance of Question 10 were changed to 0.5 mh what would be the induced voltage?

The induced EMF of an inductor is dependent on the mutual inductance. Mutual inductance is, in turn, dependent on the physical dimensions of the two coils, number of turns on each coil, premeability of the cores, and the COEFFICIENT OF COUPLING. Of the above, the effect of the first three on induced voltages, flux density, etc., has been shown previously. The last factor, coefficient of coupling, is a new term. It is dependent upon the distance between two coils and on the position of the coil axis with respect to each other. In other words, coefficient of coupling is a measure of how much of the flux from one coil cuts the turns of the other coil.

Figure 9-10 is used to clarify the above. In Figure 9-10A, the inductors are seen to be very close, in fact almost all of the flux from L_1 cuts all the turns of L_2. Under these conditions the coefficient of coupling is maximum or very near unity. Under ideal conditions all the flux from one coil cuts all the turns of the other coil and the coefficient of coupling is unity or one. This is a condition which may be approached, but never reached in practical applications.

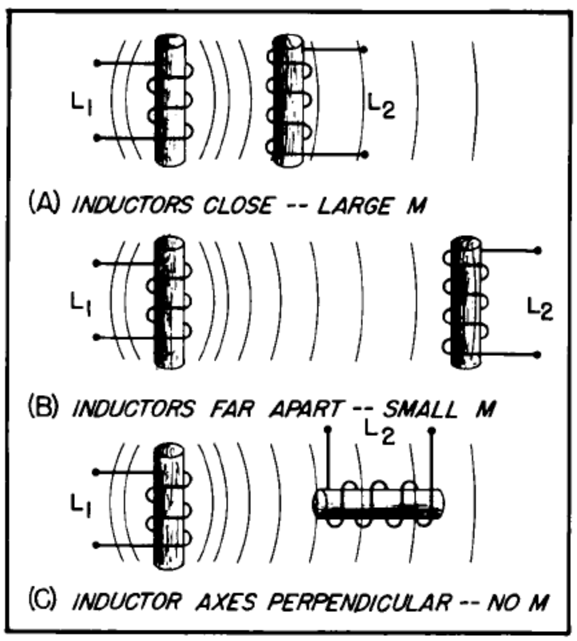

Figure 9-10 - The effect of position of coils on mutual inductance.

In Figure 9-10B the inductors are seen to be physically separated by a considerable distance. Since the strength of a flux field decreases rapidly with distance, it is easy to see how very few of the flux lines from coil L_1 will cut the turns of L_2. Therefore, the coefficient of coupling is small, making the mutual inductance small.

In 9-10C, the relative position of the axes of the two coils is seen to be perpendicular. It has been previously established that a flux line must cut a conductor at right angles in order to induce maximum voltage. In the conditions shown in Figure 9-10C, there are virtually no conductors positioned at right angles to the flux lines. If none of the flux of L_1 is able to cut turns of L_2, the coefficient of coupling is zero and the mutual inductance is zero.

The relationship between self-inductance, mutual inductance, and the coefficient of coupling (designated by the letter K) can be shown mathematically:

$$M = K\sqrt{L_1 L_2} \qquad (9\text{-}7)$$

where: L = self inductance of the coils in (henrys, millihenrys, microhenrys)

K = coefficient of coupling, expressed as a decimal

M = mutual inductance in same units as L.

Equations (9-6) and (9-7) show that mutual inductance and induced voltage vary directly as the coefficient of coupling.

9-17. Series Connected Inductors

When series connected inductors are well shielded, or located far enough apart to make the effects of mutual inductance negligible, the equivalent of total inductance of the circuit is the algebraic sum of the individual inductances. This is expressed mathematically as:

$$L_t = L_1 + L_2 + L_3 \ldots\ldots + L_n \qquad (9\text{-}8)$$

where: L_t = total inductance in henrys

L_1, etc = individual inductances in henrys

Equation (9-8) is true only when there is NO MUTUAL INDUCTANCE.

Example. Assume perfect shielding. Four coils are connected in series. Find their total inductance.

Given: $L_1 = 1.5$ henrys

$L_2 = 500$ millihenrys

$L_3 = 0.2$ henrys

$L_4 = 25,000$ microhenrys

Solution: First convert all inductances to the same units.

$L_1 = 1.5$ henrys $= 1500$ mh

$L_2 = 500$ mh $= 500$ mh

$L_3 = 0.2$ h $= 200$ mh

$L_4 = 25,000\ \mu h = 25$ mh

Equation: $$L_t = L_1 + L_2 + L_3 + L_4 \qquad (9\text{-}8)$$

Substitute values in equation:

$$L_t = 1500 + 500 + 200 + 25$$

$$L_t = 2,225 \text{ mh}$$

$$L_t = 2.225 \text{ h}$$

When two inductors in series are so arranged that there is magnetic coupling between them, equation (9-8) must be modified. The total inductance of series connected coils when M is present is expressed mathematically as:

$$L_t = L_1 + L_2 \pm 2M \qquad (9\text{-}9)$$

where: L_t = total inductance of the two coils

L = self-inductance of each coil

M = mutual inductance

The $\pm$ signs in equation (9-9) must be included due to the fact that the coils can be arranged so that their fields are either series aiding or series opposing. When the fields are series aiding, the plus sign is used. When the fields are series opposing, the minus sign is used.

Figure 9-11 illustrates two coils, series connected and wound in such a manner as to have their fields aiding. Notice the polarities of the individual coils and the overall polarity. Application of the left hand rule for coils will show how these polarities are obtained.

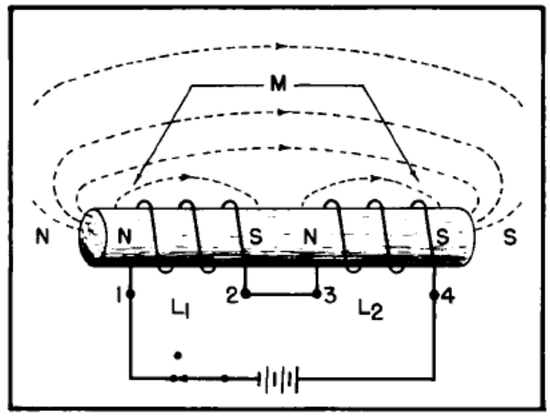

Figure 9-11 - Series inductors with aiding fields.

Example. Assume the two coils, L_1 and L_2 in Figure 9-11 are series connected and have mutual inductance. Find the total inductance.

Given: $L_1 = 20$ mh

$L_2 = 15$ mh

$M = 16$ mh

Solution: Select equation.

$$L_t = L_1 + L_2 + 2M \qquad (9\text{-}9)$$

The +2M is used because an inspection of Figure 9-11 shows the fields to be aiding.

Substitute values:

$$L_t = 20 + 15 + 2\times16$$

$$L_t = 67 \text{ mh}$$

Figure 9-12 illustrates two coils, series connected and wound in such a manner as to have their fields opposing. Again apply the left hand rule and note the polarities.

Example. Assume the two coils, L_1 and L_2 in Figure 9-12 are series connected and possess mutual inductance. Find the total inductance.

Given: $L_1 = 20$ mh

$L_2 = 15$ mh

$M = 16$ mh

A11. 0.25 millivolts.

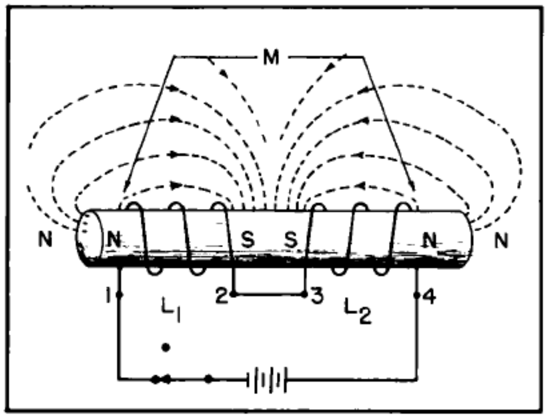

Figure 9-12 - Series inductors with opposing fields.

Solution: Select equation.

$$L_t = L_1 + L_2 - 2M \qquad (9\text{-}9)$$

The -2M is used because an inspection of Figure 9-12 shows the fields to be opposing.

Substitute values:

$$L_t = 20 + 15 - 2x16$$

Combine like terms:

$$L_t = 35 - 32$$

$$L_t = 3 \text{ mh}$$

Q12. When mutual inductance is present in series connected inductors, the total inductance cannot be found by merely adding all the inductances together. Why?

9-18. Parallel Connected Inductors

The total inductance, L_t of inductors in parallel is calculated in the same manner that the total resistance of resistors in parallel is calculated, provided the coefficient of coupling is zero. If there is NO mutual inductance then for inductors connected in parallel, the reciprocal of the total inductance equals the sum of the reciprocals of the individual inductances. Mathematically this is:

$$\frac{1}{L_t} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \cdots\cdots + \frac{1}{L_n}$$

OR:

$$L_t = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \cdots\cdots\cdots\cdots + \frac{1}{L_n}} \qquad (9\text{-}10)$$

When there are only two inductors in parallel (and NO magnetic coupling) the product over the sum method may be employed. Mathematically that is:

$$L_t = \frac{L_1 L_2}{L_1 + L_2} \qquad (9\text{-}11)$$

SERIES L-R CIRCUIT

9-19. LR Circuit with DC Source

The action of current and voltage in a purely resistive circuit is for all practical purposes instantaneous. In other words, when a switch is closed in a circuit containing only a dc source and a resistor, current and voltage reach a maximum value almost instantaneously.

It has been mentioned previously that inductance affects circuit action only when there is a CHANGE in circuit current. It can then be assumed that the technician is primarily concerned with inductance in ac and not steady state dc circuits. This assumption is correct. But since dc current and voltage can increase and decrease in magnitude it is informative to study the effects of the inductor when voltage or current is increasing or decreasing.

9-20. Growth Current in an LR Circuit

All inductors and batteries have some internal resistance. For ease of explanation this resistance can be lumped together or treated as an individual resistor. This process of lumping circuit elements is called idealization. The circuit in Figure 9-13A is idealized and consists of a perfect voltage source, perfect switch, perfect inductor, and a resistance which represents the combined inherent resistances of all the elements.

In previous sections of this chapter and particularly in section 9-10, it has been stated repeatedly that inductance is the property of a conductor that opposes an increase or decrease in current. Using the circuit of Figure 9-13A, and the waveforms of 9-13B, a voltage and current analysis of the series RL circuit will now be made.

At time zero (t_0) the switch (sw) would be in the number one (1) position. As can be seen the circuit is open. The waveforms in 9-13B show that at (t_0) there is no voltage across the coil (e_L is zero), there is no voltage across the resistor (e_R is zero), and there is no current flowing (i is zero).

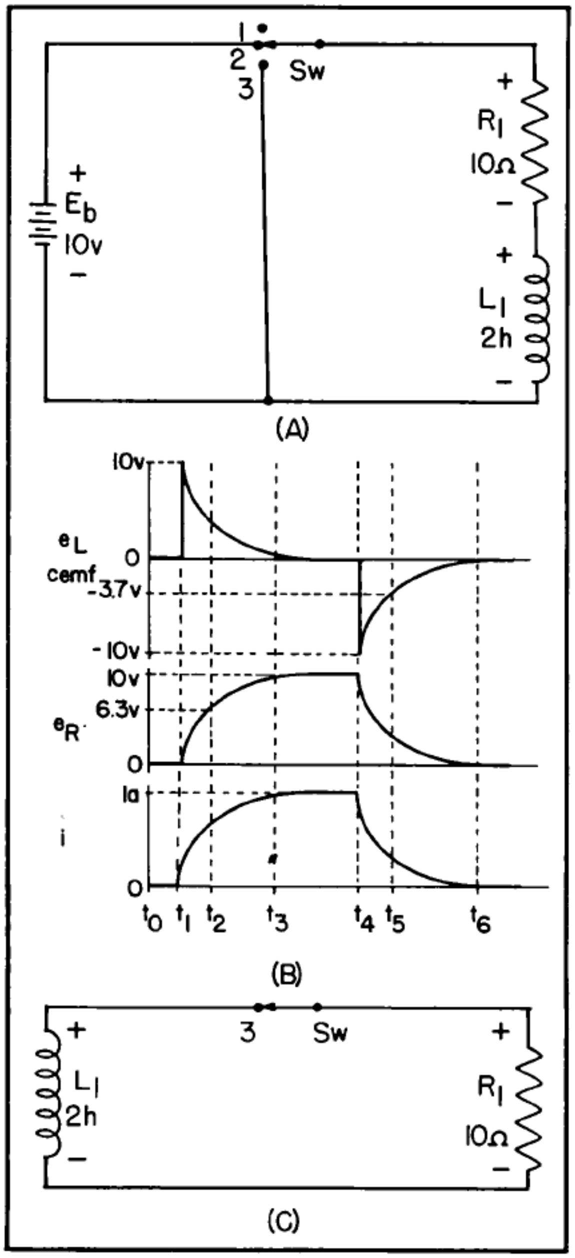

Figure 9-13 - Current, voltage, and time relationships in a series LR circuit with a dc source.

The waveforms show the above conditions remain unchanged until time one (t_1). At t_1 the switch is thrown to the number two (2) position. At this instant the current will attempt to increase to its maximum Ohm's law value, in this case:

$$I = \frac{E}{R}$$

$$I = \frac{10}{10}$$

$$I = 1 \text{ amp}$$

The current attempts to attain a value of one amp in zero elapsed time. Thus, the current is undergoing its maximum rate of change (roc) at this instant. At the exact instant (t_1) the switch is closed, there is a simultaneous displacement of electrons in all parts of the circuit, and while this displacement is immeasurable as a current flow, it does cause the coil to produce a CEMF which is almost equal to the EMF of the source. An inspection of the waveforms (Figure 9-13B) will show the above statements in a graphical form. At t_1, e_L (CEMF) has increased to nearly the source value, e_R shows no measurable voltage drop, and i shows no measurable amount of current flow. A brief instant of time after t_1, current flow starts and a measurable drop occurs across R. As part of the source voltage is now dropped across the resistor, less voltage is applied to the coil. As a result, a decrease of CEMF occurs which permits an increase of current.

Going back to t_1, it was mentioned that the rate of change of current (roc) was maximum. This can be proven by computing the rate of change of current at various instants of time as follows:

$$L = \frac{e}{\frac{\Delta i}{\Delta t}} \qquad (9\text{-}3)$$

transposing:

$$\frac{\Delta i}{\Delta t} = \frac{e}{L}$$

since:

$$\frac{\Delta i}{\Delta t} = \text{roc}$$

then:

$$\text{roc} = \frac{e}{L} \qquad (9\text{-}12)$$

where: e = induced coil voltage e_L in volts

L = self-inductance of coil in henrys

roc = rate of change of current

A12. The lines of flux from each coil cut the other coil. Since inductance is a function of flux linkages mutual inductance must be considered.

---

substituting: $$roc = \frac{10 \text{ volts}}{2 \text{ henrys}}$$

$$roc = 5 \text{ amps per sec}$$

NOTE: This roc only applies at the exact instant of time (t_1)

As the current increases, the e_R drop increases, and the induced voltage of the coil decreases. In fact, examination of the e_R waveform indicates that at time two (t_2) e_R has increased to about 6.3 volts. According to Kirchhoff's law for voltage the sum of the voltage drops ($e_L + e_R$) must equal E_b. Therefore, e_L must equal $E_b - e_R$. At t_2 then:

$$e_L = E_b - e_R$$

substituting approximate values from waveforms:

$$e_L = 10 - 6.3$$

$$e_L = 3.7\text{V (at } t_2)$$

The approximate roc at t_2 may now be found:

$$roc = \frac{e}{L} \quad (9\text{-}12)$$

Substituting: $$roc \text{ (at } t_2) = \frac{3.7}{2}$$

$$roc \text{ (at } t_2) = 1.85 \text{ amps per sec}$$

From the above it can be seen that as time increases the roc decreases. As time increases more voltage is dropped across the resistor and less across the inductor. Since:

$$\frac{\Delta i}{\Delta t} = \frac{e_L}{L}$$

it follows that $\frac{\Delta i}{\Delta t}$ decreases with time.

The action of the circuit is continued up to time three (t_3). It can now be seen from the waveforms that e_L has decreased to nearly zero, e_R is now nearly equal to the source voltage, and the circuit current has just about reached maximum. Close inspection of the waveform shows that:

$$\text{at } t_3\ e_R = 9.9\text{V}$$

$$e_L = 0.1\text{V}$$

$$i_t = 0.99 \text{ amps}$$

$$roc = \frac{e}{L} \quad (9\text{-}12)$$

$$roc = \frac{0.1}{2}$$

$$roc = 0.05 \text{ amps per sec.}$$

Actually the current will never reach the maximum value of one amp. After a predictable length of time however, the magnitude of the current is so close to its theoretical maximum value it can be considered to have reached this value. This final point is reached in Figure 9-13B very shortly after t_3. After t_3, a steady state value is established by the current, and since for all practical purposes there is no more change, there is effectively no more CEMF being produced. The energy taken to overcome the CEMF of the coil is now stored in the magnetic field which exists around the coil.

As long as the switch is maintained in position (2) the conditions of the circuit will remain as follows: e_L equal to zero, e_R equals the voltage source and is equal to its maximum Ohm's law value.

9-21. Decay Current in Series LR Circuit

At time four (t_4) the switch (sw) is moved instantaneously to position (3). Since the source E_b is now removed from the circuit the current will ATTEMPT to stop instantly. Again the term attempt has been used because of the action of the inductor in opposing any change. At t_4 the waveform depicts the coil as developing a large CEMF, only this time the polarity is in the opposite direction because the CEMF is developed by the collapsing of the magnetic field. At t_4 the circuit of 9-13A consists of a basic circuit with the resistance connected across the coil, which is now acting as the source of voltage. The equivalent circuit during decay time is shown in 9-13C. Current through the resistance is maintained in the original di-

rection due to the action of the collapsing flux field.

The waveforms in 9-13B indicate that at the instant of time t_4 the coil (acting as the source), is supplying maximum current (i at t_4 is one amp) and the circuit satisfied Kirchhoff's voltage law. (The voltage drop, e_R = 10V, is equal to the source, e_L = 10V.) The energy originally supplied to the coil is stored in the field surrounding the coil, and as the field collapses more and more of this energy is returned to the circuit. The less energy the field contains, the less rapidly it will collapse, and the less rapidly it collapses the less CEMF will be produced.

The action of the circuit during decay is shown more clearly by the waveforms. It can be seen that at t_5 the field has expended a large part of its stored energy. The coil voltage e_L has decreased to a value of approximately 3.7 volts. To satisfy Kirchhoff's voltage law the resistive voltage drop e_R must also have decreased to 3.7 volts which in turn indicates a decrease in circuit current. Since current will only be maintained during the time the field is collapsing, the waveforms indicate the above circuit action will continue until t_6, when for all practical purposes the field has completely collapsed. This time e_L has decreased to zero, e_R has decreased to zero, and i has decreased to zero. It will be noted that the waveforms of resistors voltage and circuit current are identical and therefore, in some graphs, may be illustrated by the same curve.

Q13. When is the rate of change of current maximum in a dc RL circuit?

9-22. LR Time Constants

It was stated, during the explanation of roc, that the current would reach its maximum value in a certain predictable length of time. Specifically, the time it takes the current in a circuit, containing only resistance and inductance to increase to 63.2 percent (or decrease to 36.8 percent) of its maximum value is known at the TIME CONSTANT. The time constant is determined by the ratio of circuit inductance to circuit resistance. The mathematical equation is:

$$T = \frac{L}{R} \qquad (9\text{-}13)$$

where: T = time constant in seconds

L = circuit inductance in henrys

R = circuit resistance in ohms

Example. Find the time constant of a circuit in which the inductance is 2 henrys and the resistance is 10 ohms.

Given: L = 2 henrys

R = 10 ohms

Solution: $$T = \frac{L}{R} \qquad (9\text{-}13)$$

Substituting: $$T = \frac{2}{10}$$

T = 0.2 second

Q14. What determines the time constant of an LR circuit?

The current in an LR circuit does not rise in a linear manner. The instantaneous current magnitude with respect to time follows what is called an EXPONENTIAL CURVE.

The exponential curve is a result of the fact that a current, in approaching a maximum value in a series of time constant, will only increase 63.2 percent of the remaining value in EACH time constant. In other words, if the final maximum value of current in a circuit is to be one amp, then the first time constant the current will increase to 63.2 percent of one amps or 0.632 amp. During the second time constant the CHANGE in current will be 63.2% of the difference between the final value and the value at the end of the first time constant.

1 amp - 0.632 amp = 0.368 amp

According to the explanation of the exponential curve and the definition of a time constant, the current will increase 63.2 percent of the REMAINING value during the second time constant. This will be 63.2 percent of 0.368 amp or approximately 0.233 amp.

Therefore, at the end of the two time constants the current, in its rise toward maximum, will have a value equal to the current increase in the second time constant or:

0.632 amp + 0.233 amp = 0.865 amp

Following the above procedure again the remaining value of current will be:

1 - 0.865 = 0.134 amp

63.2 percent of this value, the amount or rise in current during the 3^{rd} time constant, is:

0.632 x 0.134 = 0.085 amp

A13. During the first instant of time of an attempted increase or decrease.

A14. The LR ratio.

At the end of the third time constant the current will equal:

$$I = 0.632 + 0.233 + 0.085$$

$$I = 0.95 \text{ amps}$$

In the above calculations the increase of current in each succeeding time constant is seen to be less and less. For normal applications, AFTER FIVE TIME CONSTANTS HAVE PASSED THE CURRENT DIFFERS FROM ITS FINAL VALUE BY SUCH A NEGLIGIBLE AMOUNT THAT IT IS CONSIDERED TO HAVE REACHED ITS FINAL VALUE.

If greater accuracy is desired it may be achieved by the use of the above procedure carried out for as many time constants as needed. Mathematically this takes the form of the general equation:

$$I_t = \Delta I_1 + \Delta I_2 + \Delta I_3 \ldots\ldots + \Delta I_n \qquad (9\text{-}14)$$

where: I_t = current after desired number of time constants

ΔI_1 = 63.2 percent times the value of current remaining (between maximum value and present value) or the current increases in a particular time constant

ΔI_n = meaning the equation extended to include any integral number of time constants desired (usually five)

Q15. What percentage of the final value of current has a dc RL circuit reached after 2 time constants of growth?

9-23. Universal Time Constant Chart

Since the growth and decay of current in any LR circuit follows the exponential curve, a curve developed for a specific circuit can be made to apply to any LR circuit by merely changing the values of time and current.

Using the circuit in Figure 9-14 and the general equation (9-14) curves depicting current growth and decay will be developed, and from these curves a universal curve will be made. Figure 9-15 illustrates the specific curves.

Example. Plot the growth and decay curves for current in the circuit of Figure 9-14.

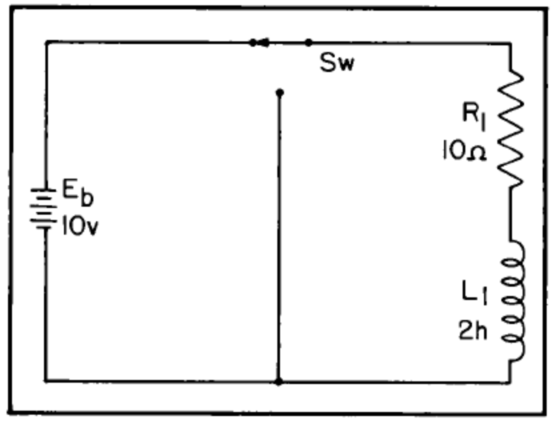

Figure 9-14 - Example LR circuit.

Given: Supply voltage (E_b) = 10 volts

Inductance L = 2 henrys

Resistance R = 10 ohms

Find: Time constant and value of current at time constants one through five.

Solution: Lay out a graph (such as Figure 9-15) and label the X-axis time and the Y-axis current. Divide the time axis into five equal segments, each one representing a time constant.

Using equation (9-13) find the value in seconds of one time constant. Then, progressing from zero write in the value of each time constant.

$$T = \frac{L}{R} \qquad (9\text{-}13)$$

$$T = \frac{2}{10}$$

$$T = 0.2 \text{ second}$$

Next find the maximum circuit current by Ohm's law:

$$I = \frac{E}{R}$$

$$I = \frac{10V}{10}$$

$$I = 1 \text{ amp}$$

Label the maximum current on the graph as one amp and divide the current axis into ten

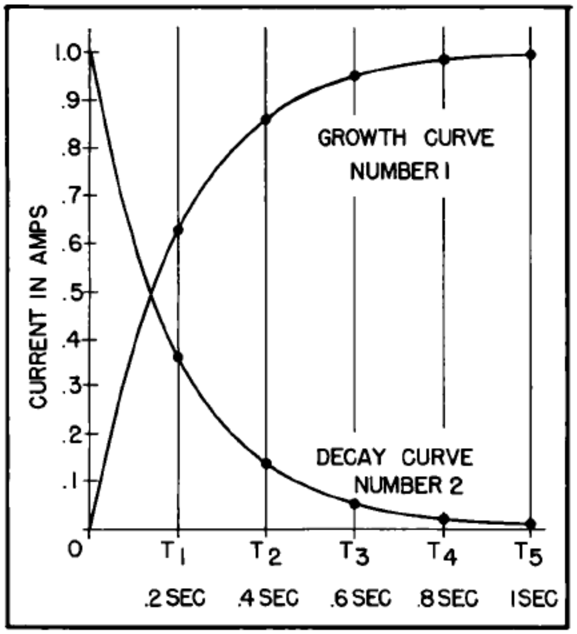

Figure 9-15 - Growth and decay curves for LR circuit.

equal segments, each one representing a tenth of an ampere.

It was established in section 9-22 that in the first time constant (0.2 sec) the current would increase to 63.2 percent of maximum or 0.632 amp. Therefore from the point t = 0.2 sec on the X-axis, follow the vertical line until a point opposite 0.632 amp is reached. Mark a dot at this point. Follow the above procedure using the values of t_2 and t_3 previously established in section 9-22.

$$t_2 = 0.865 \text{ amp}$$

$$t_3 = 0.95 \text{ amp}$$

At t_3 the value of remaining current is:

$$1 - 0.95 = 0.05 \text{ amp}$$

therefore:

$$t_4 = 0.632 \times 0.05$$

$$t_4 = 0.031 \text{ amp}$$

and:

$$I_t = \Delta I_1 + \Delta I_2 + \Delta I_3 + \Delta I_4 \qquad (9\text{-}14)$$

$$I_t = 0.632 + 0.233 + 0.85 + 0.031$$

$$I_t = 0.98 \text{ amp}$$

Plot I_t = 0.98 amp at t4 on the graph. Follow the above procedure for time constant five (t5).

$$I_t \text{ at } t_5 \text{ will equal } 0.99 \text{ amp}$$

Plot this value at t_5 on the graph. It can be seen that the current will continue to approach its maximum value in smaller and smaller increments.

Connecting the plotted values will develop the EXPONENTIAL GROWTH CURVE, number (1). Comparison of this curve with e_R and i of Figure 9-13B will show the validity of the assumption of the gradual decrease of the roc of current as time progresses.

Figure 9-16 illustrates one method by which the rate of change of current (or any quantity) may be determined at a given instant. It is desired to find the approximate rate of change of current (roc) at point one (P_1). A line is drawn tangent to the curve at P_1 and extended until it crosses the Y or current axis. A line is now dropped from P_1 perpendicular to the time axis and the point of intersection is noted. The change in time (Δt) is equal to the difference between the point at which P_1 exists along the time axis and the Y-axis (zero). Thus, Δt_1 is equal to:

$$\Delta t_1 = 0.5 - 0$$

$$\Delta t_1 = 0.5 \text{ second}$$

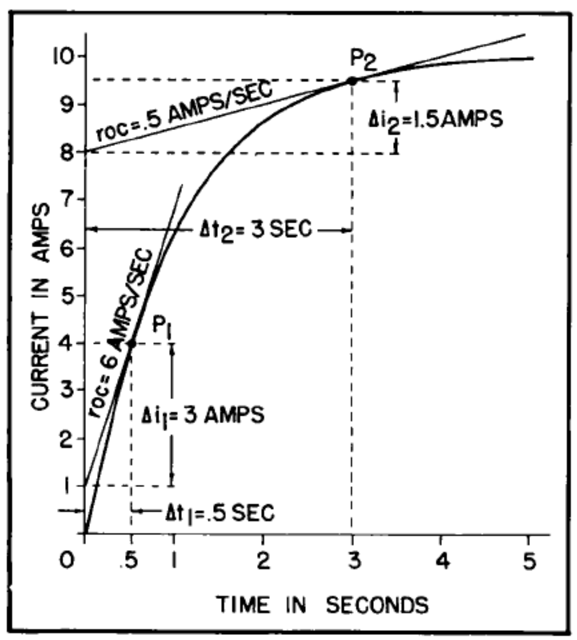

Figure 9-16 - Rate of Change.

A15. 86.5%. 63.2% for the first T plus 63.2% of the remaining 36.8% for the second T or 63.2% + 23.3% = 86.5%.

A line is now projected from P_1 perpendicular to the Y-axis and the value of current at this point is noted. The change in current (Δi) is equal to the difference between this value and the value at which the tangent line crossed the Y-axis. Thus, Δi, is:

$$\Delta i_1 = 4 - 1$$

$$\Delta i_1 = 3 \text{ amps}$$

The rate of change of current at P_1 is now determined by inserting the values in the "equation":

$$roc_i = \frac{\Delta i_1}{\Delta t_1} \quad (9\text{-}15)$$

$$roc_i = \frac{3}{0.5}$$

$$roc_i = 6 \text{ amps per sec}$$

This roc_i applies to point P_1 only. The above procedure is followed to determine the roc_i at point two (P_2).

Therefore: $\Delta i_2 = 9.5 - 8$

$$\Delta i_2 = 1.5 \text{ amps}$$

$$\Delta t_2 = 3 - 0$$

$$\Delta t_2 = 3 \text{ seconds}$$

and:

$$roc_i = \frac{\Delta i_2}{\Delta t_2}$$

$$roc_i = \frac{1.5}{3}$$

$$roc_i = 0.5 \text{ amps per sec}$$

The roc_i at point two is much smaller than at point one and since P_2 is further along the time axis than P_1, the roc_i is seen to be decreasing with time. The slope of the tangent line is an indication of the magnitude of the rate of change of a curve. The steeper the slope the larger the rate of change.

It was also stated that a time constant is the time it takes a current or voltage to decrease to 36.8 percent of its maximum value. Using the same procedure that was used in plotting the growth curve it is possible to plot the DECAY CURVE number (2) of Figure 9-15. In the first time constant interval, the total current will decrease 63.2 percent to an amount equal to 36.8 percent of the maximum value. During each successive time constant interval, the current will continue to decrease by 63.2 percent to an amount equal to 36.8 percent of the value at the beginning of the time interval. Therefore:

$$I_{t1} = 36.8\% \times I_{max}$$

$$I_{t1} = 0.368 \times 1 \text{ amp} = 0.368 \text{ amp}$$

$$I_{t2} = 36.8\% \times I_{t1}$$

$$I_{t2} = 0.368\ (0.368) = 0.135 \text{ amp}$$

$$I_{t3} = 36.8\% \times I_{t2}$$

$$I_{t3} = 0.368\ (0.135) = 0.05 \text{ amp (approx.)}$$

$$I_{t4} = 36.8\% \times I_{t3}$$

$$I_{t4} = 0.368\ (0.05) = 0.02 \text{ amp (approx.)}$$

$$I_{t5} = 36.8\% \times I_{t4}$$

$$I_{t5} = 0.368\ (0.02) = 0.01 \text{ amp (approx.)}$$

From the above example it can be seen that the value of the current for any given time constant interval can also be calculated with respect to the percent of maximum current as follows:

$$I_{t1} = 36.8\% \times I_{max}$$

$$I_{t2} = 13.5\% \times I_{max}$$

$$I_{t3} = 5\% \times I_{max}$$

$$I_{t4} = 2\% \times I_{max}$$

$$I_{t5} = 1\% \times I_{max}$$

Q16. If an LR circuit has a time constant of 20 milliseconds and a resistance of 55 ohms, what is the value of inductance?

The curves of Figure 9-15 have been converted to a UNIVERSAL TIME CONSTANT CHART in Figure 9-17. The chart can be used to indicate the current and voltage relationships at any instant of time in an LR circuit.

CURVE A: Represents inductor growth current or resistor voltage during growth of inductor current (e_Rt_1 through t_3, Figure 9-13B).

CURVE B: Represents inductor current during decay, inductor voltage during inductor current growth (e_Lt_1 through t_3, Figure 9-13B), and resistor voltage during decay of inductor current (e_Rt_4 through t_6, Figure 9-13B).

Chart I presents the magnitude of voltage and current in a series LR circuit of any integral time constant during growth and decay. The time constants give the percentage of the components voltage or current at the end of the period.

To use the chart merely determine the maximum voltage or current for a particular circuit. Then take the percentage figure listed in the time period desired and multiply the maximum by this percentage.

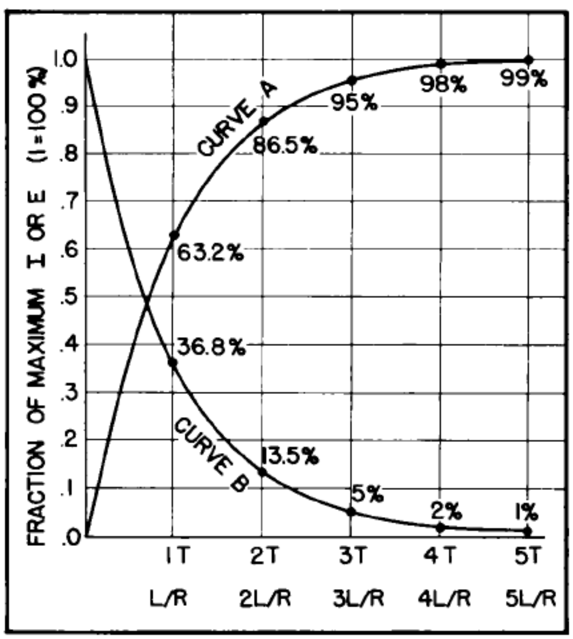

Figure 9-17 - Universal time constant chart.

Example. If the maximum coil voltage is 120 volts and the maximum circuit current is 2.6 amps, find the coil voltage after 3T and the current after 4T of growth.

Solution: Under e_L% in the 3T find the value of 5%.

$$5\% \times e_{Lmax} = 0.05 \times 120 = 6V$$

$$e_L \text{ after } 3T = 6 \text{ volts}$$

Under i% in the 4T find the value 98%.

$$98\% \times i_{max} = 0.98 \times 2.6 = 2.55 \text{ amp}$$

$$i \text{ after } 4T = 2.55 \text{ amp}$$

	GROWTH			DECAY		
T	e_L%	e_R%	i%	e_L%	e_R%	i%
0	100	0	0	100	100	100
1	36.8	63.2	63.2	36.8	36.8	36.8
2	13.5	86.5	86.5	13.5	13.5	13.5
3	5	95	95	5	5	5
4	2	98	98	2	2	2
5	1	99	99	1	1	1

NOTE: Percentages are of maximum values

Chart I - Growth and decay percentages

Q17. Given an LR dc circuit where:
E_b = 130 volts
L = 150 millihenry
R = 200 ohms
Use the universal time constant charts and estimate current after 1.5 time constants.

Q18. Determine the time constant for the circuit of Question 17.

9-24. Variation of Time Constant

The exponential rise of the growth curve is determined by the L/R ratio of the circuit. The effect of a change in inductance or resistance on the time constant should be noted. An inspection of equation (9-13) will show how the time constant is affected by L and R.

$$T = \frac{L}{R} \qquad (9\text{-}13)$$

It can be seen from the equation that the time constant is a direct function of inductance (L) and an inverse function of resistance (R). If the

A16. 1.1 henrys ($L = RT$).

A17. 0.505 amps.

A18. 750 microseconds.

maximum current value changes, due to a change in the voltage applied to the circuit, the curve will rise at a different rate. However, the time required for the current to reach its new maximum will REMAIN THE SAME. This is easily understood if it is remembered that the time constant (T) is a function of the L/R ratio. Changing the current does not change the L/R ratio, hence the time constant T remains the same. The time constant curves for all inductive circuits have the same general shape regardless of the values assigned to R, L, and I.

Q19. What will be the effect on the time constant of doubling the resistance?

9-25. Exponential Formula

If a high degree of accuracy is not required, or if computations are confined to integral time constants, the universal time constant chart may be used. In other words, values for 1.25 or 1.5 time constants may be estimated from the curve to a reasonable degree of accuracy. However, if greater accuracy is required then some other method must be employed. Instantaneous values of voltage and current at any value of time (integral or non-integral) may be determined with reasonable accuracy through the use of the exponential formula. This formula is so named because a graph of the equation produces an exponential curve.

Before proceeding with the explanation and use of the exponential formula, some of the symbols used in conjunction with it will be redefined with more clarity.

Scientists, while dealing with the mathematical analysis of natural growth phenomenon, encountered the number 2.718 with such regularity that a system of logarithms was developed by Napier with 2.718 as the base. In this system of NATURAL OR NAPIERIAN LOGARITHMS the number 2.718 is symbolized by the Greek letter epsilon (ϵ).

The L/R ratio has already been defined as the time constant of the circuit and designated by the capital letter T. This will be reemphasized in the following manner:

$$\frac{L}{R} = T = \text{ONE time constant}$$

In the discussion to follow the lower case letter "t" will be taken to mean ELAPSED TIME and TC will equal the NUMBER of time constants. Expressed mathematically:

$$TC = \frac{t}{T} = \frac{t}{\frac{L}{R}} = \frac{tR}{L} \qquad (9\text{-}16)$$

where: t = elapsed time, in seconds

T = time of one time constant, in sec.

R = resistance of circuit, in ohms

L = inductance in henrys

TC = number of time constants

The curve representing the growth current in a series LR circuit is a graph of the general exponential formula given below:

$$i = I_{max}\,(1 - \epsilon^{\frac{-Rt}{L}}) \qquad (9\text{-}17)$$

Where: i = instantaneous current in amps

I_{max} = Ohm's law value of current (after 5T), in amps.

ϵ = epsilon, the base of natural logarithms, (rounded off to 2.718)

R = resistance of circuit on ohms

t = elapsed time in seconds

L = inductance in henrys

There are various ways of solving for the value of $\epsilon^{\frac{-Rt}{L}}$. This text will use the method most convenient with the tools available, one of the tools being the table of common logarithms in Volume 8. In order to simplify the use of equation (9-17) it may be rearranged as follows:

$$i = I_{max}\,(1 - \epsilon^{\frac{-Rt}{L}}) \qquad (9\text{-}17)$$

By the laws of exponents the negative exponent of epsilon may be made positive by taking its reciprocal:

Therefore: $$i = I_{max}\,(1 - \frac{1}{\epsilon^{\frac{Rt}{L}}}) \qquad (9\text{-}18)$$

Since: $TC = \frac{Rt}{L}$

and: $\epsilon = 2.718$

therefore: $$i = I_{max} \left(1 - \frac{1}{2.718^{TC}}\right) \qquad (9\text{-}19)$$

At this point equation (9-19) could be handled with comparative ease, providing the number of time constants (TC) always consisted of an integral number. For example, raising 2.718 to the 2.78 power might present some difficulty without resorting to the use of logarithms. Using the laws of logarithms equation (9-19) may be further simplified:

$$i = I_{max} \left(1 - \frac{1}{2.718^{TC}}\right) \qquad (9\text{-}19)$$

Multiply the common log of the number (2.718) by the exponent (TC).

$$i = I_{max} \left(1 - \frac{1}{TC \times \log_{10} 2.718}\right)$$

Take the antilog of the product $TC \times 0.4343$ (since the $\log_{10} 2.718 = 0.4343$):

Finally:

$$i = I_{max} \left(1 - \frac{1}{\text{antilog } (TC \times 0.4343)}\right) \qquad (9\text{-}20)$$

where: TC = number of time constants $\left(\frac{Rt}{L}\right)$

0.4343 = common log of epsilon

all other factors are as defined in equation (9-17)

The final simplification of the exponential formula, in the form of equation (9-20) can now be used with only one operation involving the log tables (finding the antilog).

An example problem (Figure 9-18) with an integral time constant will now be solved using the modified exponential formula and the chart of Figure 9-15. This will serve the dual purpose of illustrating the use of the exponential formula and confirming the validity of the universal time constant chart.

Example. In the circuit of Figure 9-18, find the current 0.4 seconds after the switch is closed.

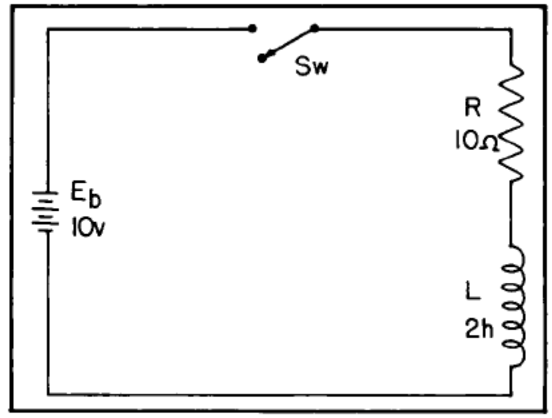

Figure 9-18 - Example circuit.

Given: $E_b = 10V$

$R = 10$ ohms

$L = 2H$

$t = 0.4$ sec

Solution by universal time constant chart:

Determine time constant (T):

$$T = \frac{L}{R} \qquad (9\text{-}13)$$

Substitute: $$T = \frac{2}{10}$$

$$T = 0.2 \text{ sec}$$

Determine number of time constants:

$$TC = \frac{t}{T} \qquad (9\text{-}16)$$

$$TC = \frac{0.4}{0.2}$$

TC = 2 time constants

Determine maximum current value (5T):

$$I = \frac{E}{R}$$

$$I = \frac{10}{10}$$

I = 1 amp

A19. Decrease T by half.

According to Figure 9-17 the current after two time constants is 86.5% of maximum:

$$I_{t2} = 86.5\% \times I_{max}$$

$$I_{t2} = 0.865 \times 1$$

$$I_{t2} = 0.865 \text{ amp}$$

The solution to the problem will now be checked using the exponential formula. All quantities are as given previously:

Determine I_{max}:

$$I = \frac{E}{R}$$

$$I = \frac{10}{10}$$

$$I = 1 \text{ amp}$$

Determine T:

$$T = \frac{L}{R} \qquad (9\text{-}13)$$

$$T = \frac{2}{10}$$

$$T = 0.2 \text{ seconds}$$

Determine TC:

$$TC = \frac{t}{T} \qquad (9\text{-}16)$$

$$TC = \frac{0.4}{0.2}$$

$$TC = 2 \text{ time constants}$$

Determine instantaneous current after two time constants:

$$i = I_{max}\left(1 - \frac{1}{\text{antilog }(TC \times 0.4343)}\right) \qquad (9\text{-}20)$$

Substitute:

$$i = 1\left(1 - \frac{1}{\text{antilog }(2 \times 0.4343)}\right)$$

$$i = 1\left(1 - \frac{1}{\text{antilog } 0.8686}\right)$$

From the log table:

$$i = 1\left(1 - \frac{1}{7.39}\right)$$

$$i = 1\,(1 - 0.1353)$$

$$i = 1\,(0.8647)$$

$$i = 0.8647 \text{ amps}$$

From the above solutions it can be seen that when the number of time constants is a whole number between one and five the universal time constant chart will give a solution very close to that of the exponential formula. However, if the number of time constants turns out to be a number containing a decimal, the solution is simplified considerably by the use of equation (9-20).

Example. Using the circuit in Figure 9-19 find the value of instantaneous current at (a) 7 milliseconds and (b) 42.5 milliseconds after the switch is moved from position (1) to position (2).

Given: $E_b = 90\text{V}$

$R_1 = 12$ ohms

$L = 220$ mh

$t_1 = 7$ m sec.

$t_2 = 42.5$ m sec.

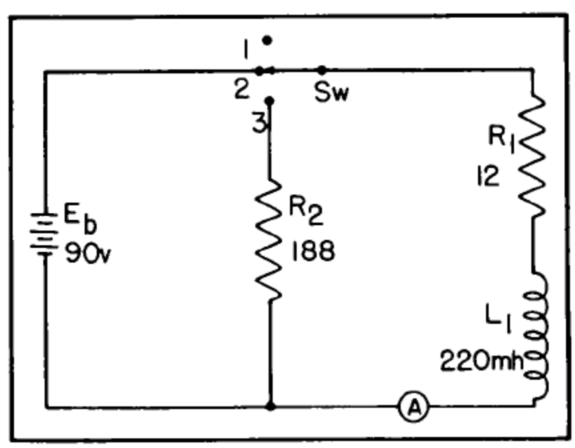

Figure 9-19 - Series LR circuit.

Solution:

Determine I_{max}: $I = \frac{E}{R_1}$

$$I = \frac{90}{12}$$

$$I = 7.5 \text{ amps}$$

Determine T:

$$T = \frac{L}{R} \qquad (9\text{-}13)$$

$$T = \frac{0.22}{12}$$

$$T = .0183 \text{ sec}$$

Determine TC for t_1:

$$TC = \frac{t}{T} \qquad (9\text{-}16)$$

$$TC = \frac{0.007}{0.0183}$$

$$TC = 0.382 \text{ time constants}$$

Determine instantaneous current for t_1:

$$i = I_{max}\left(1 - \frac{1}{\text{antilog }(TC \times 0.4343)}\right) \qquad (9\text{-}20)$$

$$i = I_{max}\left(1 - \frac{1}{\text{antilog }(0.382 \times 0.4343)}\right)$$

$$i = 7.5\left(1 - \frac{1}{\text{antilog }(0.1659)}\right)$$

$$i = 7.5\left(1 - \frac{1}{1.465}\right)$$

$$i = 7.5\,(1 - 0.682)$$

$$i = 7.5 \times 0.318$$

$$i = 2.385 \text{ or } 2.39 \text{ amps}$$

Determine TC for t_2:

$$TC = \frac{t}{T} \qquad (9\text{-}16)$$

$$TC = \frac{0.0425}{0.0183}$$

$$TC = 2.32 \text{ time constants}$$

Determine instantaneous current for t_2:

$$i = I_{max}\left(1 - \frac{1}{\text{antilog }(TC \times 0.4343)}\right) \qquad (9\text{-}20)$$

$$i = 7.5\left(1 - \frac{1}{\text{antilog }(2.32 \times 0.4343)}\right) \qquad (9\text{-}20)$$

$$i = 7.5\left(1 - \frac{1}{\text{antilog }(1.0075)}\right)$$

$$i = 7.5\left(1 - \frac{1}{10.17}\right)$$

$$i = 7.5\,(1 - 0.0983)$$

$$i = 7.5 \times 0.902$$

$$i = 6.765 \text{ amps}$$

Equation (9-20) is just one variation of the exponential formula used in LR circuits. Other variations are listed below for convenience:

EQUATIONS FOR LR CIRCUITS DURING CHARGE:

Growth: $i = I_{max}\left(1 - \frac{1}{\text{antilog}(TC \times 0.4343)}\right)$ (9-20)

Growth: $e_R = E_b\left(1 - \frac{1}{\text{antilog}(TC \times 0.4343)}\right)$ (9-21)

Decay: $e_L = E_b \times \frac{1}{\text{antilog }(TC \times 0.4343)}$ (9-22)

Where: E_b = source in volts

e_R = instantaneous resistor voltage in volts

e_L = instantaneous coil voltage in volts

All other factors as defined previously.

Q20. Given a circuit where: $E_b = 40V$, $R = 30$ ohms, $L = 0.5h$, determine the voltage across the resistance 55.8 m sec after the switch is closed.

A20. $T = \frac{L}{R} = \frac{0.5}{30} = 0.0166$

$$TC = \frac{t}{T} = \frac{0.0558}{0.0166} = 3.36$$

$$e_r = E_b \left(1 - \frac{1}{\text{antilog}(TC \times 0.4343)}\right)$$

$$= 40 \left(1 - \frac{1}{\text{antilog}(3.36 \times 0.4343)}\right)$$

$$= 40 \left(1 - \frac{1}{\text{antilog}(1.459248)}\right)$$

$$= 40 \left(1 - \frac{1}{28.78}\right)$$

$$= 40 \ (1 - 0.0347)$$

$$= 40 \times 0.965$$

$$= 38.6 \text{ volts}$$

EQUATIONS FOR LR CIRCUITS DURING EXPONENTIAL DECAY:

$$i = I_s \frac{1}{\text{antilog } (TC \times 0.4343)} \qquad (9\text{-}23)$$

$$e_R = I_s R \frac{1}{\text{antilog } (TC \times 0.4343)} \qquad (9\text{-}24)$$

$$e_L = I_s R \frac{1}{\text{antilog } (TC \times 0.4343)} \qquad (9\text{-}25)$$

Where: I_s = circuit current at start of decay period, in amps

R = circuit resistance, in ohms

All other factors as defined previously.

Example. In Figure 9-19 the switch was originally moved from position (1) to position (2) for 42.5 milliseconds. If at the end of t_2 in the growth portion of the cycle (42.5 m sec) the switch is thrown to position (3) and left there for (560) microseconds, what will be the reading on the ammeter at this time?

Given: L = 220 mh

R = 200 (R_1+R_2 for decay)

I_s = 6.765 amps (from solution of i at t_2 for growth problem

t = 560 microseconds

Solution: Since the circuit now has a different resistance for decay than it had for growth, the time constant for decay is:

$$T = \frac{L}{R} \qquad (9\text{-}13)$$

$$T = \frac{0.22}{200}$$

$$T = 0.0011 \text{ second}$$

Determine TC:

$$TC = \frac{t}{T} \qquad (9\text{-}16)$$

$$TC = \frac{560 \times 10^{-6}}{11 \times 10^{-4}}$$

$$TC = \frac{560 \times 10^{-2}}{11}$$

$$TC = 0.509 \text{ time constants}$$

Determine instantaneous current:

$$i = I_s \frac{1}{\text{antilog } (TC \times 0.4343)} \qquad (9\text{-}23)$$

$$i = 6.765 \frac{1}{\text{antilog } (0.509 \times 0.4343)}$$

$$i = 6.765 \frac{1}{\text{antilog } 0.221}$$

$$i = 6.765 \frac{1}{1.660}$$

$$i = 6.765 \times 0.603$$

$$i = 4.08 \text{ amps}$$

9-26. Inductive Kick

It has been shown that an inductor in which there is a changing current becomes a source of EMF, and that the direction of this EMF is such that it tends to oppose the change in current producing it. As a result of this action

the current in the inductor does not rise to its full value the instant the switch is closed, but rises at a rate which depends upon the L/R ratio. Likewise when the switch is opened, the source removed, and a short placed across the circuit, the current does not instantaneously fall to zero; it slowly decays at a rate determined by the L/R ratio of the discharge circuit. This action was detailed in sections 9-20 and 9-21.

The circuits so far considered have been composed of ideal components. However, components are not perfect, therefore no switch could be manufactured that would be capable of being moved from one position to another instantaneously. This leads to an explanation of the action of the induced voltage at the INSTANT a switch is opened. Because the magnitude of self-induced voltage can be extremely great even though the source voltage is very low, the development of high induced voltages when an inductive circuit is suddenly opened will now be studied.

Consider the circuit shown in Figure 9-20A consisting of a 6 volt battery, a switch, and a 30 henry coil. The resistor (R) represents the total circuit resistance, including the resistance of the coil. At the instant the switch is closed (time zero), 6 volts of CEMF will develop across the coil. This is shown in Figure 9-20B where e_L = 6V at t_0.

Since at time zero the current, for all practical purposes has not started to flow, there is

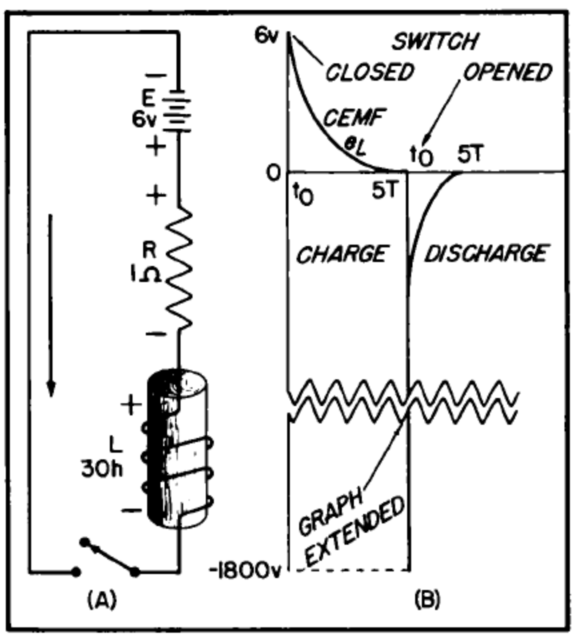

Figure 9-20 - Development of high induced voltage from low source.

no voltage drop across the resistor. With an inductance of 30 henrys and a voltage of 6 volts, the initial rate of change (roc) will be:

$$roc = \frac{e}{L} \qquad (9\text{-}12)$$

$$roc = \frac{6}{30}$$

$$roc = 0.2 \text{ amp/sec.}$$

After the interval of time $(5\frac{L}{R})$ the current has reached its final steady value. For the circuit of Figure 9-20A this will be:

$$T = \frac{L}{R} \qquad (9\text{-}13)$$

$$T = \frac{30}{1}$$

$$T = 30 \text{ seconds}$$

But: steady state value equals:

$$t = 5\frac{L}{R} = 5T$$

Where: t = elapsed time, in seconds

Therefore: steady state will be reached in

$$5 \times T = t$$

$$5 \times 30 = 150 \text{ sec}$$

The magnetic field around the coil is now fully established and steady. Figure 9-20B, shows no e_L (CEMF) at $5\frac{L}{R}$ or 5T), therefore, there must be no change in current at this time.

If the switch is now opened, one side of the battery is disconnected and there is no complete path through which the current can flow. As a result, the current supplied by the battery stops immediately. However, the inductance of the coil opposes the change. Since the current supplied by the battery is no longer available to support the magnetic field, it will start to collapse. As it collapses, the magnetic lines of force cuts the turns of the coil. Whereas expanding lines of force induced an EMF with the polarity on the coil as shown in Figure 9-20A, collapsing lines of force induce an EMF in the coil with the opposite polarity. The inductance is therefore acting as a source

of voltage attempting to maintain the same current flow in the circuit just as if the battery was still connected. It has been stated previously that the decay time will be the same as the growth time if the same L/R ratio is maintained during decay as existed during growth. For an understanding of the change in time constant, picture for a moment, the conditions existing in the circuit shown in Figure 9-21A. The current flowing in this circuit is:

$$I = \frac{E}{R}$$

$$I = \frac{6}{1}$$

$$I = 6 \text{ amps}$$

The voltage drop across the resistor (E_R) is:

$$E_{R1} = IR$$

$$E_{R1} = 6 \times 1$$

$$E_{R1} = 6 \text{ volts}$$

Now if it were possible to maintain the battery at a constant 6 volts and briefly maintain the current at 6 amperes while INSTANTANEOUSLY removing the one ohm resistor (R_1) and inserting a large resistance (R_2), as in Figure 9-21B, the resistive voltage drop would increase:

$$E_{R2} = IR$$

$$E_{R2} = 6 \times 2000$$

$$E_{R2} = 12{,}000 \text{ volts}$$

The above conditions are impossible to obtain in the resistive circuit discussed and are inserted here merely to show the possibility of obtaining a high voltage pulse when exchanging a high resistance for a low resistance and MOMENTARILY maintaining the same current. These conditions can be obtained in the circuit of Figure 9-20A. At the exact INSTANT the switch contacts represent the insertion of an extremely high resistance (open circuit) the action of the inductance is maintaining the current at very near the value built up with the small resistance in the circuit has been increased by a very large amount, the L/R time constant of the circuit for decay is not the original 30 seconds. Due to the insertion of the high resistance, the time constant has become a fraction of a second. Transposing equation (9-3) and substituting values will give an approximation of the voltage developed across the coil by the collapsing magnetic field.

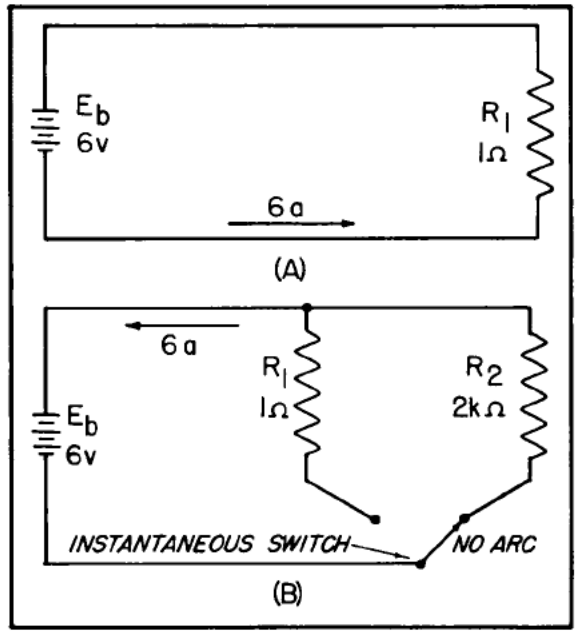

Figure 9-21 - Resistive demonstration circuit for high voltage pulse.

$$L = \frac{e}{\Delta i / \Delta t} \qquad (9\text{-}3)$$

transposing:

$$e = L\frac{\Delta i}{\Delta t}$$

NOTE: Assuming for the purpose of illustration, that 0.00001 second (Δt) after the switch is opened the current has decreased to 5.9994 amps, a change (Δi) of 0.0006 amp:

Substituting: $$e = L\frac{\Delta i}{\Delta t}$$

$$e = 30 \times \frac{0.0006}{0.00001}$$

$$e = 30 \times 60$$

$$e = 1800 \text{ volts}$$

The energy contained in the collapsing magnetic field must be dissipated somewhere within the circuit. The voltage developed in the conductor is sufficient to create an arc across the switch contacts. The energy in the magnetic field is dissipated in the heat of this arc. The energy expended in the arc can seriously burn an individual, damage the switch contacts, or break down the insulation of the coil. For these reasons care should be taken in the abrupt interruption of current in any inductive circuit.

The development of a large voltage pulse from a low voltage source (INDUCTIVE KICK) is not always a disadvantage, but is commonly used in the spark coil circuits (ignition system) of most gasoline engines.

Q21. What is the main reason that the inductive kick appears when a switch is opened in an inductive circuit?

INDUCTANCE AND ALTERNATING CURRENT

9-27. Inductive Reactance

Previously the opposition that an inductance offers to a changing current was called self-induced voltage or CEMF and has been measured in volts. However, opposition to current flow is normally measured in ohms, not in volts. Since a coil reacts to a current change by generating a CEMF, a coil is said to be reactive. The opposition of a coil is therefore called reactance (x) and is measured in ohms. Since more than one kind of reactance exists, the subscript L is added to denote INDUCTIVE REACTANCE. Thus, the opposition offered by a coil to alternating current will be termed inductive reactance, designated X_L.

Q22. Define inductive reactance.

In an alternating current circuit, the roc of the current is determined by the angular velocity of the applied voltage. The unit of measure for angular velocity is radians per second. A complete cycle (or rotation of 360 degrees) contains 2π radians. Angular velocity is designated by the Greek letter omega (ω). Angular velocity is expressed mathematically by the equation:

$$\omega = 2\pi f \qquad (8\text{-}14)$$

The student desiring to refresh his memory of the above to a greater extent is directed to review Chapter 8, sections 8-9 and 8-10.

Since the angular velocity determines the roc of current in an ac circuit the opposition, inductive reactance offered by a coil or inductive circuit will be directly proportional to omega (ω). Furthermore, inductive reactance is a direct function of inductance. The mathematical relationship of the above statements is shown by a very important equation.

$$X_L = 2\pi f L \qquad (9\text{-}26)$$

Where: X_L = inductive reactance in ohms

2π = number of radians in one cycle (6.28)

f = frequency of the applied voltage in cycles per second

L = inductance in henrys

An important statement may now be made. INDUCTANCE DOES NOT CHANGE WITH A FREQUENCY CHANGE. It should be remembered from section 9-13 that a conductor possesses inductance due to its property of opposing a current change. This inductance is constant (for a given configuration) whether there is a change in current or not. However the INDUCTIVE REACTANCE will change with a change in frequency. It was stated previously that a straight piece of wire possesses very little inductance due to minimum flux linkage between various sections of the same conductor. At extremely high frequencies even the small inductance of a piece of wire will exhibit an appreciable amount of inductive reactance. Figure 9-22 illustrates the relationship between frequency, current, and X_L in an ac circuit.

In analyzing Figure 9-22 it is seen that as the ac frequency is increased, the current decreases and the X_L increases. Therefore, it may be said that at high frequencies the inductor approaches the characteristics of an open component. At low frequencies where the current is high and the opposition is low, the inductor approaches the characteristics of a shorted component.

Example. In a purely inductive circuit containing an ac source the following quantities are known. Solve for the inductive reactance and current.

Given: ac source = 100 volts

A21. The roc has been greatly increased. The high resistance of the open switch makes the time constant of the decay circuit MUCH less than that of the growth circuit.

A22. The opposition offered by an inductance to alternating current.

L = 20 millihenrys

f = 60 cps

Find: Current (I) flowing and opposition (X_L):

$$X_L = 2\pi f L \qquad (9\text{-}26)$$

Substitute: $X_L = 6.28 \times 60 \times 2 \times 10^{-2}$

$X_L = 7.536$ ohms

Using Ohm's law for ac inductive circuits:

$$I = \frac{E}{X_L} \qquad (9\text{-}27)$$

Where: I = current in amperes

E = applied voltage, in volts

X_L = inductive reactance in ohms

Substitute: $I = \frac{E}{X_L}$ (9-27)

$I = \frac{100}{7.54}$

I = 13.26 amps

Example. Maintain the ac source voltage and inductance the same as in the previous problem and increase the frequency to 10 kc.

Given: ac source = 100 volts

L = 20 mh

f = 10,000 cps

Find: Current (I) flowing and opposition (X_L):

Solution: $X_L = 2\pi f L$ (9-26)

Substitute: $X_L = 6.28 \times 1 \times 10^4 \times 2 \times 10^{-2}$

$X_L = 6.28 \times 2 \times 10^2$

$X_L = 1256$ ohms

$I = \frac{E}{X_L}$

$I = \frac{100}{1256}$

I = 0.0795 amps

The two problems above are seen to agree with Figure 9-22. In the first problem with a low frequency the current was large (13.26 amps) while the X_L was low (7.5 Ω). Compare this to the second problem with a relatively high frequency. In this case the current decreased (79.5 ma) while the X_L increased (1256 Ω).

Q23. Explain why inductance of a coil does not change with a frequency change.

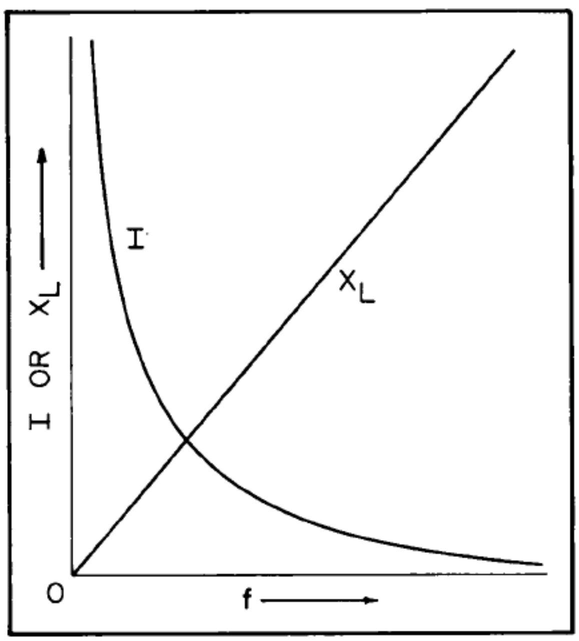

Figure 9-22 - Relationship between I, f, and X_L in an inductive ac circuit.

9-28. Phase Relationships

Any change in current through a coil causes a corresponding change in the magnetic flux around the coil. If the current change is sinusoidal, the induced voltage will also have the form of a sine wave. Because the current changes at its maximum rate when passing through its zero value (at 0°, 180°, and 360°, Figure 9-23B), the flux change is also greatest at those times. Consequently, the self-induced voltage (CEMF) in the coil is at its maximum value at these instants.

Lenz's law states that CEMF always opposes a change in current. Thus, when the current is rising in a positive direction at 0°, the CEMF is of opposite polarity to the applied EMF and opposes the rise in current. When the current is falling toward its zero value at 180°, the polarity of the CEMF is such as to keep the current from falling. Thus the CEMF can be seen to lag the current by 90°. The dc resistance of the coil is small and the principal opposition to the current flow through the coil is the induced voltage (CEMF). The applied voltage vector, E, is slightly larger than the E induced vector, E_{ind}. The reason for this is explained in section 9-20. The two vectors are diametrically opposed as indicated in the vector diagram (Figure 9-23C).

The current lags the applied voltage in an inductive circuit by an angle of 90° and leads the CEMF by an angle of 90°. The induced voltage (CEMF) is always of opposite polarity to the applied voltage.

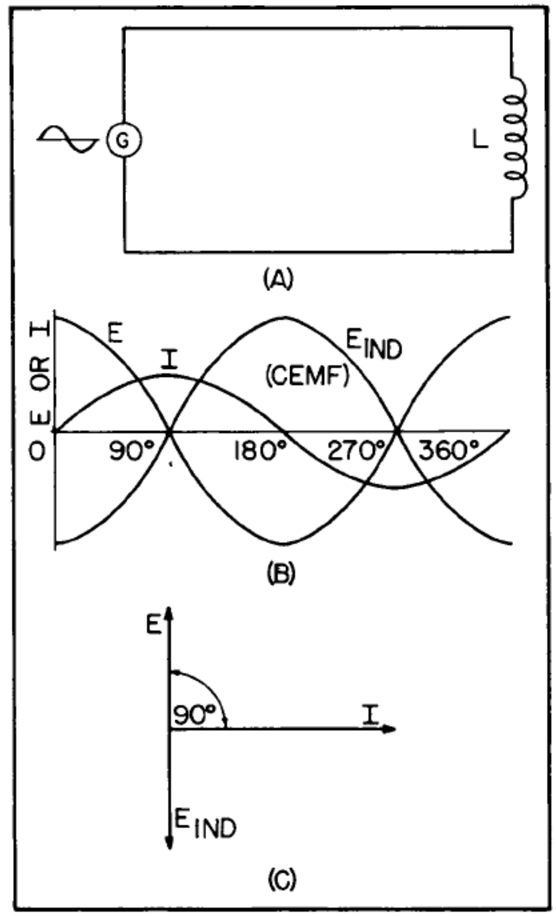

Figure 9-23 - Relation between I, E, and CEMF in an ac inductive circuit.

A23. Inductance is a function of the physical aspects of the coil. It is not dependent on voltage applied or current flow.

EXERCISE 9

1. What will be the action of a magnetic needle placed beside a wire carrying a current? Explain.
2. Why can a coil be assigned a north and south pole?
3. What do the fingers in the left hand rule for conductors indicate?
4. What is indicated by the thumb in the left hand rule for coils?
5. Define flux density. Why does increasing current increase flux density?
6. Would a core with a large or a small cross-sectional area be best if a high flux density is desired?
7. Define electromagnetic induction. Explain in your own words how it is accomplished.
8. Give the conditions which must exist for inducing an EMF.
9. Explain the left hand rule for generators in your own words.
10. What will be the effect on EMF if the direction of the flux lines is reversed?
11. Explain the action of Lenz's law in your own words.
12. What is the effect on induced EMF if the following factors are changed? (a) increase number of conductors, EMF will ________, (b) increase flux, EMF will ________, (c) increase time, EMF will ________.
13. Define CEMF.
14. Define mutual inductance. Explain.
15. Define inductance. Explain.
16. The value of a coil is measured in henrys. Give one method of determining the value of a specific coil.
17. How will increasing the turns of a coil affect inductance?
18. How will increasing the current a small amount affect the inductance of a coil?
19. How does an increase in the coefficient of coupling affect induced EMF?
20. How does the position of coupled coils affect induced EMF?
21. Why are the polarities of two coils containing mutual inductance important?
22. Explain why maximum current does not flow immediately in an LR series circuit.
23. What is the initial rate of change of current in a circuit containing a 1.5 henry coil and where E_{max} equals 25 volts?
24. Define an LR time constant.
25. Determine the time constant of an LR circuit containing a 215 microhenry coil and a 110 ohm resistor.
26. Explain the meaning of: rate of change equals 1.5 amperes per sec.
27. Explain the use of the universal time constant chart and the meaning of curves A and B.
28. Determine the voltage across the resistor of an LR circuit after 4T if the source voltage equals 210 volts.
29. How will the time constant of an LR circuit be affected if both the inductance and the resistance are decreased by half?
30. Why is the exponential formula of more value than the universal curve?
31. Determine the voltage across the resistor of an LR circuit 0.334 milliseconds after the switch is closed in a circuit containing a 150 mh coil, a 300 ohm resistor and a 120 volts source.
32. Determine the circuit current in Question 31.
33. Determine the coil voltage in Question 31.
34. Explain why a spark is formed when the contacts of an inductive circuit are opened.
35. Define inductive reactance.
36. What is the inductive reactance of a 10 henry coil at a frequency of 120 cycles per second?
37. Doubling the frequency of the voltage applied to a coil has what effect on X_L and current?
38. What is the phase of the current through a pure inductance with respect to the voltage across the inductance?
39. What type of core would one expect to find in a 20 henry coil? (air or iron)
40. Is it possible for the voltage across an inductance to exceed the applied voltage? Explain.

CHAPTER 10

CAPACITANCE

Every electronic circuit, no matter how complex, is composed of no more than three basic electrical properties; resistance, inductance, and capacitance. Therefore, a thorough understanding of each of these basic properties is a necessary step toward the understanding of electronic equipment. Since resistance and inductance have been covered, the last of the basic three, capacitance, will now be discussed.

Two conductors separated by a non-conductor exhibit the property called CAPACITANCE, because this combination can store an electric charge. Whereas inductance was defined as a property of a circuit which opposes a change in CURRENT. Capacitance is a property of a circuit which opposes a change in VOLTAGE. Where inductance stored energy in an ELECTROMAGNETIC field, capacitance stores energy in an ELECTROSTATIC field.

The study and comprehension of the effects of capacitance on current and voltage requires that the student be familiar with the use of logarithms and exponents. These subjects are discussed in detail in Volume 8.

10-1. Review of Electrostatics

In order to promote a clear understanding of capacitance, the student should be thoroughly familiar with the theories and laws of electrostatics. For convenience, the main points of electrostatics will be briefly reviewed in this section. A student desiring to refresh his memory to a greater extent is directed to review Chapter 2.

When a charged body is brought into close proximity with another charged body, there is a force that causes the bodies to attract or repel one another. If the charged bodies possess the same sign of charge, a repelling force will exist between the two bodies. If they have unlike signs, there will be a force of attraction between them. The force of attraction or repulsion is caused by the electrostatic field that surrounds every charged body. If a material is charged positively, it has a deficiency of electrons. If it is charged negatively, it has an excess of electrons. The direction of the electrostatic field is represented by lines of force drawn perpendicular to the charged surface and shown originating from the positively charged material. Each line of force is drawn in the form of an arrow and is shown pointing from positive to negative.

The force between charges is described by Coulomb's law: "The force existing between two charged bodies is directly proportional to the product of the charges and inversely proportional to the square of the distance separating them."

If a test charge is inserted in an existing electrostatic field it will move toward one or the other of the charged areas which is causing the field to exist. The direction of movement will depend on whether the test charge is positive or negative. Previously it has been shown that a positive test charge placed in a field moves in the direction that the line of force points, from positive toward negative. In this case the test charge will be an electron and since the electron is negative it will move in a direction opposite to that of the positive charge. In other words, an electron in an electrostatic field will move AGAINST the arrow from negative toward positive. The above action is illustrated by Figure 10-1.

If Coulomb's law is analyzed in connection with Figure 10-1 it can be seen that the greater the distance between the electron and the positive charge the less the force of attraction. The importance of the distance between the charges creating the field will become apparent at a later time in this chapter.

One important characteristic of electrostatic

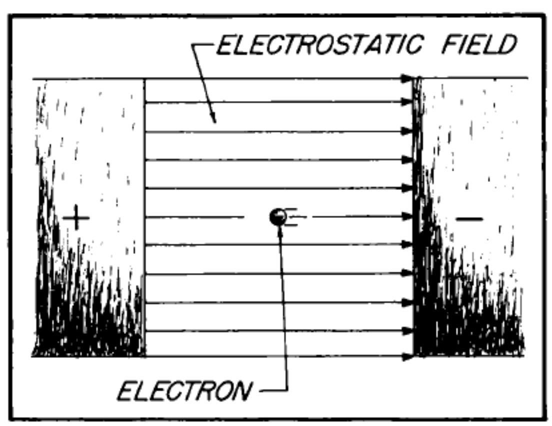

Figure 10-1 - Electron movement in an electric field.

lines of force is that they have the ability to pass through any known material.

10-2. Capacitance

CAPACITANCE is defined as the property of an electrical device or circuit that tends to oppose a CHANGE in VOLTAGE. Capacitance is also a measure of the ability of two conducting surfaces, separated by some form of non-conductor, to store an electric charge. For the present time air will be used as the insulating material between the conducting surfaces.

The device used in electrical circuits to store a charge by virtue of an electrostatic field is called a CAPACITOR. (The larger the capacitor, the larger the charge that can be stored).

The simplest type of capacitor consists of two metal plates separated by air. It has been established that a free electron inserted in an electrostatic field will move. The same is true, with qualifications, if the electron is in a bound state. The material between the two charged surfaces of Figure 10-1 (air in this case) is composed of atoms containing bound orbital electrons. Since the electrons are bound they can not travel to the positively charged surface. Therefore, the resultant effect will be a distorting of the electron orbits. The bound electrons will be attracted toward the positive surface, and repelled from the negative surface. This effect is illustrated in Figure 10-2. In (A) of Figure 10-2, there is no difference in charge placed across the plates; and the structure of the atom's orbits is undisturbed. If there is a difference in charge across the plates as shown in (B) of Figure 10-2, the orbits will be elongated in the direction of the positive charge.

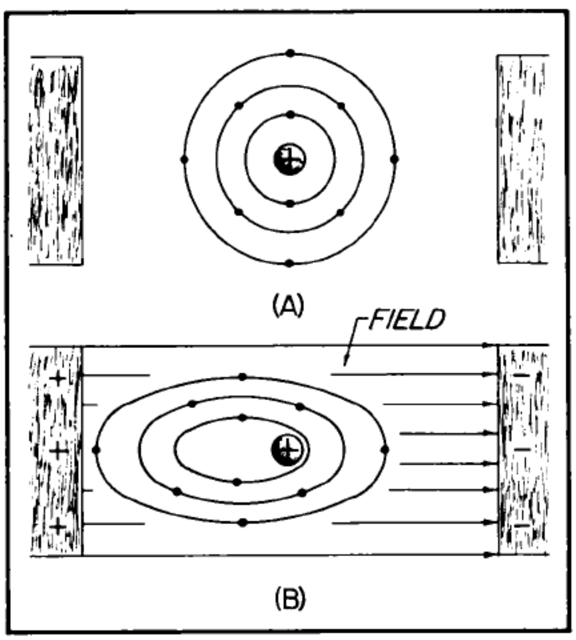

Figure 10-2 - Electron orbits with and without the presence of an electric field.

As energy is required to distort the orbits, energy is transferred from the electrostatic field to the electrons of each atom between the charged plates. Since energy cannot be destroyed, the energy required to distort the orbits can be recovered when the electron orbits are permitted to return to their normal positions. This effect is analogous to the storage of energy in a stretched spring. A capacitor can thus "store" electrical energy. An illustration of a simple capacitor and its schematic symbol is shown in Figure 10-3. The conductors that form the capacitor are called PLATES. The material between the plates is called the DIELECTRIC. In (B) of Figure 10-3 the two vertical lines represent the connecting leads. The two horizontal lines represent the capacitor plates. Notice that the schematic symbol (B) and the simple capacitor diagram (A) are similar in appearance. In a practical capacitor, the parallel plates may be constructed in various configurations (circular, rectangular, etc.); but the cross-sectional area

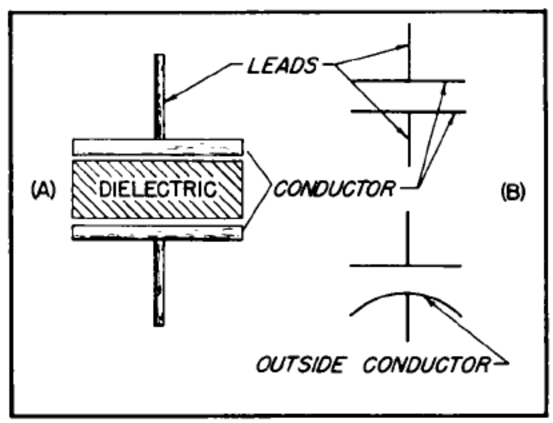

Figure 10-3 - Capacitor and schematic symbol.

of the capacitor plates is tremendously large in comparison to the cross-sectional area of the connecting conductor. This means that there is an abundance of free electrons available in each plate of the capacitor. If the cross-sectional area and plate material of the capacitor plates are the same, the number of free electrons in each plate must be approximately the same. It should be noted that there is a possibility of the difference in charge becoming so large as to cause ionization of the insulating material to occur (cause bound electrons to be freed). This places a limit on the amount of charge that can be stored in the capacitor.

Q1. Explain the effect upon the orbits of an atom if the atom is placed between two metal plates that have an equal positive charge.

10-3. Capacitive Units

Capacitance is measured in a unit called the FARAD. This unit is a tribute to the memory of Michael Faraday, a scientist who performed many early experiments with electrostatics and magnetism.

It was discovered that for a given value of capacitance, the ratio of charge deposited on one plate, to the voltage producing the movement of charge, is a constant value. This constant value is a measure of the amount of capacitance present. The symbol used to designate a capacitor is (C). The capacitance is equal to one farad when a voltage changing at the rate of one volt/per second causes a charging current of one amp to flow. This is expressed by the equation:

$$C = \frac{i}{\frac{\Delta e}{\Delta t}} \qquad (10\text{-}1)$$

Where: C = capacitance, in farads

i = instantaneous current in amps

$\frac{\Delta e}{\Delta t}$ = rate of change of voltage, in volts, with time, in seconds

Equation (10-1) may be clearer if expressed as follows:

$$\text{Capacitance equals one farad when} = \frac{\text{Charging current of one amp flows}}{\text{When voltage changes one volt in one second}}$$

The farad can also be defined in terms of charge and voltage. A capacitor has a capacitance of one farad if it will store one coulomb of charge when connected across a potential of one volt. This relationship can be expressed mathematically as:

$$C = \frac{Q}{E} \qquad (10\text{-}2)$$

Where: C = capacitance in farads

Q = charge in coulombs

E = applied potential in volts

Example. What is the capacitance of two metal plates separated by one centimeter of air, if .001 coulomb of charge is stored when a potential of 200 volts is applied to the capacitor?

Given: Q = .001 coulomb

E = 200 volts

Solution: $$C = \frac{Q}{E} \qquad (10\text{-}2)$$

Converting to power of ten:

$$C = \frac{10 \times 10^{-4}}{2 \times 10^{2}}$$

$$C = 5 \times 10^{-6}$$

$$C = .000005 \text{ farads}$$

Although this capacitance might appear rather small (five millionths of a farad), many electronic circuits require capacitors of much smaller value. Consequently the farad is a cumbersome unit, far too large for most applications. The MICROFARAD which is one millionth of a farad (1×10^{-6} farad) is a more convenient unit. The symbols used to designate microfarad are uf and MFD. In high frequency circuits even the microfarad becomes too large, and the unit MICROMICROFARAD (one millionth of a microfarad) is used. The symbols for micromicrofarads are uuf and MMFD.

To avoid confusion and the use of double prefixes the name PICOFARAD (pf) is preferred in place of micromicrofarad. In powers of ten, one picofarad (or one micromicrofarad) is equal to 1×10^{-12} farad.

In using equation (10-2) one must not deduce the mistaken idea that capacitance is dependent upon charge and voltage. Capacitance is determined entirely by physical factors such as plate area, plate spacing, etc.

Q2. If a capacitor is measured with an ohmmeter and the meter indicates zero ohms, what is the possible trouble? What could have caused this trouble?

Q3. Where is energy stored in a capacitor?

Q4. A capacitor's value is given as 3200 pf. What is its value in microfarads?

Q5. If the voltage across a capacitor is doubled, and the value of the capacitor remains constant, what is the effect upon the charge?

10-4. Charging a Capacitor

In order to better understand the action of a capacitor in conjunction with other components the charge and discharge action of a purely capacitive circuit will be analyzed first. For ease of explanation the capacitor and voltage source used in Figure 10-4 will be assumed to be perfect (no internal resistance, etc.) although this is impossible in practice.

A1. The electrons will be equally attracted by each plate, resulting in the elongation of the atom's orbits.

A2. The plates of the capacitor have become shorted. They may have become shorted because of an arc over in the dielectric.

A3. In the dielectric.

A4. 0.0032 micro farad.

A5. The charge will also double.

In Figure 10-4A an uncharged capacitor is shown connected to a four position switch. With the switch in position 1 the circuit is open and no voltage is applied to the capacitor. Initially each plate of the capacitor is a neutral body, and until a difference of potential is impressed across the capacitor no electrostatic field can exist between the plates.

To CHARGE the capacitor the switch must

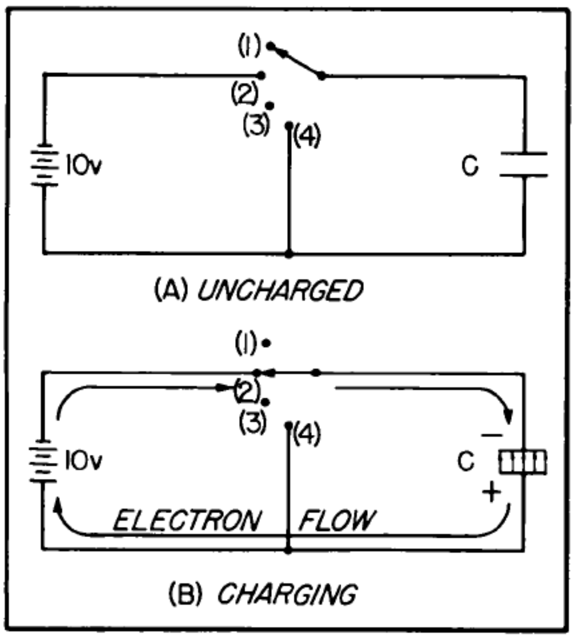

Figure 10-4 - Capacitor action.

be thrown to position 2 which places the capacitor across the terminals of the battery. Under the given conditions the capacitor would reach full charge instantaneously, however, the charging action will be spread out over a period of time in the following discussion so that a step-by-step analysis can be made.

At the instant the switch is thrown to position 2 (Figure 10-4B) a displacement of electrons will occur simultaneously in all parts of the circuit. This electron displacement is directed away from the negative terminal and toward the positive terminal of the source. An ammeter connected in series with the source will indicate a brief surge of current as the capacitor charges.

If it were possible to analyze the motion of individual electrons in this surge of charging current, the following action would be observed. (See Figure 10-5).

At the instant the switch is closed, the positive terminal of the battery extracts an electron from the bottom conductor and the negative terminal of the battery forces an electron into the top conductor. At this same instant an electron is forced into the top plate of the capacitor and another is pulled from the bottom plate. Thus, in every part of the circuit a clockwise DISPLACEMENT of electrons occurs in the manner of a chain reaction.

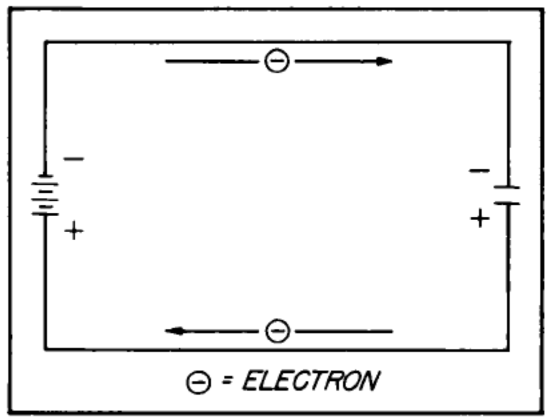

Figure 10-5 - Electron motion during charge.

As electrons accumulate on the top plate of the capacitor and others depart from the bottom plate, a difference of potential develops across the capacitor. Each electron forced onto the top plate makes that plate more negative, while each electron removed from the bottom causes the bottom plate to become more positive. Notice that the polarity of the voltage which builds up across the capacitor is such as to oppose the source voltage. The source forces current around the circuit of Figure 10-5 in a clockwise direction. The EMF developed across the capacitor, however, has a tendency to force the current in a counter-clockwise direction, opposing the source. As the capacitor continues to charge, the voltage across the capacitor rises until it is equal in amount to the source voltage. Once the capacitor voltage equals the source voltage, the two voltages balance one another and current ceases to flow in the circuit.

In studying the charging process of a cap-

acitor it must be emphasized that NO current flows THROUGH the capacitor. The material between the plates of the capacitor must be an insulator.

To an observer stationed at the source or along one of the circuit conductors, the action has all the appearances of a true flow of current even though the insulating material between the plates of the capacitor prevents having a complete path. The current which appears to flow in a capacitive circuit is called DISPLACEMENT CURRENT.

To provide a better understanding of charging action, a capacitor can be compared to the mechanical system in Figure 10-6. Part A of the diagram shows a metal cylinder containing a flexible rubber membrane which blocks off the cylinder. The cylinder is then filled with round balls as shown. If an additional ball is now pushed into the left hand side of the tube, the membrane will stretch and a ball will be forced out of the right hand end of the tube. To

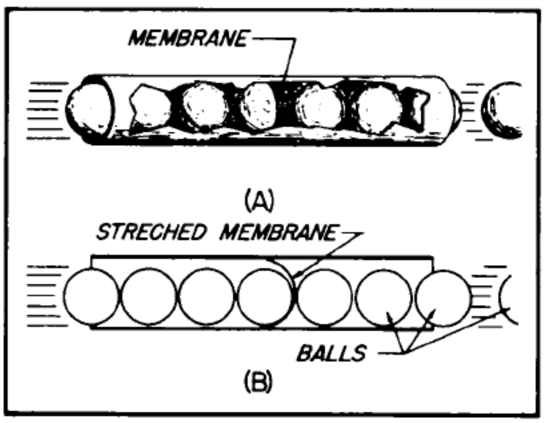

Figure 10-6 - Mechanical equivalent of a capacitor.

an observer who could not see inside the tube the ball would have the appearance of traveling all the way through the tube. For each ball inserted into the left hand side, one ball would leave the right hand side, although no balls actually pass all the way through the tube.

As more balls are forced into the tube it becomes increasingly difficult to force in additional balls, due to the tendency of the membrane to spring back to its original position.

If too many balls are forced into the tube, the membrane will rupture, and any number of balls can then be forced all the way through the tube.

A similar effect occurs in a capacitor when the voltage applied to the capacitor is too high. If an excessive amount of voltage is applied to a capacitor, the insulating material between the plates will break down and allow a current flow through the capacitor. In most cases this destroys the capacitor, necessitating its replacement.

When a capacitor is fully charged and the source voltage is equaled by the CEMF across the capacitor, the electrostatic field between the plates of the capacitor will be maximum. Since the electrostatic field is maximum the energy stored in the dielectric will be maximum.

If the switch is now opened as shown in Figure 10-7A, the electrons on the upper plate are isolated. Due to the intense repelling effect of these electrons, no electrons will return to the positive plate. Thus, with the switch in position 3, the capacitor will remain charged indefinitely. At this point it should be noted that the insulating dielectric material in a practical capacitor is not perfect and a small leakage current will flow through the dielectric. This current will eventually dissipate the charge. A high quality capacitor may hold its charge for a month or more however.

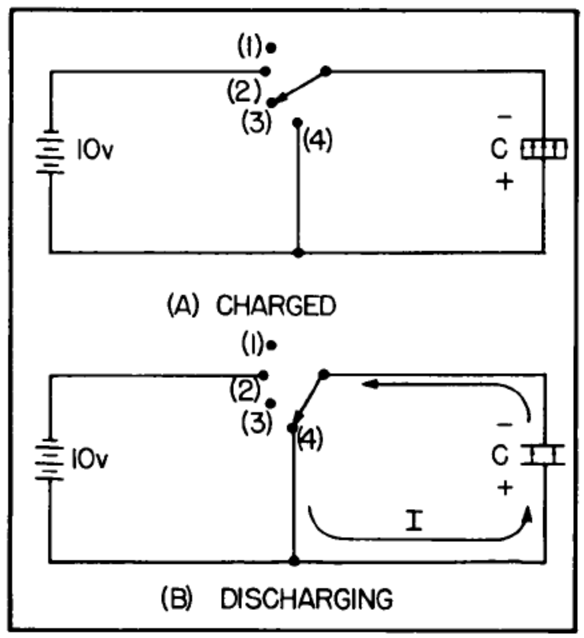

Figure 10-7 - Discharging a capacitor.

To review briefly, when the capacitor is connected across a source, a surge of charging current will flow. This charging current develops a CEMF across the capacitor which opposes the applied voltage. When the capacitor is fully charged the CEMF will be equal to the applied voltage and charging current will cease. At full charge the electrostatic field between the plates is at maximum intensity and the energy stored in the dielectric is maximum. If the charged capacitor is disconnected from the source the charge will be retained for some

period of time. The length of time the charge is retained depends on the amount of leakage current present. Since electrical energy is stored in the capacitor, a charged capacitor can act as a source.

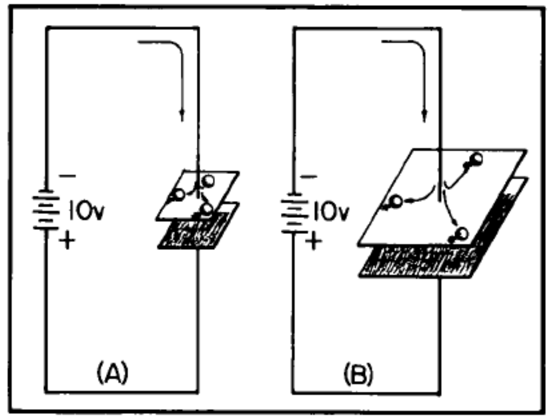

Figure 10-8 - Effects of plate area.

10-5. Discharging a Capacitor

To DISCHARGE a capacitor, the charges on the two plates must be neutralized. This is accomplished by providing a conducting path between the two plates (See Figure 10-7B). With the switch in position 4 the excess electrons on the negative plate can flow to the positive plate and neutralize its charge. When the capacitor is discharged the distorted orbits of the electrons in the dielectric return to their normal positions and the stored energy is returned to the circuit. It is important to note that a capacitor does not consume power. The energy the capacitor draws from the source is recovered when the capacitor is discharged.

FACTORS AFFECTING CAPACITY

10-6. Plate Area

To investigate the relationship between capacitance and plate area, the action of two capacitors having different plate areas will be compared. Figure 10-8A shows a relatively small capacitor to which a potential of 10 volts is applied. As shown in the diagram, three electrons have been forced onto the top plate of the capacitor by the negative terminal of the battery. Since the area of this plate is small the electrons are crowded together and repel each other (and additional electrons from the source as well) with great force. Due to this repelling effect the source has difficulty in forcing additional electrons into the capacitor. Since capacitance is the ratio of charge stored, to voltage applied ($C=Q/E$), this capacitor does not have much capacitance. If the plate area is large such as B of Figure 10-8, the electrons can spread out over the plate, and the pressure (CEMF) developed by a given number of electrons is small. A given value of source voltage can therefore force more electrons onto the large plate than onto the small plate. Thus, the capacitor with the larger plate area has a greater capacity for storing charge. Capacitance is directly proportional to plate area. Doubling the plate area will double the capacitance.

10-7. Plate Spacing

If the distance between the plates of a capacitor is changed the capacitance will change. As the distance between the plates of a capacitor is reduced the capacitance will increase. This can be explained by an examination of Figure 10-9 which shows two capacitors having equal plate areas and unequal plate spacing.

In part A of the figure, the distance between the plates is small. As a result of the close spacing of the plates, the positive charges on the top plate partially neutralize the field surrounding the negative charges on the bottom plate and the source can force more electrons onto the bottom plate. Similarly, the field about the positive charges on the top plate is partially neutralized by the electrons on the bottom plate and the source can remove more electrons from the top plate. When the plates are far apart such as in Figure 10-9, very little neutralization of field take place and the source cannot place as much charge on the capacitor.

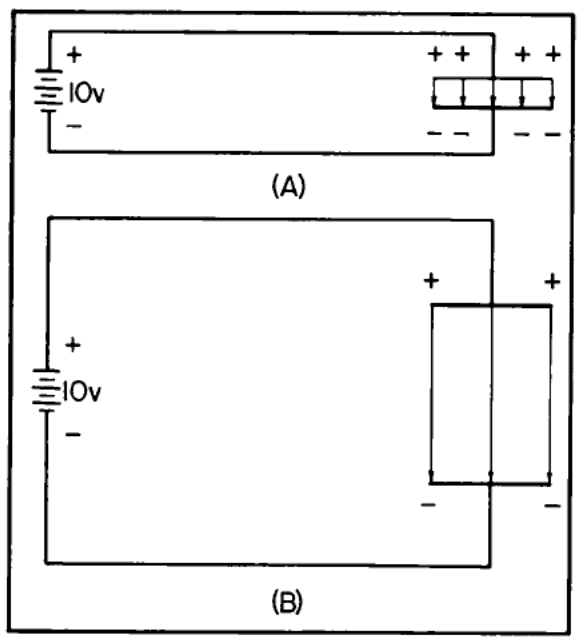

Figure 10-9 - Effects of plate spacing.

This example illustrates the fact that capacitance is inversely proportional to plate spacing. The greater the distance between the plates the smaller will be the capacitance.

10-8. Dielectric Material

The DIELECTRIC MATERIAL is the insulating material used between the capacitor plates. Up to this point in the chapter, air has been used as the dielectric material.

The amount of capacitance contained by a pair of plates is affected to a great degree by the type of dielectric material inserted between the plates. Through experimentation, scientists discovered that a given set of plates exhibit minimum capacitance when the area between the plates contains a vacuum. If a non-conductor, such as glass, is inserted between the plates in place of the vacuum, the capacitance will increase. Some modern ceramic materials can produce a capacitance several hundred times greater than that obtained with a vacuum between the same set of plates.

In order to be able to compare dielectric materials as to their ability to increase capacitance, a number is assigned to each dielectric material. This number, called the DIELECTRIC CONSTANT, tells how many times the material can increase the capacitance as compared to a vacuum dielectric.

The dielectric constants of several materials are listed in Table 10-1. Notice the dielectric constant for a vacuum. Since a vacuum is the standard of reference, it is assigned a constant of one; and the dielectric constants of all materials are compared to that of a vacuum. Since the dielectric constant of air has been

MATERIAL	CONSTANT (K)
Vacuum	1.0000
Air	1.0006
Paraffin Paper	3.5
Glass	5 - 10
Mica	3 - 6
Rubber	2.5 - 35
Wood	2.5 - 8
Glycerine (15°C)	56
Petroleum	2
Pure Water	81

Table 10-1

determined experimentally to be approximately the same as that of a vacuum, the dielectric constant of AIR is also considered to be equal to one. The formula used to compute the value of capacitance using the physical factors just described is:

$$C = 0.2249\left(\frac{KA}{d}\right) \qquad (10\text{-}3)$$

Where: C = capacitance, in picofarads (10^{-12})

A = area of one plate, in square inches

d = distance between plates, in inches

K = dielectric constant of insulating material.

0.2249 = a constant resulting from conversion from Metric to British units.

Example. Find the capacitance of a parallel plate capacitor with paraffin paper as the dielectric.

Given: $K = 3.5$

$d = 0.05$ inch

$A = 12$ square inches

Solution:

$$C = 0.2249\left(\frac{KA}{d}\right) \qquad (10\text{-}3)$$

$$C = 0.2249\left(\frac{3.5 \times 12}{0.05}\right)$$

$$C = 189 \text{ picofarads}$$

Using equation (10-3) it is easy to visualize the effects on capacitance of the physical factors involved. It can be seen that capacitance is a direct function of the dielectric constant and the area of the capacitor plates, and an inverse function of the distance between the plates.

Q6. Compare field intensity with capacitance.

10-9. Types of Capacitors

Capacitors are classified into two general types: VARIABLE and FIXED. Variable capacitors are those that are constructed in a fashion that allows the value of capacitance to be varied over a prescribed range. There are two types of variable capacitors: the ROTOR-STATOR type and the TRIMMER type.

The rotor-stator types use air as the dielectric. The amount of capacitance is varied by changing the position of the rotor plates (movable plates). This changes the effective plate area of the capacitor. When the rotor plates are fully meshed between the stator plates, the capacitance is maximum. The rotor-stator type is illustrated in Figure 10-10. This type of variable capacitor finds wide application in table model radios.

The trimmer type of variable capacitor con-

A6. Capacitance is directly proportional to field intensity.

---

sists of two plates separated by a dielectric other than air. The capacitance is varied by changing the distance between the plates. This is ordinarily accomplished by means of a screw which forces the plates closer together. This type of variable capacitor is shown in Figure 10-11.

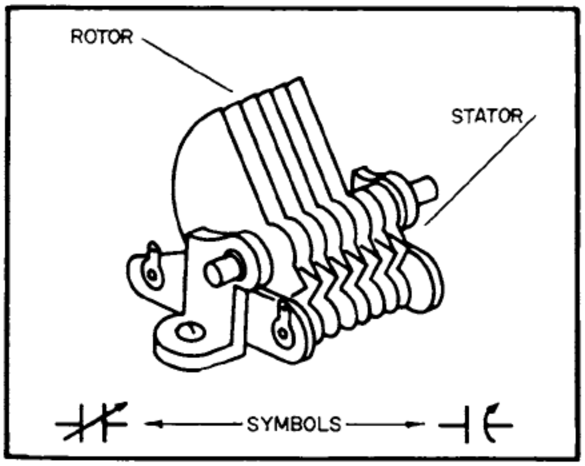

Figure 10-10 - Rotor-stator type variable capacitor.

Fixed capacitors are categorized by the type of dielectric used. The following is a description of some of the more common types of fixed capacitors.

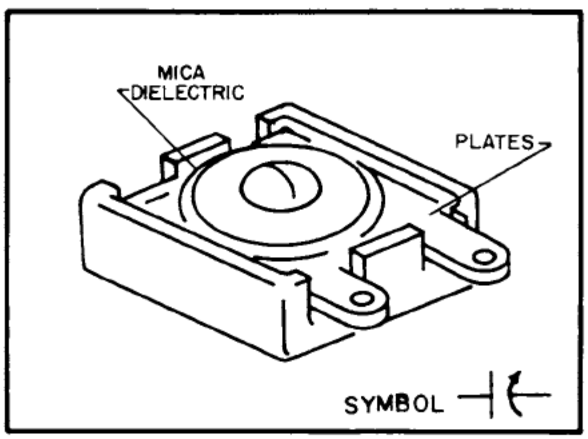

Figure 10-11 - Trimmer capacitor

A PAPER CAPACITOR is one that uses paper as its dielectric. The construction of the typical paper capacitor is shown in Figure 10-12. It consists of flat thin strips of metal foil conductors, separated by the dielectric material. In this capacitor the dielectric used is waxed paper. Paper capacitors usually range in value from about 300 picofarads to about 4 microfarads. Normally, the voltage limit across the plates rarely exceeds six hundred volts. Paper capacitors are sealed with wax to prevent the harmful effects of moisture.

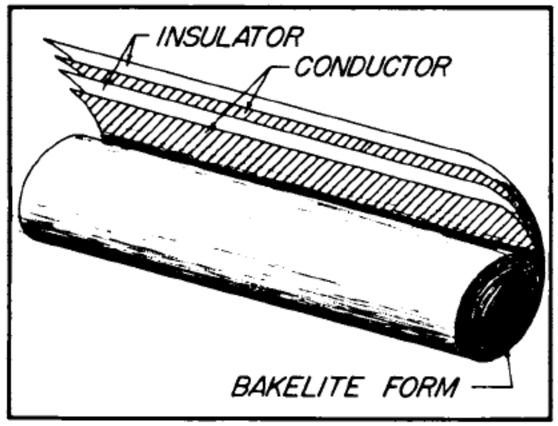

Figure 10-12 - Paper capacitor

MICA CAPACITORS consist of alternate layers of mica and plate material. Their capacitance is of a small value, usually in the picofarad range. Although small in physical size, the mica capacitors have a high voltage handling capacity. Figure 10-13 shows a cutaway view of a mica capacitor.

OIL CAPACITORS are often used in radio transmitters where high output power is desired. Oil-filled capacitors are nothing more than paper capacitors that are immersed in oil. The oil impregnated paper has a high dielectric constant which lends itself well to the production of capacitors that have a high value. Many capacitors will use oil with another dielectric material to prevent arcing between the plates. If an arc should occur between the plates of an oil-filled capacitor, the oil will tend to reseal the hole caused by the arc. These types of capacitors are often called SELF-HEALING capacitors.

CERAMIC CAPACITORS are so named because of the use of ceramic dielectrics. One type of ceramic capacitor uses a hollow ceramic cylinder as both the form on which to construct the capacitor and as the dielectric material. The plates consist of thin films of metal deposited on the ceramic cylinder.

A second type of ceramic capacitor is manufactured in the shape of a disk. After leads are attached to each side of the capacitor, the capacitor is completely covered with an insulating moisture-proof coating. Ceramic capacitors usually range in value between one picofarad and 0.01 microfarad and may be used with voltages as high as thirty thousand volts.

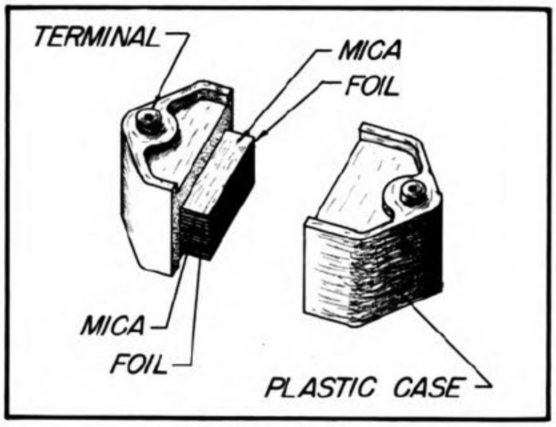

Figure 10-13 - Mica Capacitor (molded type)

ELECTROLYTIC CAPACITORS are used where a large amount of capacitance is required. As the name implies, electrolytic capacitors contain an electrolyte. This electrolyte can be in the form of either a liquid (wet electrolytic capacitor) or a paste (dry electrolytic capacitor). Wet electrolytic capacitors are no longer in popular use due to the care needed to prevent spilling of the electrolyte.

Dry electrolytic capacitors consist essentially of two metal plates between which is placed the electrolyte. In most cases the capacitor is housed in a cylindrical aluminum container which acts as the negative terminal of the capacitor (See Figure 10-14). The positive terminal (or terminals if the capacitor is of the multisection type) is in the form of a lug on the bottom end of the container. The size and voltage rating of the capacitor is generally printed on the side of the aluminum case.

Internally, the electrolytic capacitor is constructed similarly to the paper capacitor. The positive plate consists of aluminum foil covered with an extremely thin film of oxide which is formed by an electrochemical process. This thin oxide film acts as the dielectric of the capacitor. Next to, and in contact with the oxide, is placed a strip of paper or gauze which has been impregnated with a paste-like electrolyte. The electrolyte acts as the negative plate of the capacitor. A second strip of aluminum foil is then placed against the electrolyte to provide electrical contact to the negative electrode (electrolyte). When the three layers are in place they are rolled up into a cylinder as shown in Figure 10-14.

Electrolytic capacitors have two primary disadvantages in that they are POLARIZED, and they have a LOW LEAKAGE RESISTANCE. Should the positive plate be accidentally connected to the negative terminal of the source, the thin oxide film dielectric will dissolve and the capacitor will become a conductor (i. e., it will short). The polarity of the terminals is normally marked on the case of the capacitor. Since electrolytic capacitors are polarity sensitive, their use is ordinarily restricted to dc circuits or circuits where a small ac voltage is superimposed on a dc voltage. Special electrolytic capacitors are available for certain ac applications, such as motor starting capacitors. Dry electrolytic capacitors vary in size from about four microfarads to several thousand microfarads, and have a voltage limit of approximately five hundred volts.

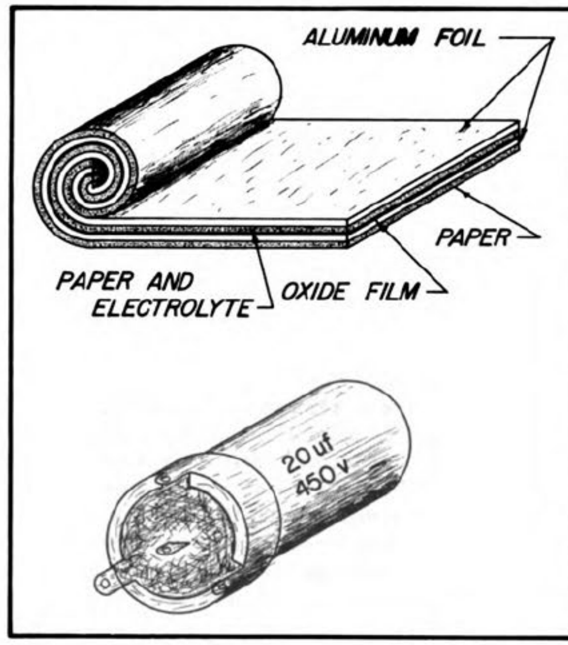

Figure 10-14 - Construction of an electrolytic capacitor.

The type of dielectric used and its thickness govern the amount of voltage that can safely be applied to a capacitor. If the voltage applied to a capacitor is high enough to cause the atoms of the dielectric material to become ionized, an arc over will take place between the plates. If the capacitor is not self-healing, its effectiveness will be impaired. The maximum safe voltage of a capacitor is called its WORKING VOLTAGE and is indicated on the body of the capacitor. The working voltage of a capacitor is determined by the type and thickness of the dielectric. If the thickness of the dielectric is increased, the distance between the plates is also increased and the working voltage will be increased. Any change in the distance between the plates will cause a change in the capacitance of a capacitor. Because of the possibility of voltage surges (brief high amplitude pulses) a margin of safety should be allowed between the circuit voltage

and the working voltage of a capacitor. The working voltage should always be higher than the maximum circuit voltage.

Q7. Compare the capacitance values of a rotor-stator type capacitor when fully meshed and when fully unmeshed. Explain why the values differ.

Q8. What is meant by the term dielectric strength?

Q9. What type of capacitor would you choose to use under the following conditions: ten volt, 0.001 microfarad?

10-10. Color Codes for Capacitors

Although the value of a capacitor may be indicated by printing on the body of a capacitor, many capacitive values are indicated by the use of a color code. The colors used to represent the numerical value of a capacitor are the same as those used to identify resistance values. There are two color coding systems that are currently in popular use: the Joint Army-Navy (JAN) and the Ratio Manufacturers' Association (RMA) systems.

In each of these systems a series of colored dots (sometimes bands) is used to denote the value of the capacitor. Mica capacitors are marked with either three dots or six dots. Both systems are similar, but the six dot system contains more information about the capacitor, such as working voltage, temperature coefficient, etc. Capacitors are manufactured in various sizes and shapes. Some are small tubular resistor like devices, and others are molded rectangular flat components. Figure 10-15 shows some of the more common shapes of capacitors. An explanation of capacitor color codes is provided in Volume 8.

Q10. From the technician's standpoint, why would the six dot color code be preferred over the three dot?

10-11. Capacitors in Series

A circuit consisting of a number of capacitors in series is similar in some respects to one containing several resistors in series. In a series capacitive circuit the same displacement current flows through each part of the circuit and the applied voltage will divide across the individual capacitors.

Figure 10-16 shows a circuit containing a source and two series capacitors. When the switch is closed, current will flow in the direction indicated by the arrows on the diagram.

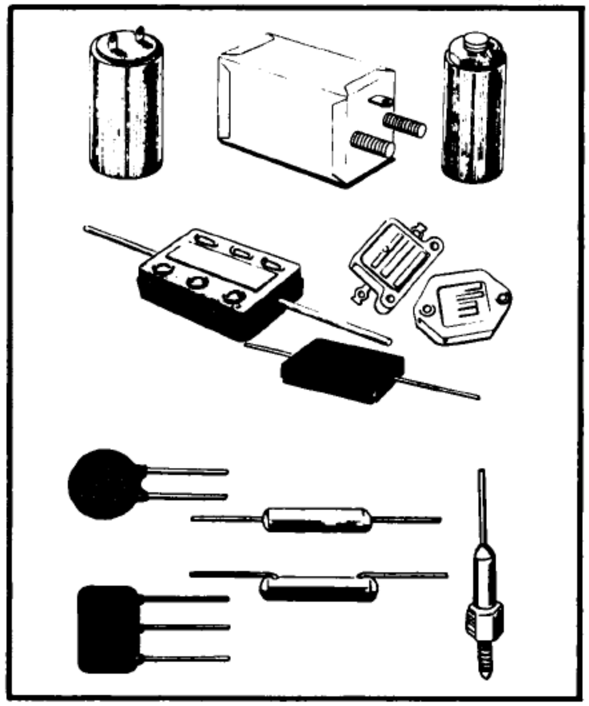

Figure 10-15 - Various shapes of capacitors

Since there is only one path for current, the amount of charge current in motion is the same in all parts of the circuit. This current is of brief duration and will flow only until the total voltage across the capacitors is equal to the source voltage.

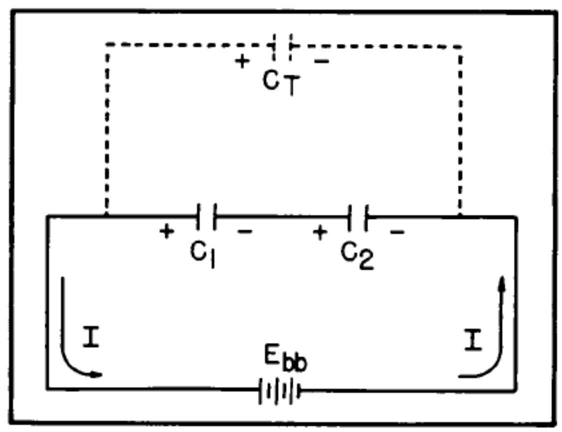

Figure 10-16 - Series Capacitive circuit.

Since the charge (Q) is the same in all parts of the circuit:

$$Q_t = Q_1 = Q_2 = \cdots\cdots Q_n \qquad (10\text{-}4)$$

also:

$$C = \frac{Q}{E} \qquad (10\text{-}2)$$

transposing: $$E = \frac{Q}{C} \qquad (10\text{-}5)$$

Since the sum of the capacitor voltages must equal the source voltage (Kirchhoff's law):

$$E_t = E_1 + E_2 + \cdots E_n \qquad (6\text{-}11)$$

Substituting equation (10-5) into (6-11)

$$\frac{Q_t}{C_t} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \cdots\cdots\frac{Q_n}{C_n} \qquad (10\text{-}6)$$

Since by equation (10-4) all the charges are the same, dividing each term of (10-6) by Q_t yields:

$$\frac{1}{C_t} = \frac{1}{C1} + \frac{1}{C_2} + \cdots\cdots\frac{1}{C_n} \qquad (10\text{-}7)$$

Taking the reciprocal of both sides:

$$C_t = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \cdots\cdots\frac{1}{C_n}} \qquad (10\text{-}8)$$

Where: C_t, C_1, etc. are in farads.

Equation (10-8) is the general equation used to compute the total capacitance of capacitors connected in series. Notice the similarity between this equation and the one used to find equivalent resistance of parallel resistors. If the circuit contains only two capacitors the product over the sum formula can be used.

$$C_t = \frac{C_1 C_2}{C_1 + C_2} \qquad (10\text{-}9)$$

Where: C_t, C_1, etc. are in farads.

As might be anticipated from the above equations, the total capacitance of series connected capacitors will always be smaller than the smallest of the individual capacitors.

Example. Determine the total capacitance of a series circuit containing three capacitors of 0.01 uf, 0.25 uf, and 50,000 pf respectively.

Given:

$C_1 = 0.01$ uf

$C_2 = 0.25$ uf

$C_3 = 50,000$ pf

Solution:

$$C_t = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \qquad (10\text{-}8)$$

$$C_t = \frac{1}{\frac{1}{.01uf} + \frac{1}{.25uf} + \frac{1}{50,000pf}}$$

Converting to powers of ten:

$$C_t = \frac{1}{\frac{1}{1x10^{-8}} + \frac{1}{25x10^{-8}} + \frac{1}{5x10^{-8}}}$$

$$C_t = \frac{1}{100x10^6 + 4x10^6 + 20x10^6}$$

$$C_t = \frac{1}{124 x 10^6}$$

$$C_t = 0.008 \text{ uf}$$

The total capacitance of 0.008 uf is slightly smaller than the smallest capacitor (0.01 uf).

Q11. Explain why the total capacitance of capacitors in series is smaller than the smallest capacitor.

10-12. Capacitors in Parallel

When capacitors are connected in parallel one plate of each capacitor is connected directly to one terminal of the source, while the other plate of each capacitor is connected to the other terminal of the source. In Figure 10-17, since all the negative plates of the capacitors are connected together, and all the positive plates are connected together, C_t appears as a capacitor with a plate area equal to the sum of all the individual plate areas. According to equation (10-3) capacitance is a direct function of plate area. Connecting capacitors in parallel effectively increases plate area and thereby the capacitance.

For capacitors connected in parallel the total charge is the sum of all the individual charges.

$$Q_t = Q_1 + Q_2 + Q_3 + \cdots\cdot Q_n \qquad (10\text{-}10)$$

Transposing formula (10-2):

$$Q = CE \qquad (10\text{-}11)$$

Substitute: (10-11) into (10-10)

$$C_tE = C_1E + C_2E + C_3E$$

Divide both sides by E:

$$C_t = C_1 + C_2 + C_3 + \cdots\cdots C_n \qquad (10\text{-}12)$$

A7. Maximum capacitance when fully meshed, minimum capacitance when fully unmeshed. The effective area is maximum when plates are fully meshed.

A8. It is the measure of a dielectric's ability to sustain an electrostatic field.

A9. Either paper or mica.

A10. The six dot gives more information.

A11. In effect, the distance between the plates is widened, while the area of the plates remains the same.

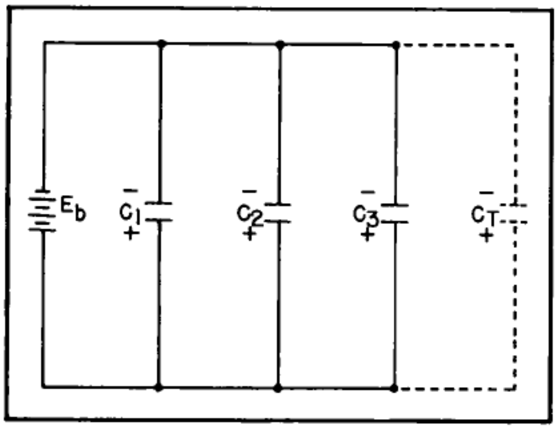

Figure 10-17 - Parallel capacitive circuit.

Where: all capacitances are in the same units.

Example. Determine the total capacitance in a parallel capacitive circuit.

Given: $C_1 = 0.03$ uf

$C_2 = 2$ uf

$C_3 = 0.25$ uf

Solution: $C_t = C_1 + C_2 + C_3$ (10-12)

$C_t = 0.03 + 2 + 0.25$

$C_t = 2.28$ uf

Q12. Explain why the total capacitance of capacitors in parallel is larger than the largest capacitor.

10-13. Series Parallel Configuration

If capacitors are connected in a combination of series and parallel the total capacitance is found by applying equations (10-8) and (10-12) to the individual branches.

Example. Determine the total capacitance of the circuit in Figure 10-18.

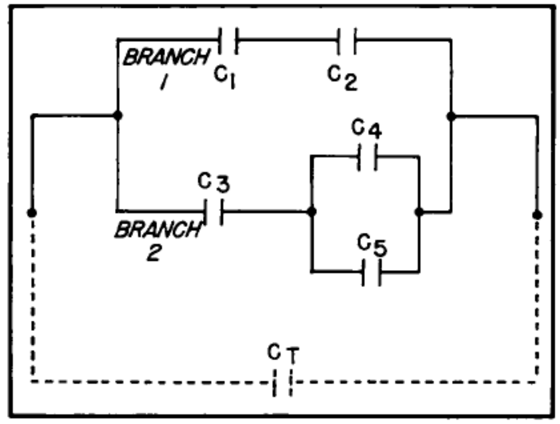

Figure 10-18 - Series parallel capacitance configuration.

Given: $C_1 = 0.06$ uf

$C_2 = 0.002$ uf

$C_3 = 3$ uf

$C_4 = 0.005$ uf

$C_5 = 0.001$ uf

Find: $C_t = ?$

Solution: Simplify the circuit into separate branches.

BRANCH 1 - consists of a series combination C_1 and C_2. To determine total capacitance of this branch only:

$$\text{Branch 1:} \quad C_{t1} = \frac{C_1 C_2}{C_1 + C_2} \qquad (10\text{-}9)$$

$$C_{t1} = \frac{0.06 \times 0.002}{0.06 + 0.002}$$

$$C_{t1} = \frac{0.00012}{0.062}$$

$$C_{t1} = 0.00193 \text{ uf}$$

BRANCH 2 - consists of a series-parallel combination. Solve for total capacitance of Branch 2 only. The equivalent capacitance (C_{eq}) of the parallel combination of C_4 and C_5 must be determined first by the use of equation (10-12) for

parallel configurations:

$$C_{eq} = C_4 + C_5$$

$$C_{eq} = 0.005 + 0.001$$

$$C_{eq} = 0.006 \text{ uf}$$

Branch 2 is now reduced to an equivalent series circuit consisting of C_3 and the equivalent capacitance of C_4 and C_5.

The circuit in Figure 10-18 has been simplified considerably and appears as the circuit in Figure 10-19.

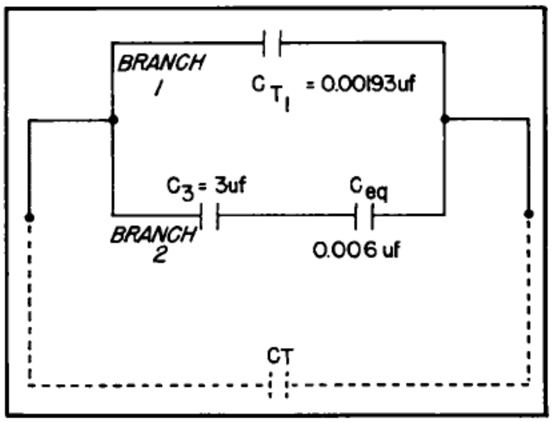

Figure 10-19 - Simplified series - parallel capacitance configuration.

To illustrate the use of the reciprocal method the total capacitance of branch 2 is determined using equation (10-8):

$$C_{t2} = \frac{1}{\frac{1}{C_3} + \frac{1}{C_{eq}}} \qquad (10\text{-}8)$$

$$C_{t2} = \frac{1}{\frac{1}{3} + \frac{1}{0.006}}$$

$$C_{t2} = \frac{1}{0.333 + 166.666}$$

$$C_{t2} = 0.00598 \text{ uf}$$

NOTE: Equation (10-9) could also have been used to solve the previous problems.

The entire problem is now reduced to a simple parallel combination consisting of C_{t1} (capacity branch 1) and C_{t2} (capacity branch 2). Equation (10-12) is now used to determine the otal capacitance of the original circuit.

$$C_t = C_{t1} + C_{t2}$$

$$C_t = 0.00193 + 0.00598$$

$$C_t = 0.00791 \text{ uf}$$

10-14. Series RC Circuit

It has been stated previously that a capacitor will charge to the source potential instantly if there is no resistance in the circuit. In practice all circuits and components contain some amount of resistance. The presence of resistance affects the action of the capacitor.

For ease of explanation all circuit resistances will be lumped together. Figure 10-20 will be used to explain the overall action of a series RC circuit before going into a detailed analysis.

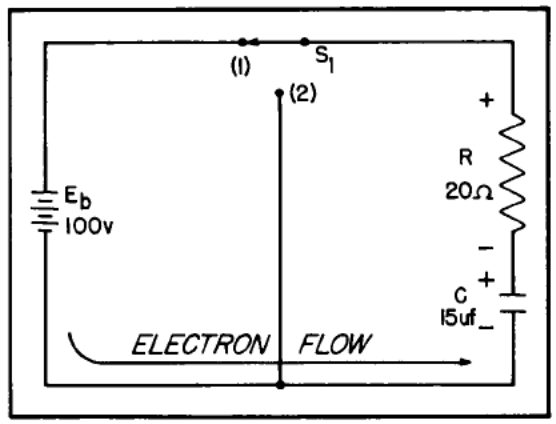

Figure 10-20 - Series RC circuit during charge.

With switch S_1 open, there is no current flow in the circuit and no voltage drop across either component. At the instant switch S_1 is moved to position (1) the current flow in the circuit tends to increase to a very large value because the capacitor, which has no charge at the first instant of time, offers very little opposition to current flow. The only component in the circuit limiting the flow of current is the resistor. The ohmic value of the resistor and the magnitude of the applied voltage will determine what the value of the current will be at the first instant of time. Because the opposition offered to the flow of current by the capacitor is so small at the first instant, little or no voltage will be developed across it and the entire applied voltage will be dropped across the resistor. The voltage drop across the resistor and the value of current flowing in the circuit at this time may be computed through the use of Ohm's law.

$$I = \frac{E}{R}$$

$$I = \frac{100}{20}$$

A12. The plate area of the combination is greater than the plate area of any one capacitor.

$$I = 5 \text{ amps}$$

Therefore the maximum current in the circuit of Figure 10-20 is 5 amps and will occur at the instant the switch is closed. The full source voltage will be dropped across the resistor at this instant. As the charge on the capacitor increases, the opposition offered by the capacitor to the flow of current steadily increases, reducing current. The decreased circuit current will cause the voltage drop across the resistor to decrease. At all times the sum of the voltage drops across the resistor and capacitor must equal the applied voltage (Kirchhoff's law). This is expressed mathematically as:

$$E_b = e_c + e_r \qquad (10\text{-}13)$$

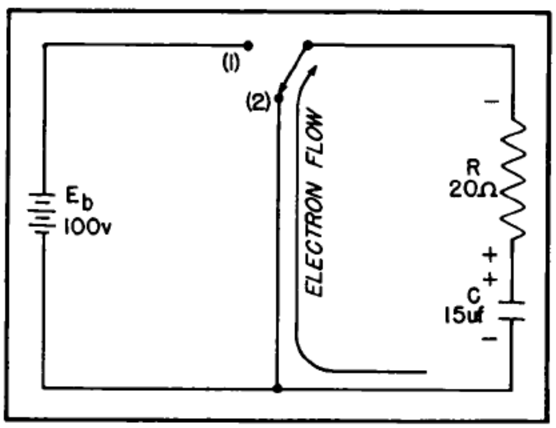

Figure 10-21 - Series RC circuit during discharge.

When the capacitor is fully charged, the circuit current drops to zero, therefore the voltage across the resistor also drops to zero. The capacitor voltage is equal to the applied voltage. Because the presence of the resistor limits the charging current, the capacitor requires a definite amount of time to reach full charge. When the voltage on the capacitor is equal to the source voltage, the EMF of the source is balanced by the CEMF of the capacitor. This leaves no EMF to cause current flow. The circuit will remain in this condition as long as the switch is in position (1).

Figure 10-21 will be used to describe the action at the time when the switch is moved to position (2). When the switch is moved to position (2) the source is removed from the circuit and the charged capacitor becomes the source. At the first instant the capacitor has the same 100 volt potential as the source. The resistance is the same value, therefore the same Ohm's law value of 5 amps, will flow momentarily. A brief instant later the charge of the capacitor will have decreased. By Ohm's law if the resistance remains constant and the voltage decreases then the current must decrease. The resistor, being connected directly across the capacitor, will have a voltage drop which will decrease exactly in step with the capacitor voltage. The discharge action will continue until the capacitor voltage (hence the circuit current and resistor voltage) has decreased to zero. Due to the fact that the same value of resistance was used in the charge path and discharge path, the time for charge and discharge is the same.

10-15. RC Time Constants

It has been mentioned briefly that a definite amount of time is required for a capacitor to charge through a series resistor. The amount of time required to charge a capacitor to 63.2% of its maximum voltage is called the TIME CONSTANT (T) of the circuit. Numerically the time constant of the circuit is equal to the product of the resistance and capacitance.

Expressed mathematically:

$$T = RC \qquad (10\text{-}14)$$

Where: T = one time constant in seconds

R = circuit resistance in ohms

C = circuit capacitance in farads

Example. Determine the time constant of a circuit containing a 15 microfarad capacitor and a 20 ohm resistor.

Given: $R = 20$

$C = 15$ uf

$E_b = 100$ volts

Solution:

$$T = RC \qquad (10\text{-}14)$$

$$T = 20 \times 15 \times 10^{-6}$$

$$T = 300 \times 10^{-6}$$

$$T = 0.3 \text{ millisecond}$$

The time constant (T) of the example problem is three tenths of a millisecond. By the definition of a time constant this means that the capacitor will charge to 63.2% of a hundred volts in 0.3 millisecond. The charge on the capacitor (e_1) after one time constant will be:

$$e_1 = 63.2\% \times E_b$$

$$e_1 = 0.632 \times 100$$

$$e_1 = 63.2 \text{ volts}$$

During the second time constant the capacitor will charge to 63.2% of the remaining value of voltage between E_{max} (in this case E_b) and its present charge. Therefore, in the second time constant the capacitor will gain an additional charge (e_2) of:

$$e_2 = 63.2\% \times (E_b - e_1)$$

$$e_2 = 0.632 \times (100 - 63.2)$$

$$e_2 = 23.26 \text{ volts}$$

The total charge on the capacitor will now equal the charge attained during the first time constant plus the charge accumulated during the second time constant.

$$e_t = e_1 + e_2$$

$$e_t = 63.2 + 23.3$$

$$e_t = 86.5 \text{ volts}$$

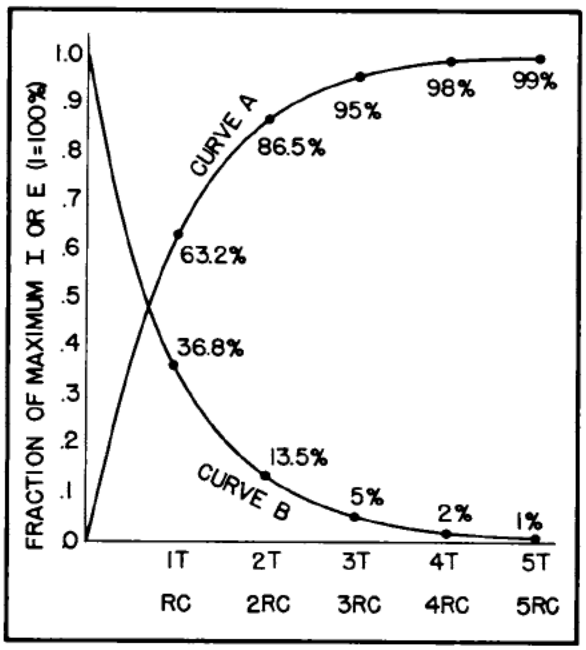

Figure 10-22 - Universal time constant chart.

The charge on the capacitor after two time constants will be 86.5% of maximum or 86.5 volts.

Q13. Determine the time constant of a circuit containing a 200 picofarad capacitor and a 2500 ohm resistor.

10-16. Universal Time Constant Chart

A review of section 9-23 of Chapter 9 will reveal the similarity between RL and RC growth percentages in relation to time. The growth and decay in BOTH inductive and capacitive circuits follows the same exponential curve. The same universal time constant curve that was developed for inductive circuits can be used in capacitive circuits.

Such a chart appears in Figure 10-22.

Curve A: Represents capacitor voltage on charge.

Curve B: Represents circuit current on charge, capacitor voltage on discharge, resistor voltage charge.

Table 10-2 shows the value of voltage and current existing in an RC circuit at the end of each of the first five time constants for both charge and discharge conditions.

	CHARGE			DISCHARGE		
T	e_c%	e_r%	i%	e_c%	e_r%	i%
0	0	100	100	100	100	100
1	63.2	36.8	36.8	36.8	36.8	36.8
2	86.5	13.5	13.5	13.5	13.5	13.5
3	95	5	5	5	5	5
4	98	2	2	2	2	2
5	99	1	1	1	1	1

TABLE 10-2

Charge and Discharge Percentage Chart.

To review the use of the universal curve and the use of the percentage chart, the following example is presented.

Example. Using the circuit in Figure 10-23 find the circuit current and capacitor voltage 30 microseconds after the switch is closed. What is the resistor voltage 120 microseconds after the switch is closed?

Given: C = 2 uf

R = 30 ohms

E_b = 100 volts

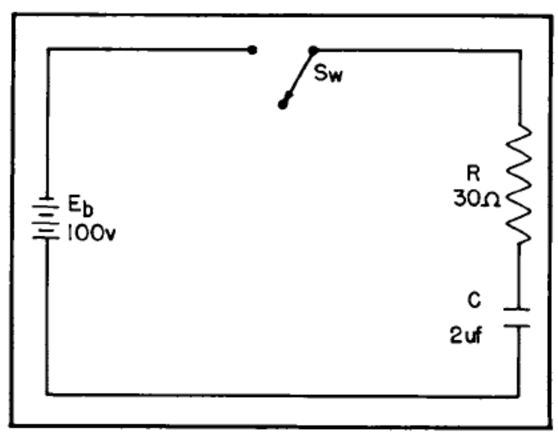

Figure 10-23 - Series RC Circuit.

A13. 0.5 microsecond

Find: $I_m = ?$

$T = ?$

i at t = 30 u sec = ?

e_c at t = 30 u sec = ?

e_r at t = 120 u sec = ?

Where: t = elapsed time

i_m = maximum current

T = time constant

Solution: Since the capacitor offers negligible opposition at the instant the switch is closed, the maximum circuit current will be determined by the source voltage and circuit resistance (R).

Determine I_m:

$$I_m = \frac{E_b}{R}$$

$$I_m = \frac{100}{30}$$

$$I_m = 3.333 \text{ amps}$$

Determine the time constant:

$$T = RC \qquad (10\text{-}14)$$

$$T = 2\times10^{-6}\times30$$

$$T = 60 \times 10^{-6}$$

$$T = 60 \text{ microseconds}$$

Determine the number of time constant using the equation:

$$TC = \frac{t}{T} = \frac{t}{RC} \qquad (10\text{-}15)$$

Where: TC = the number of time constants

T = one time constant, in seconds

t = the elapsed time in seconds

R = resistance in ohms

C = capacitance in farads

$$TC = \frac{t}{T} \qquad (10\text{-}15)$$

$$TC = \frac{30\times10^{-6}}{60\times10^{-6}}$$

$$TC = 0.5 \text{ time constant}$$

Determine the current and capacitor voltage in the circuit after 30 microseconds. From equation (10-15) it was found that 30 microseconds is one-half of a time constant.

To determine circuit current at 0.5T, use curve B of the universal time constant chart. Follow up the 0.5T line until it intersects curve B. This occurs at a Y-axis value of slightly more than 0.6 (actually 0.606). Therefore, current at 0.5T is:

$$i = 60.6\% \times I_m$$

$$i = 0.606 \times 3.33$$

$$i = 2.019 \text{ amps at } 0.5T$$

To determine e_c at 0.5T follow the same procedure but use curve A. Curve A crosses 0.5T at approximately 0.39 (actually 0.3935).

$$e_c = 39.4\% \times E_m$$

$$e_c = 0.394 \times 100$$

$$e_c = 39.4 \text{ volts at } 0.5T$$

To determine resistor voltage after 120 microseconds:

$$TC = \frac{t}{T}$$

$$TC = \frac{120 \times 10^{-6}}{60 \times 10^{-6}}$$

$$TC = 2 \text{ time constants}$$

Use the percentage chart. Follow down the column marked $e_r\%$ for charge to the block opposite 2T. The voltage e_r is then:

$$e_r = 13.5\% \times E_m$$

$$e_r = 0.135 \times 100$$

$$e_r = 13.5 \text{ volts at } 2T$$

Q14. Estimate the percentage of total current after 2.2 time constants of charge.

Q15. What is the difference in meaning of the symbols: T and $\frac{t}{RC}$

10-17. Exponential Formula

With RC circuits (as with RL circuits) the problem of accuracy and ease of computation exists when calculations must be made for a non-integral time constant.

In Chapter 9 a simplified equation (9-20) for current was derived from a general exponential equation (9-17). Simplified equations for capacitive circuits are derived from a general exponential equation in a similar manner. To avoid repetition, only the general and simplified equation will be listed here.

General Exponential Equation:

$$e_c = E_b\,(1 - \epsilon^{\frac{-t}{RC}}) \qquad (10\text{-}16)$$

Where: e_c = instantaneous capacitor voltage in volts

E_b = source voltage in volts

ϵ = epsilon, the base of natural logarithms (2.718)

t = elapsed time in seconds

R = resistance in ohms

C = capacitance in farads

Equation for RC circuits during Charge:

$$e_c = E_b \left(1 - \frac{1}{\text{antilog}\,(TC \times 0.4343)}\right) \quad (10\text{-}17)$$

$$e_r = E_b \frac{1}{\text{antilog}\,(TC \times 0.4343)} \quad (10\text{-}18)$$

$$i = I_m \times \frac{1}{\text{antilog}\,(TC \times 0.4343)} \quad (10\text{-}19)$$

Where: e_c = instantaneous capacitor voltage in volts

e_r = instantaneous resistor voltage in volts

E_b = source voltage, in volts

I_m = maximum current in amps

i = instantaneous current in amps

TC = number of time constants

$0.4343 = \log_{10}\epsilon$

Equations for RC circuits during discharge:

$$e_c = E_c \times \frac{1}{\text{antilog}\,(TC \times 0.4343)} \quad (10\text{-}20)$$

$$e_r = E_c \times \frac{1}{\text{antilog}\,(TC \times 0.4343)} \quad (10\text{-}21)$$

$$i = \frac{E_c}{R} \times \frac{1}{\text{antilog}\,(TC \times 0.4343)} \quad (10\text{-}22)$$

Where: e_r, e_c, TC, R and 0.4343 are as previously defined.

E_c = the capacitor voltage at the start of discharge in volts.

Example. Using the circuit of Figure 10-24 determine the values of e_c, e_r, and i, 1.559 microseconds after the switch is moved from position (1) to position (2).

Given: E_b = 80 volts

R = 38 ohms

C = 0.03 microfarads

t = 1.559

Find: T = ? TC = ? e_r = ? I_m = ? e_c = ? i = ?

Solution: Determine time constant:

$$T = RC$$

$$T = 38 \times 0.03 \times 10^{-6}$$

$$T = 1.14 \times 10^{-6}$$

$$T = 1.14 \text{ microseconds}$$

Determine I_m:

$$I_m = \frac{E_b}{R}$$

$$I_m = \frac{80}{38}$$

$$I_m = 2.11 \text{ amps}$$

Determine number of time constants:

$$TC = \frac{t}{T}$$

$$TC = \frac{1.559 \times 10^{-6}}{1.14 \times 10^{-6}}$$

$$TC = \frac{1.559}{1.14}$$

$$TC = 1.368 \text{ time constants}$$

Summarizing results:

T = 1.14 microseconds

I_m = 2.11 amps

TC = 1.368 time constants

Determine e_c, e_r and i at 1.368 TC:

$$e_c = E_b \left(1 - \frac{1}{\text{antilog}\,(TC \times 0.4343)}\right) \quad (10\text{-}17)$$

$$e_c = 80 \left(1 - \frac{1}{\text{antilog}\,(1.368 \times 0.4343)}\right)$$

$$e_c = 80\,(1 - 0.2541)$$

$$e_c = 59.67 \text{ volts}$$

$$e_r = E_b \times \frac{1}{\text{antilog}\,(TC \times 0.4343)} \quad (10\text{-}18)$$

$$e_r = 80 \times \frac{1}{\text{antilog}\,(1.368 \times 0.4343)}$$

A14. Approximately 10%.

A15. T represents one time constant and $\frac{t}{RC}$ represents the number of time constants.

$e_r = 80 \times 0.2541$

$e_r = 20.328$ volts

$$i = I_m \times \frac{1}{\text{antilog } (TC \times 0.4343)} \qquad (10\text{-}19)$$

$i = 2.11 \times 0.2541$

$i = 0.536$ amps

Summarizing final results for charging circuit.

$e_c = 59.67$ volts

$e_r = 20.328$ volts

$i = 0.536$ amps

Example. Using the circuit of Figure 10-24 and the conditions existing at the end of 1.368 TC of charge, determine e_c, e_r, and i, 0.4 microsecond after the switch is moved to position (3).

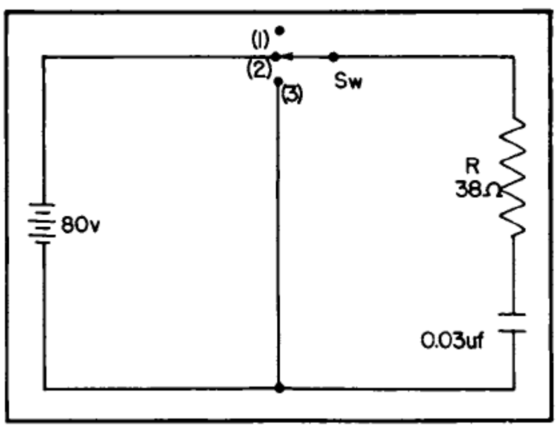

Figure 10-24 - RC circuit for charge and discharge

Given: R = 38 ohms

C = 0.03 microfarads

t = 0.4 microsecond

T = 1.14 microseconds

$E_c = 59.67$ volts

Find: $e_r = ?$ $e_c = ?$ $i = ?$

Solution:

NOTE: The time constant (T) for discharge will be the same as that of charge because the value of resistance and capacitance has not been changed.

$$TC = \frac{t}{T} \qquad (10\text{-}15)$$

$$TC = \frac{0.4 \times 10^{-6}}{1.14 \times 10^{-6}}$$

TC = 0.35 time constant

Determine e_c, e_r, and i:

$$e_c = E_c \times \frac{1}{\text{antilog } (0.35 \times 0.4343)} \qquad (10\text{-}20)$$

$$e_c = 59.67 \times \frac{1}{\text{antilog } (0.35 \times 0.4343)}$$

$$e_c = 59.67 \times \frac{1}{1.42}$$

$e_c = 59.67 \times 0.7046$

$e_c = 42.07$ volts

$$e_r = E_c \times \frac{1}{\text{antilog } (TC \times 0.4343)} \qquad (10\text{-}21)$$

$$e_r = 59.67 \times \frac{1}{\text{antilog } (0.35 \times 0.4343)}$$

$e_r = 42.07$ volts

e_r and e_c are the same because with the switch in position (3) the capacitor is the source for the circuit and since the algebraic sum of the voltage drops must equal the source, then $e_r = e_c$.

$$i = \frac{E_c}{R} \times \frac{1}{\text{antilog } (TC \times 0.4343)} \qquad (10\text{-}22)$$

$$i = \frac{59.67}{38} \times \frac{1}{\text{antilog } (0.35 \times 0.4343)}$$

$i = 1.57 \times 0.7046$

$i = 1.106$ amps

Q16. Determine the elapsed time the switch has been closed in a circuit with the following value: $E_b = 25$ volts, R = 2 ohms, C = 0.05 microfarads and TC = 2.6. Hint: $TC = \frac{t}{T}$

10-18. Effects of Varying R and C

It is apparent from the equation

$$T = RC$$

that the time constant is a direct function of resistance and capacitance. Doubling either R or C will double the time it takes the capacitor to reach 63.2% of its maximum charge. In other words if the source voltage remains the same (Figure 10-25) but either the resistance or the capacitance is doubled, the capacitor will charge to the same final value (10V) but will take twice as long to reach this value. Figure 10-25 illustrates the effect of doubling the resistance.

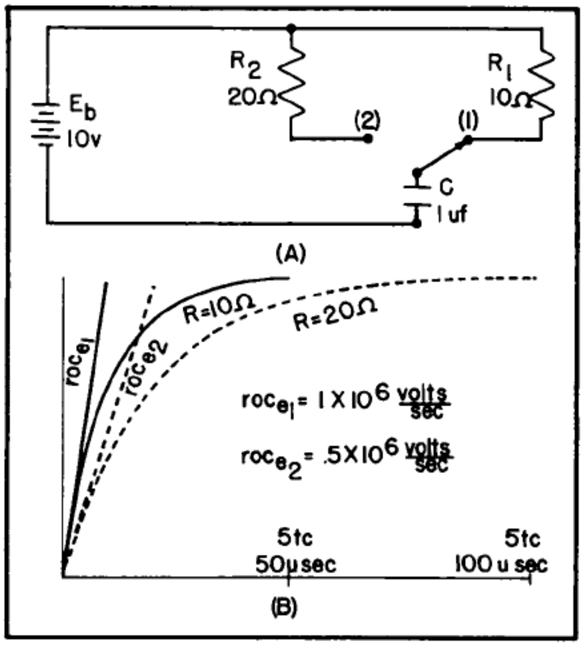

Figure 10-25 - Variation of charging rate due to varying R or C.

When the switch is in position (1) the circuit consists of a 1 microfarad capacitor, a 10 volt source, and a 10 ohm resistor. The charge curve of the circuit under these conditions is shown by the solid line in Figure 10-25B. The initial rate of change (roc) is seen to be equal to one million and is found by the following:

$$C = \frac{i}{\frac{\Delta e}{\Delta t}} \qquad (10\text{-}1)$$

Transposing: $\frac{\Delta e}{\Delta t} = \frac{i}{C}$

Since: $roc_e = \frac{\Delta e}{\Delta t}$

Therefore: $roc_e = \frac{i}{C}$ (10-23)

Where: roc_e = rate of change of voltage per change in unit of time

i = instantaneous current in amps

C = capacitance in farads

Since the initial current is determined by Ohm's law:

$$I_m = \frac{E_b}{R_1}$$

$$I_m = \frac{10}{10}$$

$$I_m = 1 \text{ amp}$$

Then the rate of change at the instant the switch is closed will be:

$$roc_e = \frac{i}{C} \qquad (10\text{-}23)$$

$$roc_e = \frac{1}{1 \times 10^{-6}}$$

$$roc_e = 1 \times 10^6 \text{ volts/sec}$$

NOTE: Maximum current will flow at the first instant.

The full charge time (5T) will be 50 microseconds.

If the capacitor is discharged and the switch moved to position (2) the circuit will contain a 20 ohm resistor. The resistance has been doubled. By equation (10-14) it can be seen that the time constant has been doubled, therefore the full charge time (5T) will be doubled. The dotted line in Figure 10-25B represents the circuit when R = 20 ohms and the full charge time is seen to be 100 microseconds.

If the circuit takes longer to reach the same charge it would seem that the rate of change should be affected. The manner in which the rate of change is affected is shown by the following. The initial current (which is also I_m) will now be:

$$I_m = \frac{E_b}{R_2}$$

$$I_m = \frac{10}{20}$$

$$I_m = 0.5 \text{ amp}$$

The initial roc_e will be:

$$roc_e = \frac{i}{C} \qquad (10\text{-}23)$$

$$roc_e = \frac{0.5}{1 \times 10^{-6}}$$

A16. 0.26 microsecond

$$roc_e = 0.5 \times 10^6 \text{ volts per sec.}$$

The rate of change has decreased to half its former value and is shown in Figure 10-25B by the dotted line marked roc_{e2}.

The above action would be the same if the resistance were maintained constant and the capacitance were doubled. An important point is illustrated by this example.

The roc DECREASES when the charge time INCREASES hence roc is an inverse function of charge time, and therefore an inverse function of the values of R and C.

Rate of change is not only dependent on the charge time, but on the value of source voltage as well.

Figure 10-26 illustrates the effect of an increased source voltage on rate of change.

With the switch in position (1) the circuit consists of a 10 volt source, a 20 ohm resistor and a one microfarad capacitor. The initial current will be 0.5 amp and the charge time (5T) will be 100 microseconds. As illustrated by the dotted lines in Figure 10-26B, the roc will be 0.5 million volts per second.

If the source voltage is doubled but the values of resistance and capacitance is held constant, the full charge time (5T) will remain the same. The capacitor will now have to charge to a higher voltage in the same amount of time. The effect on the rate of change is shown by the following:

$$I_m = \frac{E_b}{R}$$

$$I_m = \frac{20}{20}$$

$$I_m = 1 \text{ amp}$$

Since the initial current has increased, the rate of change will be:

$$roc_e = \frac{i}{C} \qquad (10\text{-}23)$$

$$roc_e = \frac{1}{1 \times 10^{-6}}$$

$$roc_e = 1 \times 10^6 \text{ volts per sec.}$$

It can be seen that the rate of change has increased to one million volts/sec. This is shown by the solid lines in Figure 10-26B. Increasing the source voltage caused the rate of change to increase. Therefore rate of change is a direct funtion of applied voltage.

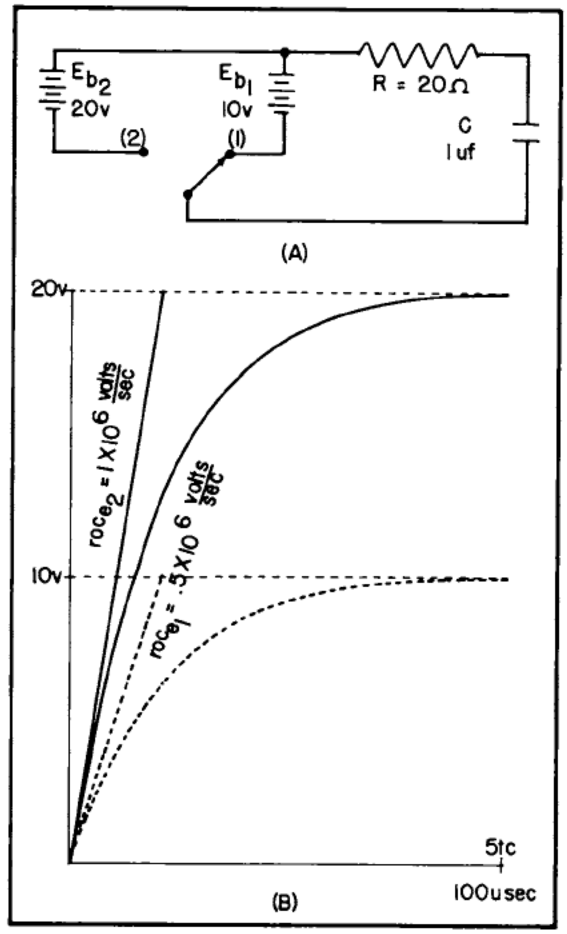

Figure 10-26 - Variation of charging rate due to varying source voltage.

At a future time circuits will be studied which depend on the actions just discussed for proper operation.

Q17. Explain the effect on roc of doubling both R and C.

10-19. Capacitive Reactance

The definition of capacitance was stated as the ability to OPPOSE a change in applied voltage. It has been shown that when the applied voltage is changed the capacitor charges or discharges until the voltage on the capacitor is equal to the new value of applied voltage. At the time when the capacitor voltage is equal to the source voltage no more current flows. Since a capacitor reacts to a voltage change by producing a CEMF, a capacitor is said to be reactive. The opposition of a capacitor is

therefore called REACTANCE (X) and is measured in ohms. In order to distinguish capacitive reactance from inductive reactance (X_L) the subscript c is added to the symbol X. The opposition offered by a capacitor to alternating current is termed CAPACITIVE REACTANCE and designated by X_C.

Although no current actually flows through a capacitor, CIRCUIT CURRENT will exist whenever a capacitor charges or discharges. If a capacitor is connected across an alternating voltage source, an alternating current will flow as the capacitor tries to charge and discharge in step with the voltage. If a sine wave of voltage is applied to a capacitor a sine wave of current will result. Since current in a capacitive circuit is maximum when the rate of change of voltage is maximum, the current waveform will be offset 90 degrees from the voltage waveform. This is illustrated in Figure 10-27A. Notice that when the voltage is passing through zero (maximum rate of change) the current is maximum. When the voltage is at its peak value (minimum rate of change) the current is zero, thus, IN A CAPACITIVE CIRCUIT THE CURRENT LEADS THE VOLTAGE BY 90 DEGREES. This phase relationship is shown vectorially in Figure 10-27B.

It was stated in Chapter 8, section 8-9 and 8-10, that a complete cycle (or rotation of 360 degrees) contains 2π radians and that radians per second is the unit of measure for angular velocity (ω). Expressed mathematically:

$$\omega = 2\pi f \qquad (8\text{-}14)$$

Capacitive reactance is an inverse function of angular velocity and capacitance. Expressed mathematically:

$$X_c = \frac{1}{2\pi f C} \qquad (10\text{-}24)$$

where: X_c = capacitive reactance in ohms

2π = a constant

f = frequency in cycles

C = capacitance in farads

Example. What is the capacitive reactance of a two microfarad capacitor at a frequency of 4 kilocycles?

Given: $2\pi = 6.28$

f = 4 kilocycles

C = 2 microfarads

Solution: $$X_c = \frac{1}{2\pi f C} \qquad (10\text{-}24)$$

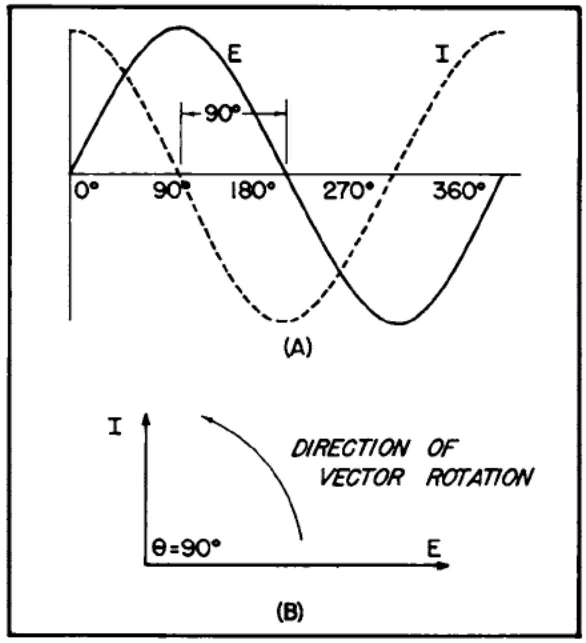

Figure 10-27 - Phase relations of E and I in a pure capacitive circuit.

$$X_c = \frac{1}{6.28\,(4\times10^3)(2\times10^{-6})}$$

X_c = 19.8 ohms

Capacitive reactances in series or parallel are computed in much the same way as resistances in series or parallel. If two capacitors each possessing twenty-five ohms of reactance are connected in series, the total reactance is fifty ohms. If the same two reactances are connected in parallel, the total reactance would be twelve and one half ohms. In equation form:

For series: $$X_{ct} = X_{c1} + X_{c2} + \cdots X_{cn} \qquad (10\text{-}25)$$

For parallel:

$$X_{ct} = \frac{1}{\frac{1}{X_{c1}} + \frac{1}{X_{c2}} + \cdots \frac{1}{X_{cn}}} \qquad (10\text{-}26)$$

Example. Using the circuit of Figure 10-28 determine the total capacitive reactance between points X and Y. Use the same method of analysis as would be used for resistors.

Solution: Determine X_{ceq} between X_{c2} and X_{c3}:

A17. Rate of change would decrease. Doubling both components would increase the charge time (5T) to four times as long.

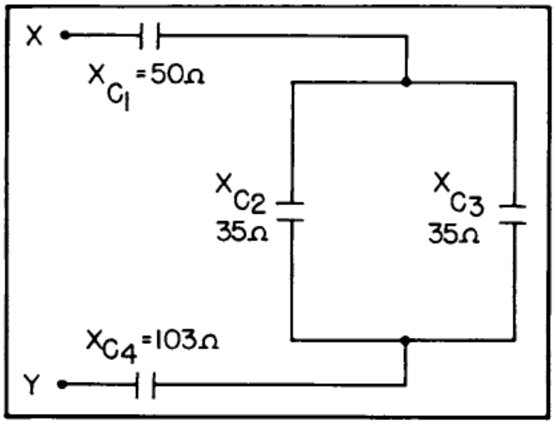

Figure 10-28 - Series-parallel capacitive network.

$$X_{ceq} = \frac{1}{\frac{1}{X_{c2}} + \frac{1}{X_{c3}}} \qquad (10\text{-}26)$$

$$X_{ceq} = \frac{1}{\frac{1}{35} + \frac{1}{35}}$$

$$X_{ceq} = 17.5 \text{ ohms}$$

Determine X_{ct} between X and Y.

$$X_{ct} = X_{c1} + X_{c4} + X_{ceq}$$

$$X_{ct} = 50 + 103 + 17.5$$

$$X_{ct} = 170.5 \text{ ohms}$$

Figure 10-29 illustrates the relationships between capacitive reactance, current, and frequency in an ac capacitive circuit.

From analyzing Figure 10-29, it can be seen that as frequency increases X_c decreases and current increases. At high frequencies a capacitor has a low opposition to current. Thus, as frequency is increased a capacitor's characteristics approach those of a short circuit. On the other hand as the frequency is decreased the opposition (X_C) increases and the current decreases. At the point marked zero frequency (represents dc) the capacitor exhibits the characteristics of an open circuit, i. e., extremely high opposition and no circuit current.

Q18. Explain why current will decrease when frequency is decreased in a capacitive circuit.

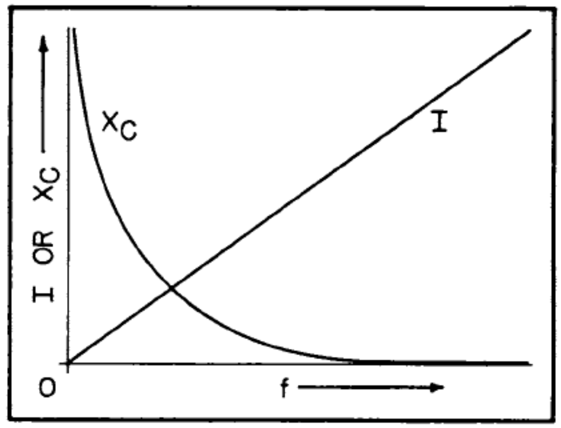

Figure 10-29 - X_C, I, and F relationships in a capacitive ac circuit.

10-20. Capacitive Circuit Analysis

As stated previously the capacitive reactance in an ac circuit can be treated in much the same way as resistances. To solve a capacitive network the same general laws and equations are used, except that in some cases the notation in the formulas are changed to comply with the quantities under consideration. For example, Ohm's law for a capacitive circuit is expressed mathematically as:

$$I_c = \frac{E_C}{X_C} \qquad (10\text{-}27)$$

In the following network the circuit will be solved using Thevenin's theorem. If a review of the application of this theorem is needed the reader is directed to section 7-17.

Example. Determine the voltage across and the current through the capacitive load (X_{c4}) in the circuit of Figure 10-30A.

Given: $C_1 = X_{c1} = 20$ ohms

$C_2 = X_{c2} = 30$ ohms

$C_3 = X_{c3} = 8$ ohms

$C_4 = X_{c4} = 5$ ohms

$E_a = 50$ volts

Solution: Remove the load (X_{c4}) and compute the open circuit voltage (Thevenin's voltage E_{th}). The circuit will appear as in Figure 10-30B. Since X_{c1} and X_{c2} now form a capacitive voltage divider across E_a, E_{Xc2} (which is equal to E_{th}) can be found by a variation of the voltage divider formula:

$$E_L = \frac{E^{th}R_L}{Z^{th}+R_L} \qquad (7\text{-}10)$$

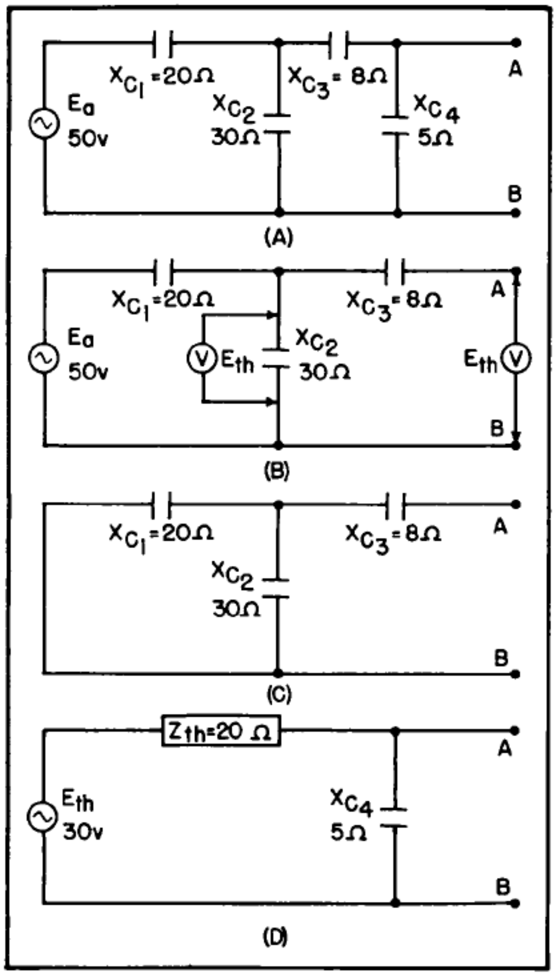

Figure 10-30 - Capacitive circuit analysis.

Determining values:

$$E_{th} = \frac{E_a X_{c2}}{X_{c1}+X_{c2}}$$

$$E_{th} = \frac{50 \times 30}{20 + 30}$$

$$E_{th} = 30 \text{ volts}$$

Replacing E_a with a short (because E_a is the voltage generator) and computing the total reactance (Z_{th}) looking back from terminals A and B. The equivalent circuit appears in Figure 10-30C.

$$Z_{th} = X_{c3} + \frac{X_{c1} X_{c2}}{X_{c1}+X_{c2}}$$

$$Z_{th} = 8 + \frac{20 \times 30}{50}$$

$$Z_{th} = 8 + 12$$

$$Z_{th} = 20 \text{ ohms}$$

Reconnecting the load with Z_{th} and E_{th} as in Figure 10-30D the circuit may now be solved using the voltage divider equation.

$$E_L = \frac{E_{th} X_{c4}}{Z_{th}+X_{c4}}$$

$$E_L = \frac{30 \times 5}{25}$$

$$E_L = 6 \text{ volts}$$

The load current may now be found by use of Ohm's law (ac):

$$I_L = \frac{E_L}{X_{c4}}$$

$$I_L = \frac{6}{5}$$

$$I_L = 1.2 \text{ amps}$$

10-21. Power in a Capacitive Circuit

With no voltage applied to the capacitor the electrons in the dielectric maintain normal orbits. When a potential difference is applied the electron orbits are elongated toward the positive charge. When the polarity of the potential reverses, the electrons will distort their orbits toward the opposite plate. If a sine wave voltage is applied between the capacitor plates the orbital electrons will oscillate back and forth in a direction parallel to the electrostatic lines of force.

Figure 10-31A shows a capacitive circuit and Figure 10-31B indicates the sine wave of charging current, applied voltage, and instantaneous power. The effective voltage is 70.7 volts. The effective current is 7.07 amperes. Because the circuit is purely capacitive the phase angle between current and voltage is 90°.

The equation for power is:

$$p = e \times i$$

Where: p = instantaneous power in watts

e = instantaneous voltage in volts

i = instantaneous current in amps

A18. X_C (opposition) increases.

The power curve in Figure 10-31B is the result of plotting all the instantaneous ei products through the cycle.

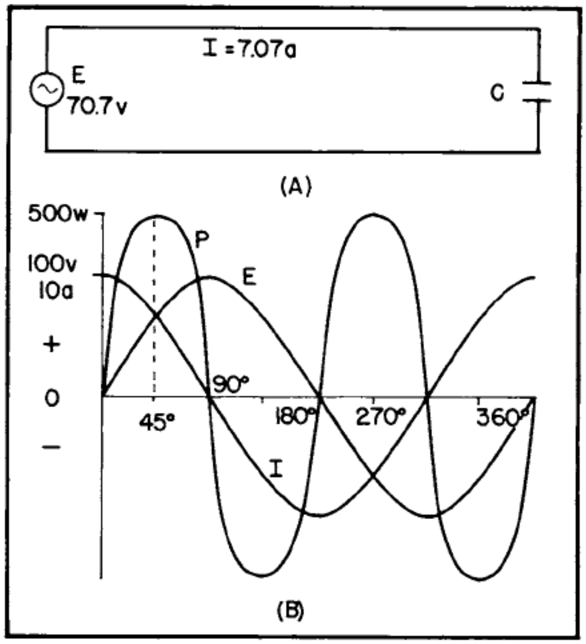

Figure 10-31 - E, I, and P relationships in a capacitive circuit.

At 0° the current is 10 amperes but the voltage is zero. Therefore, the power is:

$$p = e \times i$$

$$p = 0 \times 10$$

$$p = 0 \text{ watts}$$

At 45° the voltage is 70.7 volts while the current is 7.07 amps. The power at 45° is therefore:

$$p = e \times i$$

$$p = 70.7 \times 7.07$$

$$p = 499.849 \text{ VARS}$$

This process is continued until the power curve is graphed. It will be noticed that there are equal amounts of power on the negative and positive side of the reference line. It will also be noticed that the "negative power" portion of the cycle takes place during the times that E is decreasing toward zero (capacitor discharging).

The "negative power" represents energy returned to the circuit by virtue of the elongated electron orbits returning to normal. Since there is as much negative power as positive power, this is a lossless circuit and the average power is zero. This means that A CAPACITIVE CIRCUIT DOES NOT DISSIPATE POWER.

EXERCISE 10

1. Describe the physical construction of a simple capacitor.
2. What would happen to the value of a capacitor if the plate area were increased?
3. What would happen to the value of a capacitor if the distance between the plates were decreased?
4. Describe a dielectric material.
5. Name four common dielectric materials.
6. What is the dielectric constant of a material?
7. What type of energy is stored in a capacitor?
8. In what physical part of a capacitor is energy stored?
9. Two parallel plates have a capacitance of 200 picofarads. If the capacitance is found to be 900 picofarads after a sheet of insulating material is inserted between the plates what is the dielectric constant of the insulating material?
10. What two factors determine the working voltage of a capacitor?
11. How much charge is stored in a 0.1 ufd capacitor charged to 200 volts?
12. How much voltage is present across a 0.01 ufd capacitor containing a charge of 0.001 coulomb?
13. What is the total capacitance of a series circuit containing two capacitors of 2 and 4 microfarads respectively?
14. What is the total capacitance of a parallel circuit containing two capacitors of 10 and 30 microfarads respectively?
15. If capacitors of 2 and 4 microfarads respectively are connected in series across 120 volts dc, find the charge on each capacitor, the total charge, and the voltage across each of the two capacitors.
16. If capacitors of 2 and 4 microfarads respectively are connected in parallel across 120 volts dc, find the charge on each capacitor, the total charge, and the voltage across each capacitor.
17. When capacitors are connected in series, in what proportion does the voltage divide across the individual capacitors?
18. When capacitors are connected in parallel, how does the charge divide between the capacitors?
19. A molded mica capacitor is marked from left to right in the following colors: red, blue and green. What is the capacitance range and the working voltage?
20. A tubular ceramic capacitor is marked with six bands which are colored from left to right: blue, white, yellow, silver, yellow, and black. What is the capacitance range and the working voltage?
21. What is meant by the term "time constant" of an RC circuit?
22. How many time constants are assumed to be necessary to fully charge a capacitor?
23. What percentage of the maximum voltage will appear across a capacitor at the beginning of the third time constant?
24. What is the time constant of a series circuit containing series resistors of 47 and 33 kilohms respectively and a capacitance of 0.05 microfarad?
25. Describe the voltage change which occurs across the resistor as the capacitor in a series RC circuit accumulates a charge.
26. What value of resistance is required to provide a time constant of 200 microseconds in a circuit containing a 100 picofarad capacitor?
27. What value of resistance is necessary to limit the maximum charging current of a 10 microfarad capacitor to 10 amperes, when charging from a 200 volt source?
28. A two hundred and fifty ohm resistor and a twenty microfarad capacitor are connected in series. What is the voltage across the resistor 0.001 seconds after voltage is applied from the source?
29. A resistor of six thousand ohms and a capacitor of two microfarads are connected in series. What is the total current after 0.0006 second has elapsed? What is the voltage across the capacitor after 0.002 second?
30. Explain the meaning of capacitive reactance.
31. What effect does source frequency have on capacitive reactance?
32. Compute the capacitive reactance of a 100 picofarad capacitor at a frequency of 200 kilocycles.
33. At what frequency would a 0.005 microfarad capacitor have a reactance of 1000 ohms?
34. A certain capacitor has a reactance of 5000 ohms at a frequency of 600 kilocycles. What would be its reactance at a frequency of 200 kilocycles?
35. What would be the total reactance of two 0.05 microfarad capacitors connected in series across a 200 cycle source? How much voltage would appear across each capacitor if the source potential were 60 volts?
36. What is the equivalent reactance of six 0.03 microfarad capacitors connected in parallel at a frequency of 1 kilocycle?
37. What is the total reactance of a circuit in which the applied voltage is 100 volts and the source current is 40 ma?

38. What is the absolute minimum working voltage required for a capacitor which is to be connected across a 120 volt ac line?
39. What is the phase relationship between current and voltage in a capacitive circuit?
40. Does a capacitor consume power? Explain.
41. What is leakage current?
42. Name two factors which cause a power loss in a practical capacitor.
43. Determine the voltage across (X_{cL}) in Figure 10-32 by any method (Norton's, Thevenin's, Ohm's law, etc.)

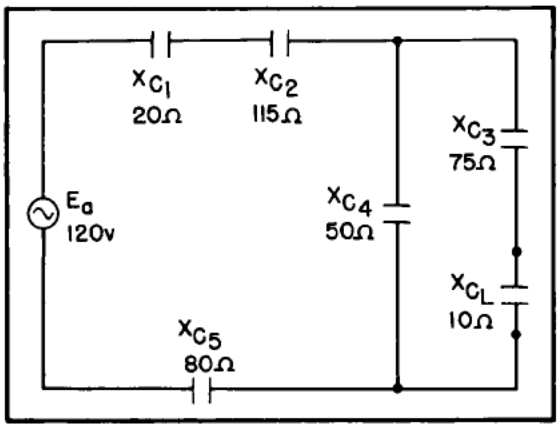

Figure 10-32

CHAPTER 11

SERIES CIRCUITS AND RESONANCE

ı previous chapters the properties of R, C, nd L have been analyzed on an individual basis. ince few electronic circuits exist which contain nly one type of component there will normally e a combination of at least two and usually ll three of these basic circuit elements. The ffects produced when an alternating voltage sine wave) is applied, to series circuits is onsidered in this chapter.

The chapter will commence with a brief reiew of resistive, inductive and capacitive lternating current circuits. Next, an RCL eries configuration will be completely ana-yzed. Finally, non-resonant and resonant onditions will be discussed for the series RCL ircuit.

During the analysis of these circuits, exensive use will be made of graphing procedures, omplex numbers and the trigonometric func-.ons. These subjects are treated in detail in 'olume 8.

Computations for a resistive ac circuit are arried out in a fashion very similar to the rocedure used for a dc circuit. All of the laws eveloped for series dc circuits can be applied › series ac circuits. However, ac circuit nalysis requires a study of certain concepts 'hich do not apply to dc circuits.

Before proceeding, a few points concerning ıe measurement of voltage and current in ac ircuits will be restated.

NSTANTANEOUS VALUE: The voltage or curent that exists in a circuit at any instant of .me is called the instantaneous voltage or curent. Instantaneous values of voltage and curent are indicated by a lower case e and i respect-vely.

'EAK VALUE: The maximum instantaneous alue of voltage that occurs in one cycle is desgnated E_p or E_{max}. If the quantity considered ; current, the designation is I_p or I_{max}.

'EAK TO PEAK VALUE: Alternating current r voltage is sometimes described in terms of peak to peak value. This value is measured rom the peak of the positive portion of the cycle › the peak of the negative portion of the cycle and is designated E_{p-p} or I_{p-p}.

AVERAGE VALUE: The average value of a sine wave of voltage or current is taken to be the average of one alternation of the sine wave. Average values are designated E_{avg} or I_{avg}.

EFFECTIVE (RMS) VALUE: The effective value of voltage or current in an ac circuit (sine wave of voltage) is equivalent to the value of dc needed to cause an equal amount of heating in a resistance. Effective values are designated E or I.

Figure 11-1 illustrates a comparison between the various values of a sine wave.

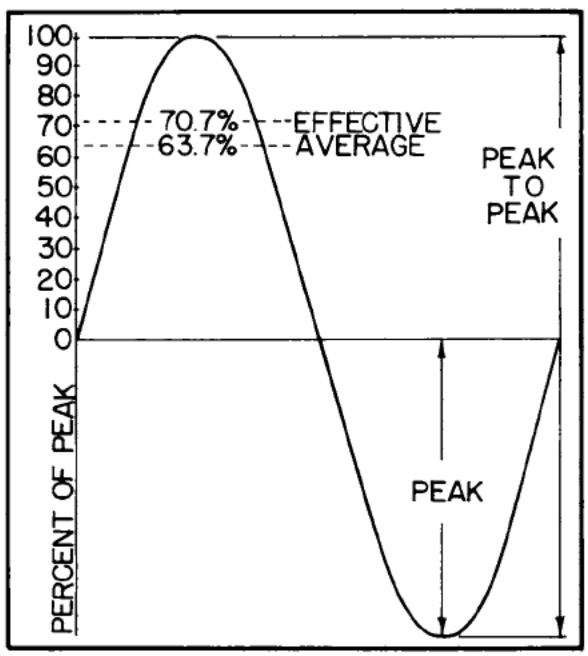

Figure 11-1 - Various values of a sine wave.

The student desiring to refresh his memory on alternating current is directed to review Chapter 8.

RESISTIVE AC CIRCUITS

<u>11-1. i, e and R</u>

Figure 11-2A shows a basic ac circuit in which a resistor (R) is connected across an ac

source (E_p). Since the circuit contains only resistance, the current will at all times be in step with the voltage. To illustrate that the voltage and current are in phase, a sine wave diagram is shown in Figure 11-2B. This diagram shows that as the applied voltage increases, the current through the resistor and hence, the voltage across the resistor increases. At every instant the current and voltage pass through corresponding points of their respective cycles. In other words when the applied voltage is maximum, the resistor current and voltage will be maximum.

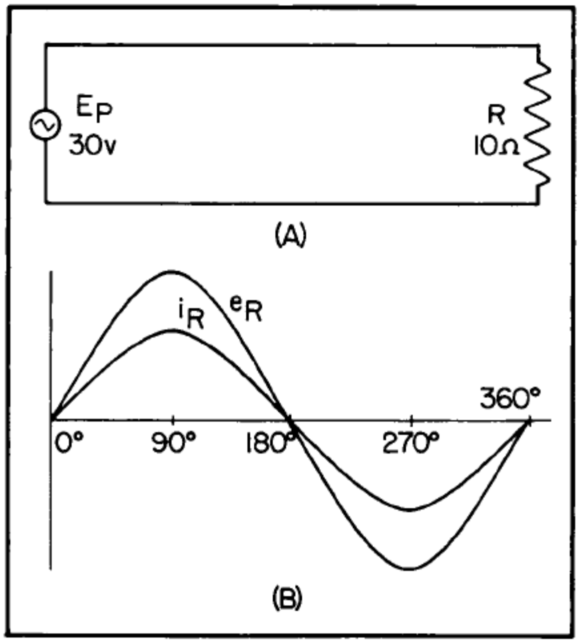

Figure 11-2 - Resistive circuit and sine wave.

In Figure 11-2 the peak voltage applied to the resistor is 30 volts. Therefore, at 90° the instantaneous voltage across the resistor will equal the peak voltage of 30 volts. The instantaneous current at 90° is found by application of Ohm's law:

$$i = \frac{e_p}{R}$$

$$i = \frac{30}{10}$$

$$i = 3 \text{ amps}$$

In Figure 11-2B it is seen that the peak values of current and voltage occur at 90° and 270°. At any other instant the current and voltage will be a value between peak and zero.

Q1. A dc voltage will cause a given amount of heat to be dissipated by a resistor. How would an ac voltage that caused the same amount of heat to be dissipated be designated?

Q2. If E_{avg} = 1.264V, what would be the peak to peak or E_{p-p} value?

11-2. I, P, E, and R

Figure 11-3 illustrates a resistive circuit with the RMS value of the source voltage given. It was previously stated that most meters are calibrated to read effective or RMS values of current and voltage.

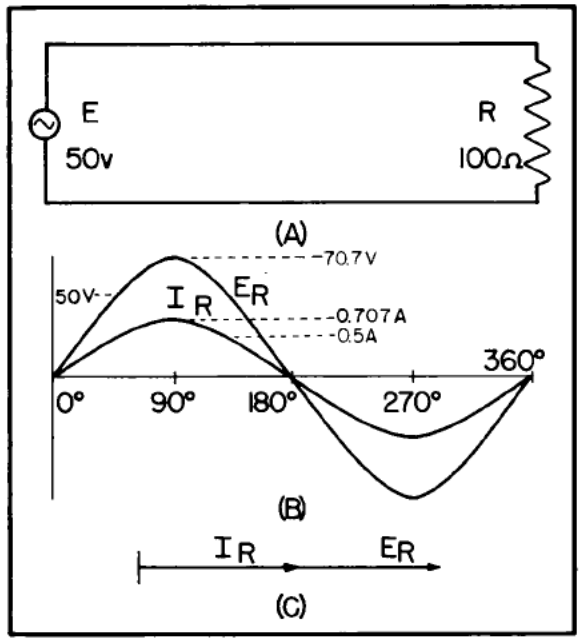

Figure 11-3 - Resistive ac circuit and vector diagram.

If a voltmeter were placed across R the voltage indicated would be lower than the maximum voltage. To show the reason for this statement it will be necessary to determine the maximum or peak voltage that will appear across the resistor. In the case of Figure 11-3A the resistor voltage will be equal to the applied voltage.

Therefore: $$E_{max} = E \times 1.414 \qquad (8\text{-}18)$$

$$E_{max} = 50 \times 1.414$$

$$E_{max} = 70.7 \text{ volts}$$

From the above it can be seen that the voltmeter would indicate 50 volts effective while the peak voltage would be 70.7 volts. An ammeter inserted in series with R would indicate the effective current as:

$$I = \frac{E}{R}$$

$$I = \frac{50}{100}$$

$$I = 0.5 \text{ amp}$$

The peak current may then be determined:

$$I_{max} = I \times 1.414 \qquad (8\text{-}19)$$

$$I_{max} = 0.5 \times 1.414$$

$$I_{max} = 0.707 \text{ amp}$$

Instantaneous power in the simple resistive circuit is the product of instantaneous voltage and instantaneous current. Expressed in the form of an equation:

$$P = e\,i \qquad (11\text{-}1)$$

Where: p = instantaneous power in watts
i = instantaneous current in amps
e = instantaneous voltage in volts

Figure 11-4 illustrates a simple resistive circuit and its e, i, p, E, I and P relationships. In (B) of Figure 11-4, equation 11-1 is used to determine the instantaneous power. The power curve resulting from the negative values of current and voltage is shown positive because the multiplication of two negative values results in a positive quantity. (Power is dissipated regardless of which direction current flows through a resistor). The power will be positive as long as voltage and current act in the SAME direction. Since all alternations of the power curve are identical, the mean or average power is the value half-way between the maximum and the minimum values of power. Thus, the average power can be determined to be half the maximum or peak power. Mathematically:

$$P_{avg} = \frac{E_m x I_m}{2} \qquad (11\text{-}2)$$

Figure 11-4C shows the average power to also be equal to the product of the voltage and current when these quantities are stated in effective value. This can be proven mathematically as:

$$P_{avg} = \frac{E_m \times I_m}{2}$$

Since $E_{max} = 1.414 \times E$ and $I_{max} = 1.414 \times I$

$$P_{avg} = \frac{(1.414E) \times (1.414I)}{2}$$

Since $1.414 = \sqrt{2}$

$$P_{avg} = \frac{\sqrt{2}E \times \sqrt{2}I}{2} = \frac{2\,EI}{2}$$

$$P_{avg} = EI$$

Where: P_{avg} = average power in watts
E = RMS voltage in volts
I = RMS current in amps

NOTE: In a resistive circuit true power and apparent power are the same. In other words, with no reactive components in the circuit there is no power returned to the source. All power applied to the resistance is dissipated. Therefore, the equations for average (true) power given above are accurate for resistive circuits only.

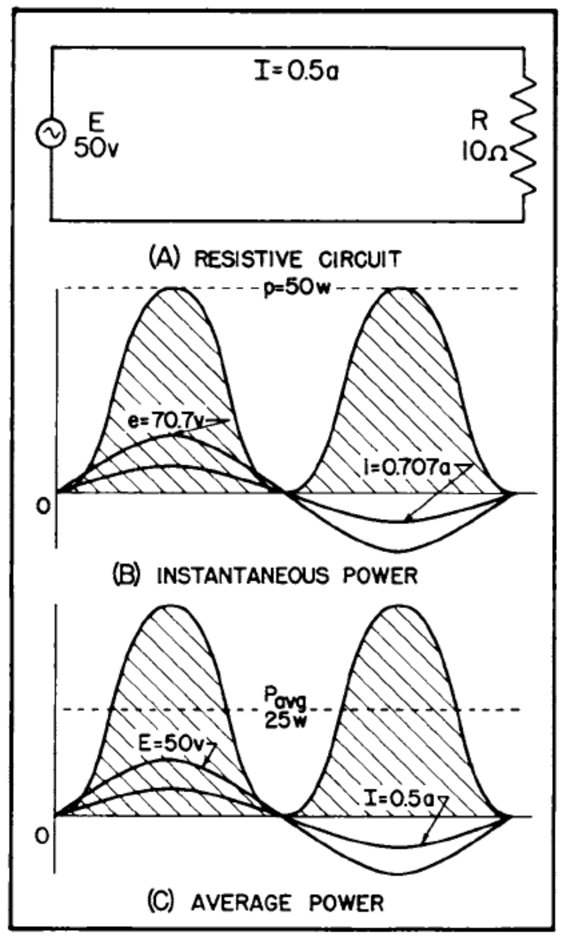

Figure 11-4 - Current, voltage and power relationships in resistive ac circuit.

A1. Effective or RMS.

A2. 4 volts. $E_p = 1.57 \times E_{avg} \cong 2$ volts
$E_{p-p} = 2 \times E_p = 4$ volts

Q3. A voltmeter connected to a 60 volts peak ac source will read what value of voltage?

Q4. Explain why, in an ac circuit, power dissipated in a resistor is always positive.

INDUCTIVE AC CIRCUITS

11-3. i, e and Reactance

Figure 11-5A shows a basic circuit in which an inductor (L) is connected across an ac source (E). Part B of the figure shows the sine wave diagram for the inductive circuit illustrated. Notice that the current which flows as a result of connecting the coil across the source lags 90 degrees behind the voltage applied. The 90 degree relationship between E and I can be explained as follows: If a graph were made depicting the rate of change of the current sine wave, it would appear as in Figure 11-5C. At time one (t_1) in the figure the current is undergoing its maximum rate of change (roc) and since current is increasing in a positive direction the roc is plotted as maximum positive. When roc of current is maximum the induced voltage of the coil is also maximum. This is shown at point (1) of Figure 11-5B. As the current approaches t_2 the roc of the current will decrease. Therefore, the curve indicating the roc is shown decreasing toward zero. Since the induced voltage is a function of the rate of change of current, the voltage waveform in part B will also decrease toward zero. At t_2 (90°) the current has reached its maximum value for the positive alternation and for an instant the roc of current is zero. Therefore, the roc curve will be zero and the induced voltage waveform will be zero (point 2, Figure 11-5B). At t_3 in part C, the current will be zero but the roc will again be maximum. Since the current is going in a negative direction the roc will be maximum negative. A maximum roc of current again will produce a maximum induced voltage, as at point (3) in part B. The current sine wave has now completed half a cycle (0° to 180°). The action described above will be repeated during the negative half cycle (180° to 360°) of the current sine wave. A comparison of parts B and C of Figure 11-5 will reveal that the induced voltage waveform and the roc of current curve are identical and that both take the form of a cosine wave (displaced from the sine wave by 90°). Thus, the current passes through the various parts of its cycle 90 degrees later in time than does the voltage.

This 90 degree relationship between current and voltage can readily be illustrated by the use of vectors as shown in Figure 11-5D. In this diagram the vector representing current is seen to lag the voltage vector by 90 degrees as the vectors rotate in a counter-clockwise direction.

In an inductive circuit, the amount of current flow is very much dependent on the source fre-

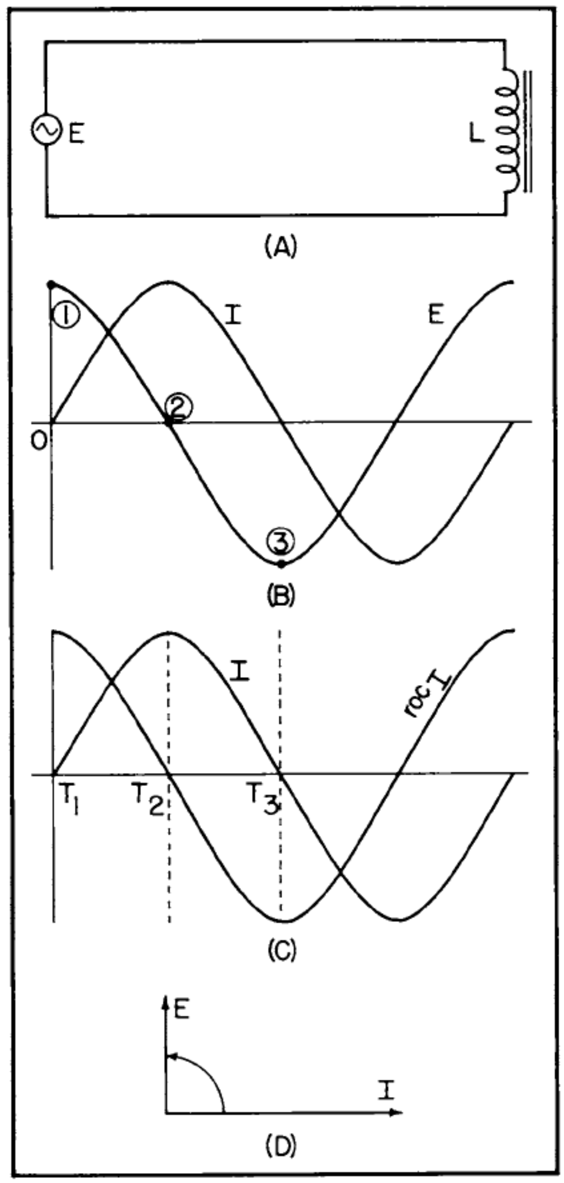

Figure 11-5 - E, I and roc of current relationships for inductive circuits.

quency. This is a direct result of the type of opposition a coil presents to the circuit current. In Chapter 9 this opposition was found to consist of a counter EMF whose magnitude is dependent on the rate of change of circuit current. A change in applied frequency alters the rate of change in current. The higher the frequency, the more rapid is the change in current and the greater is the counter EMF (opposition) of the coil.

In reviewing the action of a coil it will be recalled that the opposition a coil presents to ac is called inductive reactance for which the symbol is X_L:

$$X_L = 2\pi f L \qquad (9\text{-}26)$$

Since the current in an inductive circuit is directly proportional to applied voltage, and inversely proportional to the inductive reactance, Ohm's law can be applied to an inductive circuit. As an equation:

$$I_L = \frac{E_L}{X_L} \qquad (9\text{-}27)$$

Where: I_L = current through the coil in amperes
E_L = voltage across the coil in volts
X_L = inductive reactance of the coil in ohms

E_L and I_L can be peak, peak to peak, or effective values, however, E_L and I_L must be in the same type of unit. Thus, if E_L is an effective value of voltage, I_L must be an effective value of current.

Example. A coil having an inductive reactance of 50 ohms is connected across an ac source of 50 volts RMS. What is the peak value of current, and the instantaneous value of voltage at 45 degrees?

Given: E_a = 50 volts RMS
X_L = 50 ohms
θ = 45 degrees

Find: I_{max} = ?

e at 45^o = ?

Solution:

$$I_L = \frac{E_L}{X_L} \qquad (9\text{-}27)$$

$$I_L = \frac{50}{50}$$

$$I_L = 1 \text{ amp (RMS)}$$

Convert I_L to peak value:

$$I_{max} = I \times 1.414 \qquad (8\text{-}19)$$

$$I_{max} = 1 \times 1.414$$

$$I_{max} = 1.414 \text{ amps}$$

To determine instantaneous voltage, first convert E_a to peak value:

$$E_{max} = E \times 1.414 \qquad (8\text{-}18)$$

$$E_{max} = 50 \times 1.414$$

$$E_{max} = 70.7 \text{ volts}$$

Determine e at 45 degrees:

$$e = E_{max} \times \sin\theta \qquad (8\text{-}1)$$

$$e = 70.7 \times 0.707$$

$$e = 49.98 \text{ volts}$$

Q5. The roc of current in an inductive ac circuit is maximum during what period of the sine wave?

Q6. What is the relationship between the current and voltage when the roc of current is maximum?

Q7. Is the opposition of a coil highest at low frequencies or high frequencies?

11-4. Power in an Inductive Circuit

TRUE POWER is power actually dissipated by the resistance of a circuit. The true power in a dc circuit and a resistive ac circuit is equal to the product of voltage and amperes. When an ac circuit contains reactance this is no longer true.

In a circuit containing inductance only, the true power (P_t) is zero. The current (I) lags the applied voltage (E) by 90 degrees. The true power is the average of E x I taken over one complete cycle. The APPARENT POWER (power the circuit would apparently consume, but actually does not) is the product of RMS volts and RMS amperes. Thus, the apparent power of an inductive circuit (such as in Figure 11-6A) with an RMS voltage of 100V and an RMS current of 10 amps is:

$$P_a = E \times I$$

$$P_a = 100 \times 10$$

$$P_a = 1000 \text{ volt amperes}$$

A3. 42.42 volts, $E = 0.707\ E_p$.

A4. Power is dissipated by the resistor regardless of the direction of current flow. Current and voltage are always acting in the same direction.

A5. When it (the current) is going through zero.

A6. The voltage is maximum when the roc of current is maximum.

A7. High frequencies.

However, the power absorbed by the coil during the time the current is rising is returned to the source during the time the current is decreasing, so the average power is zero.

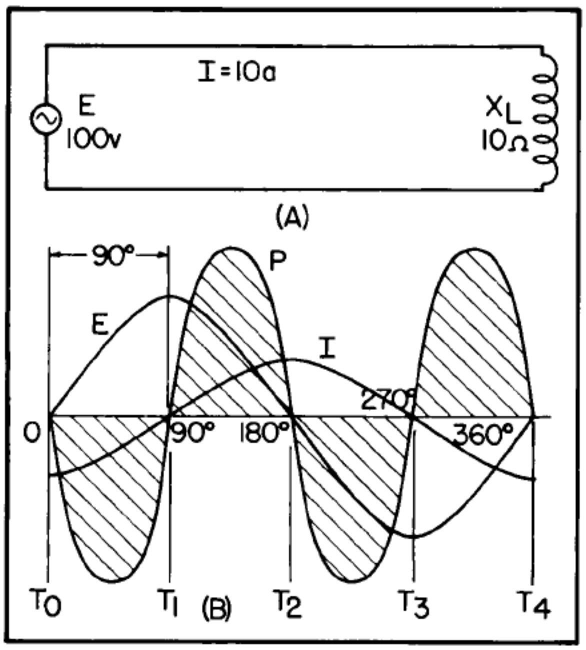

Figure 11-6 - Power in an inductive circuit.

The product of the instantaneous values of current and voltage yield a double frequency power curve (P) of Figure 11-6B.

The shaded area of the power curve in Figure 11-6B (above the zero reference line) represents the energy absorbed by the coil (positive energy) and the shaded areas of the power curve below zero represent the energy returned to the source by the collapsing magnetic field (negative energy).

From zero to 90 degrees (t_0 - t_1) current is negative and approaching zero; the magnetic field of the coil is collapsing and the energy in the field is being returned to the source (negative energy). The product of negative current and positive voltage yields negative power. The power curve is shown as negative power for the first 90 degrees in Figure 11-6B.

From 90 degrees to 180 degrees (t_1 - t_2) the current is positive and increasing; energy from the source is being stored in the expanding field of the coil (positive energy). The product of positive current and positive voltage is positive power.

From 180 degrees to 270 degrees (t_2 - t_3) current is positive and falling. At this time the voltage is negative. Again energy is being returned to the source by the collapsing coil field; the product of positive current and negative voltage is negative power.

From 270 degrees to 360 degrees (t_3 - t_4) current is again negative and approaching zero. Positive energy is being stored in the magnetic field. The power is positive.

Thus, when current is rising, power is being supplied by the source and stored in the magnetic field; when current is falling power is being returned to the source from the collapsing magnetic field. In the theoretically pure inductance, shown in Figure 11-6, the supplied power is equal to the returned power. Thus, average power (true power) is zero.

The ratio of true power to the apparent power in an ac circuit is called the POWER FACTOR. It may be expressed as a percentage or as a decimal. The mathematical expression is:

$$P.F. = \frac{P_t}{P_a} \qquad (11\text{-}3)$$

Where: P_t = true power in watts
P_a = apparent power (ExI) in volt-amperes
P.F. = power factor ($\cos\theta$)

The apparent power in a purely inductive circuit is called REACTIVE POWER and the unit of reactive power is the VAR. This unit is derived from the first letters of the words, volt-amperes-reactive. The power factor for the circuit of Figure 11-6A is:

$$P.F. = \frac{P_t}{P_a}$$

since: $P_t = 0$

$P_a = 1000$ VA

then: $$P.F. = \frac{0}{1000}$$

$$P.F. = \text{zero}$$

It can be seen (Figure 11-6B) that there are two cycles of the power curve for each cycle of

the voltage (or current) curve. Therefore, the power frequency is double the current or voltage frequency.

Q8. Explain the difference between true power and apparent power.

Q9. What happens to the positive power absorbed by a coil when the current is rising?

Q10. Define the term power factor.

CAPACITIVE AC CIRCUITS

11-5. i, e, and Reactance

Figure 11-7A shows a basic circuit in which a capacitor (C) is connected across an ac source (E). Part B of the figure shows the sine wave diagram for the capacitive circuit illustrated. Notice that the current which flows as a result of connecting the capacitor across the source leads the voltage by 90 degrees. The 90 degree relationship between E and I can be explained as follows: a graph of the rate of change of voltage appears as in Figure 11-7C. At zero degrees the roc of voltage is maximum. At 90 degrees the roc of voltage is zero. At 180 degrees the roc of voltage is again maximum. The roc of voltage (E) takes the form of a cosine wave. A comparison of Parts B and C will reveal that the current waveform and the roc curve of the voltage are identical. Thus, the voltage passes through the various parts of its cycle 90 degrees later in time than does the current.

This 90 degree relationship between current and voltage can readily be illustrated by the use of vectors as shown in Figure 11-7D. In this diagram the vector representing voltage is shown to be 90 degrees behind the current vector, as the vectors rotate in a counterclockwise direction.

In a capacitive circuit, the amount of current flow is very much dependent on source frequency. A change in source frequency alters the rate of change of voltage. The higher the frequency the more rapid is the change in voltage and the smaller is the opposition of the capacitor.

In reviewing the action of a capacitor it will be recalled that the opposition a capacitor presents to an alternating current is called capacitive reactance for which the symbol is X_C. Repeating the formula for X_C:

$$X_C = \frac{1}{2\pi f C} \qquad (10\text{-}24)$$

Equation (10-24) shows that X_C is an inverse function of frequency. Also the current in a capacitive circuit is inversely proportional to capacitive reactance. Repeating Ohm's law for ac circuits:

$$I_C = \frac{E_C}{X_C} \qquad (10\text{-}27)$$

By use of the two equations above it can be seen that an increase of frequency will decrease X_C. A decrease in X_C will cause an increase in current. Therefore, an increase of frequency in a capacitive circuit will increase current. When using equation (10-27) E_C and

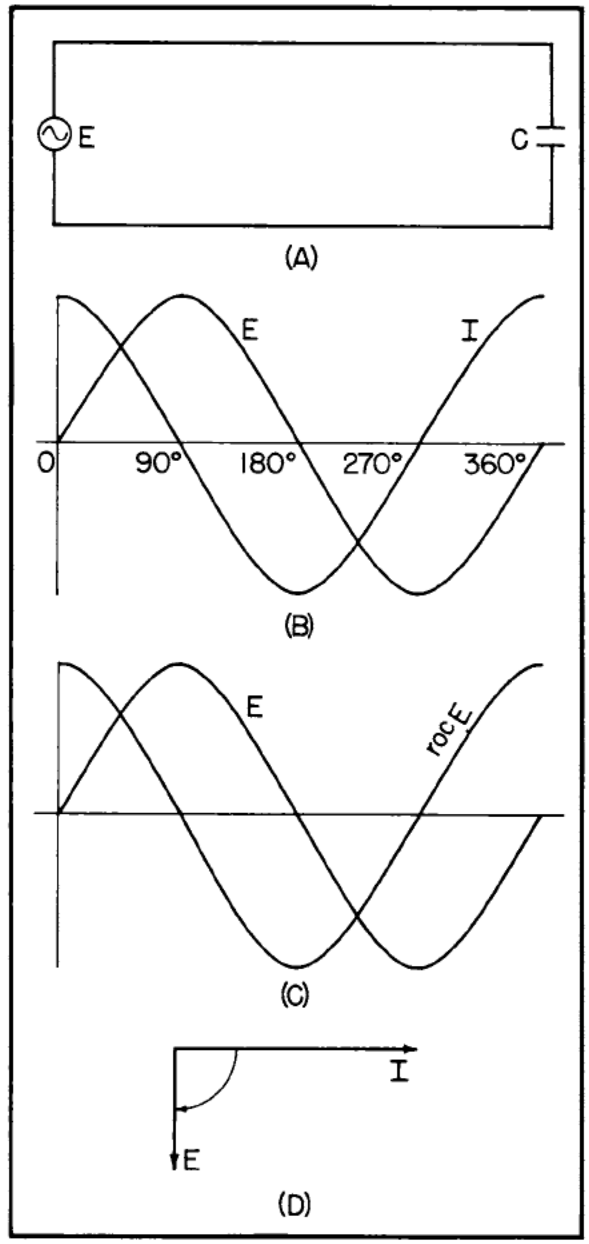

Figure 11-7 - E, I and roc of voltage relationships for capacitive ac circuit.

A8. Apparent power is the power that the circuit appears to consume and true power is the power the circuit actually consumes.

A9. It is returned to the circuit when the current is falling.

A10. The ratio of true power to apparent power in an ac circuit.

I_C may be peak, peak to peak, or effective values, as long as both are in the same type of unit.

Example. A capacitor having an X_C of 60 ohms is connected across an ac source of 120 volts RMS. What is the peak value of current and the instantaneous value of voltage at 38 degrees?

Given: E_a = 120 volts RMS
X_C = 60 ohms
θ = 38 degrees

Find: I_{max} = ?

e at 38° = ?

Solution:

$$I_C = \frac{E_C}{X_C} \qquad (10\text{-}27)$$

$$I_C = \frac{120}{60}$$

$$I_C = 2 \text{ amps RMS}$$

Convert I_C to peak value:

$$I_{max} = I \times 1.414 \qquad (8\text{-}19)$$

$$I_{max} = 2 \times 1.414$$

$$I_{max} = 2.828 \text{ amps}$$

To determine instantaneous voltage, first convert E_a to peak value:

$$E_{max} = E \times 1.414$$

$$E_{max} = 120 \times 1.414$$

$$E_{max} = 169.68 \text{ volts}$$

Determine e at 38 degrees:

$$e = E_{max} \times \sin\theta \qquad (8\text{-}2)$$

$$e = 169.68 \times 0.6157$$

$$e = 104.45 \text{ volts}$$

Q11. Will the capacitive reactance be highest at the high frequencies, or at the low frequencies?

Q12. What will be the effect on current in a purely capacitive circuit if the applied frequency is decreased?

11-6. Power in a Capacitive Circuit

Figure 11-8A shows a capacitive circuit and Figure 11-8B indicates the sine waveforms of charging current, applied voltage, and instantaneous power. The current (I) leads the applied voltage by 90 degrees. Since the circuit contains only capacitance, and the losses are neglected, the true power consumed by the circuit is equal to zero.

The apparent power of the circuit in Figure 11-8A is:

$$P_a = E \times I$$

$$P_a = 70.7 \times 7.07$$

$$P_a = 500 \text{ VA}$$

Figure 11-8B will be used in explaining the action of the circuit in relation to power.

During the first quarter cycle (0° to 90°) the applied voltage rises from zero to maximum and the capacitor receives a charge. The product of positive voltage and positive current results in positive power. The power curve being positive indicates energy is being stored in the electrostatic field of the capacitor.

During the second quarter cycle (90° - 180°)

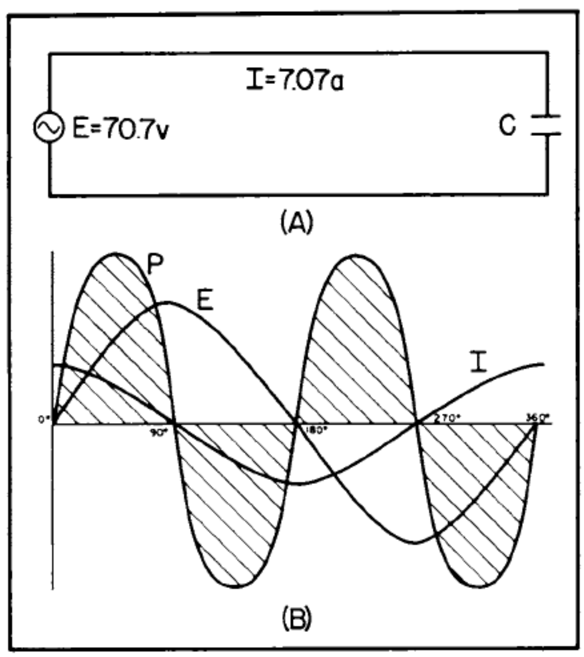

Figure 11-8 - Sine curves of E, I, and P.

the applied voltage is decreasing and the capacitor is discharging. In the act of discharging, the capacitor is returning energy to the source. The return of energy to the source is represented by the negative part of the power curve.

From 180° to 270° the capacitor stores energy and from 270° to 360° the energy is returned to the source. In the capacitive circuit, as was the case with reactive power in the inductive circuit, the shaded areas of negative power are equal to the shaded areas of positive power. This means that all the power absorbed from the source by the capacitor is returned to the source. Therefore, true power is zero.

Q13. What is the true power in a circuit containing a pure capacitor across an ac source?

AC SERIES RL CIRCUITS

11-7. Series RL Circuit

Figure 11-9 shows a circuit composed of a coil and a resistor connected in series, and placed across an ac source. As might be expected, this type of circuit displays both resistive and reactive characteristics.

Basically the analysis of a series RL circuit with an ac source is carried out in this manner. The total opposition of the circuit is computed. Then circuit current and individual voltage drops are solved for. The phase difference between the applied voltage and the circuit current must then be computed.

In an RL circuit, the total opposition cannot be called resistance or reactance, since it contains some of each. The total opposition in this type of circuit is called IMPEDANCE and is represented by the letter Z. Like resistance and reactance, impedance is measured in ohms.

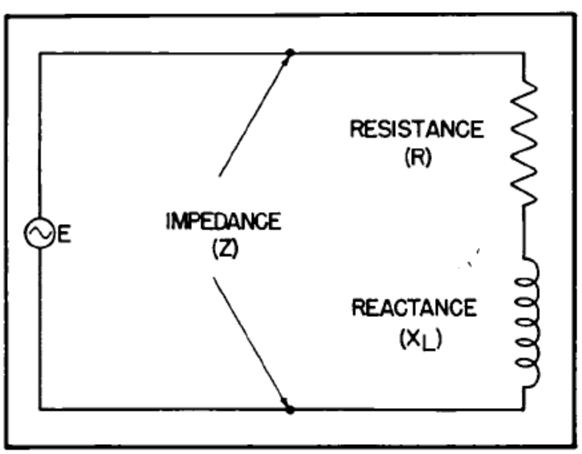

Figure 11-9 - AC series RL circuit.

11-8. Voltage Vector Diagram

A number of different methods are available for the solution of series RL circuits. In this section a series RL circuit will be solved by graphical methods. As with any graphical solution, the accuracy of the results are limited by the degree to which the values can be estimated from the graph.

Several vector diagrams can be drawn for a series RL circuit. One of these diagrams, called the IMPEDANCE VECTOR DIAGRAM, is used when it is desired to add the resistance to the reactance to obtain the impedance of the circuit. When the resistor and inductor voltages are to be added to obtain the total voltage, a VOLTAGE VECTOR DIAGRAM is used.

In order to plot a vector diagram, some circuit quantity must be chosen as a reference. The other circuit quantities are then plotted with respect to the reference. In a series circuit the same current flows through each part of the circuit. The current is therefore an ideal quantity to which the phase of the circuit voltages may be compared.

Figure 11-10B shows the voltage vector diagram for the series RL circuit in Part A of the figure. In this diagram the current vector is used as the reference and is drawn in standard position on the X-axis. Since the voltage drop across the resistor is in phase with the current through the resistor, the vector representing E_R is placed directly on top of the vector representing I. The voltage across the inductor leads the circuit current by 90 degrees. This

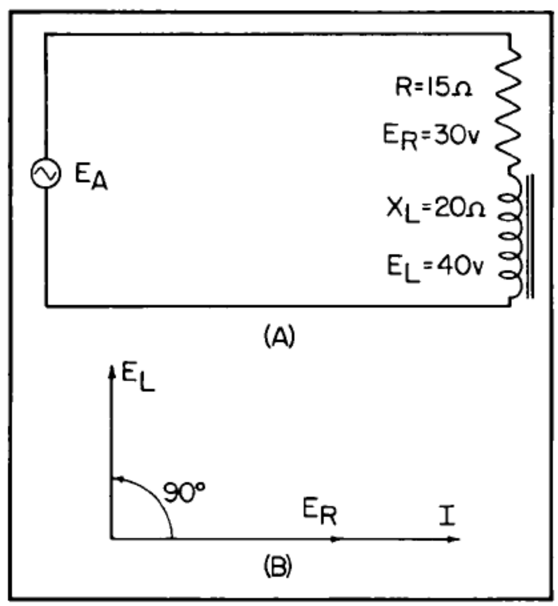

Figure 11-10 - Series RL circuit and vector diagram.

A11. Low frequencies.

A12. Current will decrease because X_C increased.

A13. Zero, because all the power consumed is returned.

---

is indicated by placing the vector representing E_L 90 degrees ahead of I (pointing upward on the Y axis).

To obtain the applied voltage E_a, vectors E_R and E_L are added as shown in Figure 11-11. Notice that the vectors representing E_R and E_L have been drawn to scale, with each division of the graph equal to 10 volts. The vector sum is now obtained by the parallelogram method of vector addition. When the length of the resultant vector E_a is measured it is found to be approximately 5 divisions, indicating the total voltage to be 50 volts. NOTICE THAT THE TOTAL VOLTAGE CANNOT BE OBTAINED BY ALGEBRAIC ADDITION OF E_R AND E_L. If E_R and E_L are added algebraically the incorrect total of 30 plus 40, or 70 volts is obtained. Algebraic addition of RMS values can only be used when quantities to be added are in phase, or 180° out of phase

In addition to providing the magnitude of the applied voltage, the vector diagram in Figure 11-11 also shows the phase angle of the circuit. If the angle theta (θ) were measured with a protractor it would be found to have a value of approximately 53 degrees. Thus, the circuit current lags the applied voltage by 53 degrees. The angle (θ) between applied voltage and circuit current is called the CIRCUIT PHASE ANGLE. In addition to the circuit phase angle, the diagram also shows the angle between E_a and E_R, and the angle between E_a and E_L.

Q14. What is the purpose of an impedance vector diagram?

Q15. What quantity is used as a reference for a voltage vector diagram of a series circuit?

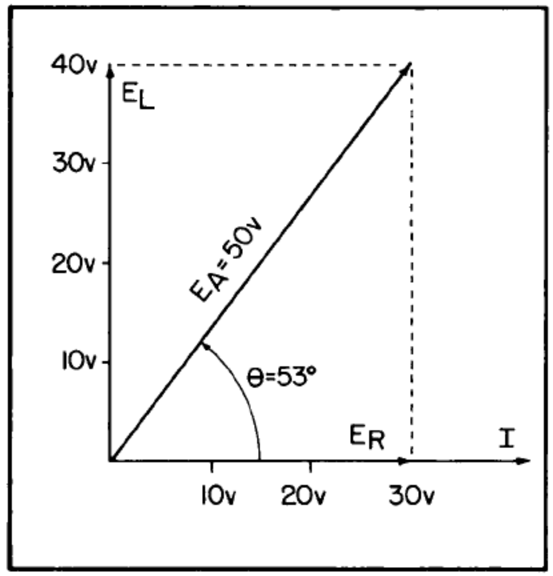

Figure 11-11 - Vector addition of voltage drops.

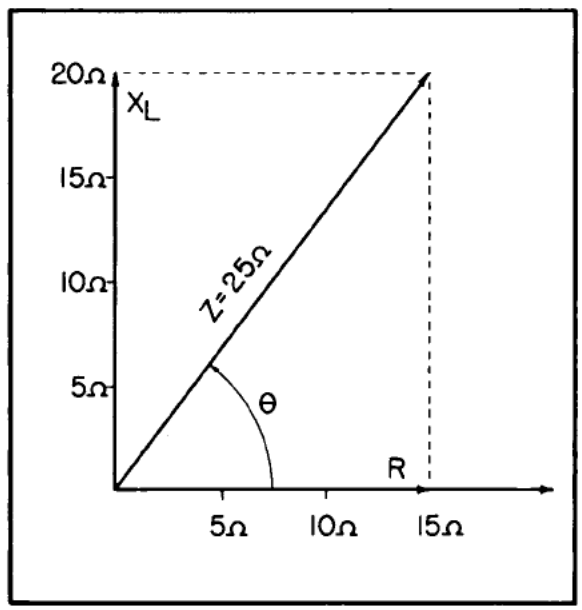

Figure 11-12 - Impedance vector diagram.

11-9. Impedance Vector Diagram

Except for the fact that oppositions are used instead of voltages, the impedance vector diagram is identical in appearance to the voltage vector diagram. Figure 11-12 shows the impedance vector diagram for a series RL circuit in which R is 15 ohms and X_L is 20 ohms. Since resistance produces a voltage drop that is in phase with circuit current, the resistance vector is plotted on the same axis as the current vector. Inductive reactance produces a voltage drop which leads the current by 90 degrees, therefore, the vector representing X_L is plotted pointing upward along the Y-axis. These two vectors are then added to obtain the impedance (Z). Measuring the length of the resultant vector in Figure 11-12 shows the impedance to be approximately 25 ohms. Again notice that

the two quantities cannot simply be added together to obtain the impedance. Since R and X_L are 90 degrees out of phase they must be added vectorially.

Once the impedance and the applied voltage are known the circuit current is easily obtained by the use of Ohm's law. The equation for current is:

$$I = \frac{E_a}{Z} \qquad (11\text{-}4)$$

where: I = circuit current in amperes
E_a = the applied voltage in volts
Z = circuit impedance in ohms

At best, a graphical solution of a circuit gives only an approximation of the actual values. If accurate values are needed the solution must be obtained by mathematical means. Two powerful mathematical tools will be introduced at this point because of their importance in ac circuit analysis.

11-10. Pythagorean Theorem

About 500 years BC a Greek philosopher and mathematician by the name of Pythagoras discovered a unique relationship between the lengths of the sides of a right triangle. His discovery stated in the form of a theorem is:

PYTHAGOREAN THEOREM: In any right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

This theorem can be illustrated by considering the right triangle shown in Figure 11-13. The three sides of the triangle are labeled A, B, and C. Side C is directly opposite to the right angle and is called the HYPOTENUSE. Using the designations from Figure 11-13 the Pythagorean theorem can be stated as:

$$C^2 = A^2 + B^2$$

or taking the square root of both sides:

$$C = \sqrt{A^2 + B^2} \qquad (11\text{-}5)$$

This formula can be readily applied to either of the vector diagrams for the series RL circuit. If the resultant vector in Figure 11-12 is determined by the triangle method of vector addition, the diagram will appear as shown in (A) of Figure 11-14. Notice that the impedance vector Z corresponds to side C of the right triangle in (B) of Figure 11-14. Similarly vector R corresponds to side A and vector X_L corresponds to side B of the right triangle. The letters R, X_L and Z can be substituted into equation (11-5) in place of the letters A, B, and C respectively.

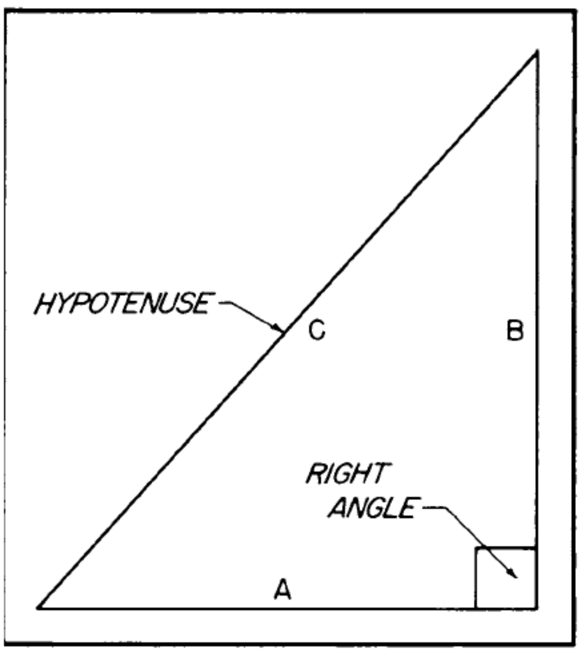

Figure 11-13 - Right triangle relationships.

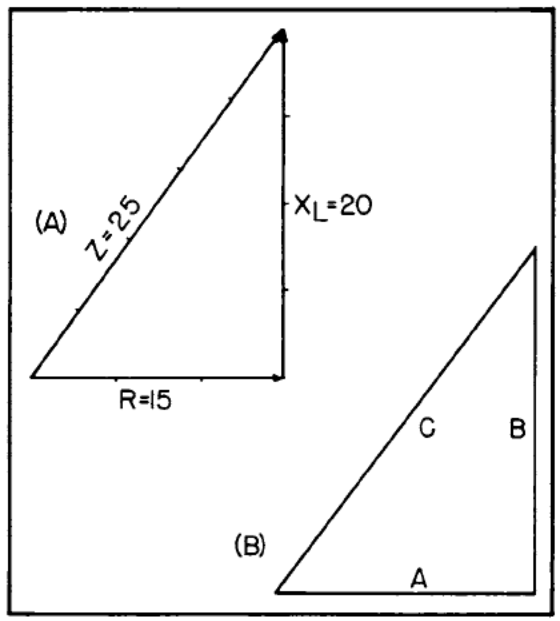

Figure 11-14 - Impedance triangle.

The equation would now appear as:

$$Z = \sqrt{R^2 + X_L{}^2} \qquad (11\text{-}6)$$

Inserting values of X_L and R from Figure 11-14A:

$$Z = \sqrt{15^2 + 20^2}$$

$$Z = \sqrt{225 + 400}$$

$$Z = \sqrt{625} = 25 \text{ ohms}$$

A14. Determine the impedance by graphically adding the resistance and reactance of a circuit.

A15. Resistor voltage because it is in phase with the circuit current, which is the common quantity.

Q16. A circuit has an impedance of 50 ohms and an inductive reactance of 40 ohms. Determine the resistance.

VECTOR NOTATION

This section is presented with the intent of supplying the student with mathematical tools that will ease the study of advanced ac circuits. A carpenter could build a house with only a hammer and hand saw for tools. But, it would be foolish for him to do so when there are so many labor saving devices, such as power saws, available. Advanced ac circuits could be analyzed by long and laborious calculations. Fortunately this is not necessary because mathematics provides labor saving devices. Some of these, such as complex numbers, j operator, polar notation, etc., will now be discussed.

11-11. j Operator

The sign of any number may be changed, without affecting the magnitude, by multiplying the number by -1. For example, +20 amps when multiplied by -1 becomes -20 amps. A -30 amps when multiplied by -1 becomes a +30 amps. This is due to the fact that when quantities with unlike signs are multiplied the sign of the resultant is always negative and when quantities with like signs are multiplied the sign of the resultant is always positive. It will be noted in both cases above, that while the sign was changed, the magnitude was not affected.

The application of this may be shown when applied to a vector diagram. If the 30 volt vector in standard position in Figure 11-15 were multiplied by -1 the vector would assume the position shown by the dotted line. Figure 11-15 indicates that multiplication by -1 will reverse the direction of a vector quantity. In other words, the vector will undergo a 180 degree counter-clockwise rotation. The counter-clockwise direction of rotation is a matter of convention. If the vector representing the -30 volts were to be multiplied by -1 it would continue its counter-clockwise rotation and return to its original position. It has been shown that in analyzing ac circuits there are numerous occasions where a vector rotation of only 90 degrees is

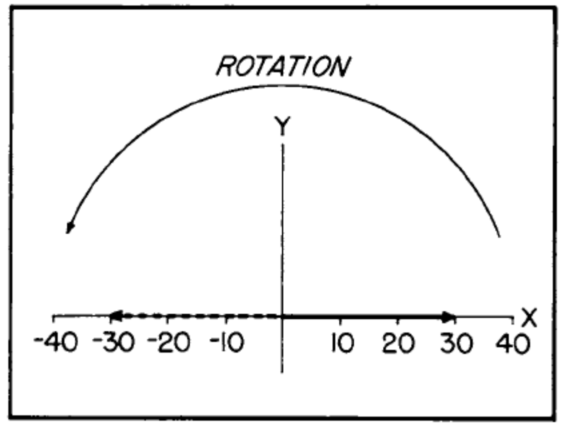

Figure 11-15 - Vector diagram.

encountered. Two examples of a 90 degree phase difference are:

1. Voltage leading current by 90° in an inductive circuit.

2. Current leading voltage by 90° in a capacitive circuit.

It has been found in mathematics that a quantity multiplied by the square root of minus one ($\sqrt{-1}$) will be rotated by 90 degrees counter-clockwise (by convention).

Example. Figure 11-16 shows a vector representing 30 volts multiplied by $\sqrt{-1}$ rotated to a position pointing upward along the Y-axis.

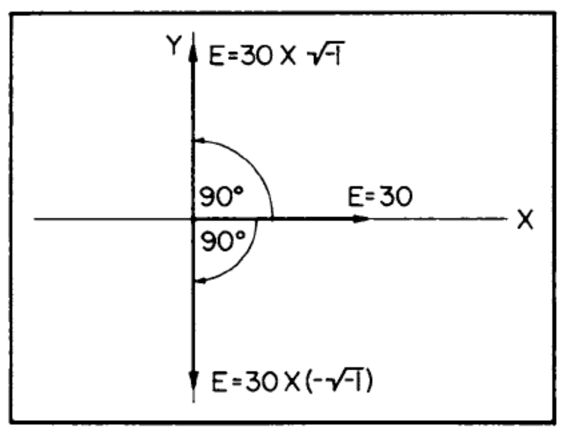

Figure 11-16 - Vector rotation.

Multiplying the vector quantity 30 volts by minus the square root of minus one:

$$30\,(-\sqrt{-1})$$

rotated the vector 90 degrees in a clockwise

lirection. This is shown in Figure 11-16 with he vector pointing downward along the Y-axis.

The symbol $\sqrt{-1}$ is rather clumsy to use in :alculations. Therefore, it is customary in :lectronics to use the letter j to represent the ;ymbol $\sqrt{-1}$. Thus, the letter j is a symbol for ι vector rotation of 90 degrees and is known as he j operator.

Therefore: $\sqrt{-1} = +j$ = 90 degrees counter-clockwise rotation of a vector.

$-\sqrt{-1} = -j$ = 90 degrees clockwise rotation of a vector.

In order to indicate that a vector quantity is o be rotated 90° (plotted along the Y-axis) it is nerely necessary to indicate multiplication by he j operator. For example, a vector magni-ude of 30 volts would indicate a vector plotted n the standard position as in Figure 11-17.

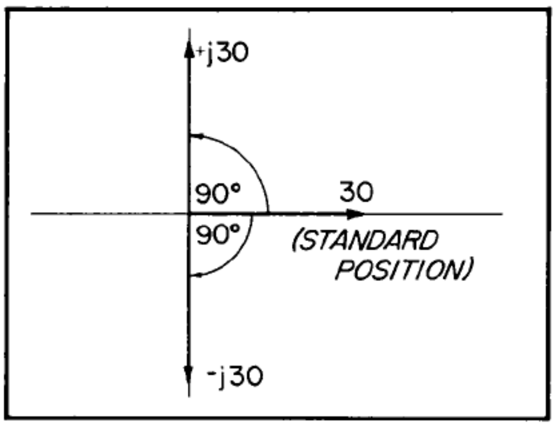

Figure 11-17 - Vector rotation by j operation.

Figure 11-17 also illustrates multiplication by +j and -j.

Successive applications of the j operator to a vector will produce a counter-clockwise rotation in successive 90 degree steps.

Figure 11-18 is used to show the effect on a vector quantity of successive multiplication by a j operator.

In Figure 11-18A, the vector quantity with a magnitude of 8 units and plotted in standard position, is multiplied by j. The result is a 90 degree counter-clockwise rotation. If the vector j8 is multiplied by j the result will be the same as if the original vector were operated by j x j. Since j is equal to $\sqrt{-1}$ and the square root of a number squared is the number itself:

$$(\sqrt{-1})^2 = -1$$

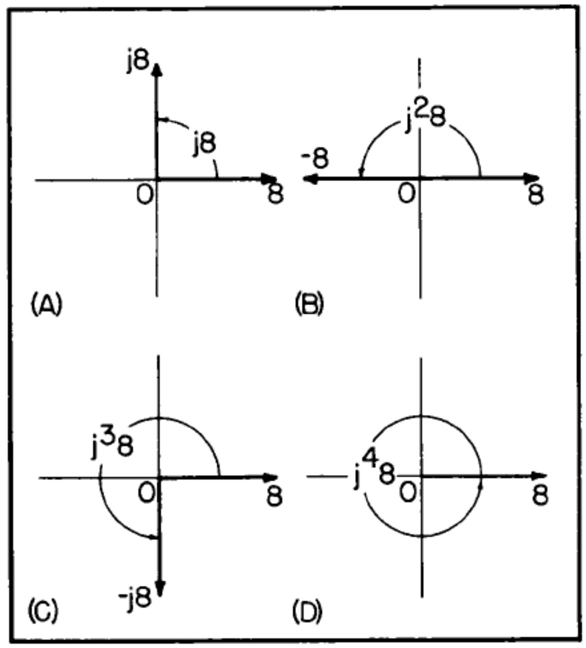

Figure 11-18 - j operator multiplication.

Then: $\sqrt{-1} \times \sqrt{-1} = -1$

or: $j \times j = -1$

finally: $j^2 = -1$

This means that multiplying a quantity by j^2 is the same as multiplying by -1 and will result in a 180 degree rotation. This is illustrated in Figure 11-18B where the vector has been rotated 180 degrees in a counter-clockwise direction and is now -8. If part B were again multiplied by j this would be the same as multiplying by j^3 because:

$$j \times j \times j = j^3$$

Since: $j \times j = -1$

then: $(j \times j) \times j = -1 \times j$

or: $j^3 = -\sqrt{-1} = -j$

Therefore, in part C of Figure 11-18 the result of $j^3 8$ is -j8. The original vector has now been rotated 270 degrees counter-clockwise.

Multiplying the vector -j8 by j (part C) would be the same as multiplying the original vector by j^4.

Since: $j \times j \times j \times j = j^4$

and: $j \times j = -1$

A16. 30 ohms. Transpose equation (11-6) to solve for resistance.

Then: $j^4 = (-1)(-1)$

$j^4 = 1$

Therefore, multiplying a vector quantity by j^4 causes a complete 360 degree rotation as shown in Figure 11-18D.

Successive applications of the -j operator will cause clockwise rotation of the vector in 90 degree steps. Rotation of a vector by j and -j operators is tabulated in Table 11-1.

OPERATOR	MATHEMATICAL EQUIVALENT	DIRECTION AND DEGREE OF ROTATION
j	$\sqrt{-1}$	90°cc
j^2	-1	180°cc
j^3	$-\sqrt{-1}$	270°cc
j^4	1	360°cc
$-j$	$-\sqrt{-1}$	90°c
$(-j)^2$	-1	180°c
$(-j)^3$	$\sqrt{-1}$	270°c
$(-j)^4$	1	360°c

NOTE: cc - counter-clockwise
c - clockwise

TABLE 11-1
Relation of j operator to vector rotation.

In all cases it will be noticed that application of the j operator rotated the vector but did not alter the magnitude.

Q17. What would be the result of a vector quantity of 25 multiplied by j^6? In other words, $j^6 \times 25 = ?$

Q18. What would be the number of degrees of rotation of a vector multiplied by j^7?

11-12. Rectangular Notation

When the sum of two "in phase quantities", such as a 5 ohm resistor and a 10 ohm resistor is desired, the total is written as:

$$R_t = R_1 + R_2$$

$$R_t = 5 + 10$$

$$R_t = 15 \text{ ohms}$$

This is simply an algebraic addition. When the sum of a non-reactive component and a reactive component is desired the equation cannot be written in the form used above because algebraic addition is no longer valid. "Out of phase" quantities must be combined by some method of addition other than algebraic. In section 11-9 a resistance of 15 ohms and a reactance of 20 ohms were added, by vector addition, to obtain an impedance of 25 ohms. In Figure 11-19, the resistance is plotted in standard position while the reactive component (X_L) is plotted 90 degrees counter-clockwise from the standard position.

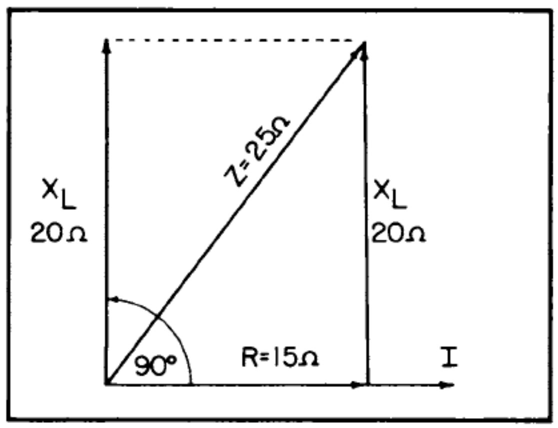

Figure 11-19 - Impedance vector diagram.

The vector position of X_L may be indicated by the j operator. That is, if the 20 ohms of X_L were multiplied by j it would indicate that X_L was 90 degrees counter-clockwise from the standard position. Therefore, by use of the j operator the vector addition of two quantities at right angles may be expressed mathematically as:

$$Z = R \pm jX \qquad (11\text{-}7)$$

where: Z = impedance in ohms
R = resistance in ohms
$\pm jX$ = reactance in ohms. Plus j is used for X_L and minus j is used for X_C.

Inserting values from Figure 11-19 in equation (11-7):

$$Z = 15 + j20 \text{ ohms}$$

Thus, the impedance may be read as: "Impedance" equals the resistive component of 15 ohms added vectorially to the reactive component of 0 ohms. Note that the right side of the equation is in the proper form to be inserted in equation (11-6) to solve for impedance.

Equation (11-7) is a COMPLEX NUMBER. In electronics a complex number is an expression containing both reactive and non-reactive components. Quantities that are at right angles to each other cannot be added or subtracted in the usual sense of the word. Their sum or difference can only be indicated. Thus, the plus sign in equation (11-7) only indicates addition. When a quantity is written in the form of equation (11-7) it is said to be written in RECTANGULAR NOTATION, or rectangular form. Complex numbers may be combined directly when in rectangular form.

Example. Three impedances are to be added to find their total.

Z_1 consists of a 20 ohm resistor in series with 60 ohms of inductive reactance.
Z_2 consists of a 10 ohms resistor and 10 ohms of inductive reactance in series.
Z_3 consists of a 50 ohm resistor and 20 ohms of capacitive reactance in series.

Solution: Write all impedances in rectangular form:

$$Z_t = R \pm jX \qquad (11\text{-}7)$$

$$Z_1 = 20 + j60$$
$$Z_2 = 10 + j10$$
$$Z_3 = 50 - j20 \text{ (note the } -j \text{ due to } X_C)$$
$$\overline{Z_t = 80 + j50}$$

The resistive components are added arithmetically:

$$20 + 10 + 50 = 80$$

The reactive components are added algebraically with due regard for the signs of the j operators:

$$(+j60)+(+j10)+(-j20) = +j50$$

The three original impedances are now described by a total impedance of 80 ohms of resistance added to 50 ohms of inductive reactance.

$$Z_t = 80 + j50 \text{ (rectangular notation)}$$

In order to determine the actual magnitude of the resultant vector quantity Z_t, either vector addition or the Pythagorean theorem can be used. By substitution of the known quantities into the impedance formula:

$$Z_t = \sqrt{R^2 + X_L^2} \qquad (11\text{-}6)$$

$$Z_t = \sqrt{80^2 + 50^2}$$

$$Z_t = 94.3 \text{ ohms}$$

In rectangular form the vector (Z_t) is described in terms of the two sides of a right triangle, the hypotenuse of which is the vector (Figure 11-19). Thus, rectangular notation describes the resultant vector quantity to a limited degree. It supplies information as to how much of the resultant vector is due to resistance and how much is due to reactance, but, it does not directly indicate the actual magnitude of the resultant vector, nor the angle of the resultant vector with respect to the reference line.

Q19. Describe in words the expression:
$$Z = 325 + j32 \text{ ohms}$$

Q20. What are the main disadvantages to describing a vector in rectangular notation?

11-13. Polar Notation

Up to this point methods have been discussed by which a resultant vector could be determined. Now a system of notation is needed that will express the information required to accurately describe the resultant vector. The vector OB (Figure 11-20) is described in rectangular form by the complex number 3 + j4. In other words, the information contained in the rectangular form is sufficient to plot the vector diagram Figure 11-20. From this diagram, the length and angle of the resultant vector with respect to the reference line, may be determined by physical measurement.

The vector OB (Figure 11-20) can be described in a combined form if its length and angle

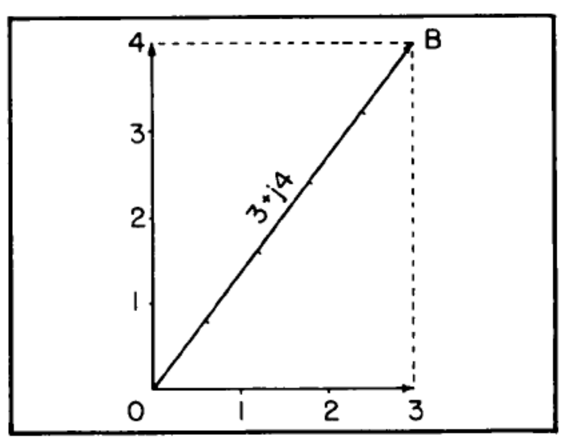

Figure 11-20 - Vector diagram, rectangular form

A17. Minus 25. The vector quantity 25 would be rotated one and a half times and end up 180 degrees from the standard position.

A18. 630 degrees. One and three-quarters rotations.

A19. Impedance equals the resistance of 325 ohms added to the inductive reactance of 32 ohms.

A20. The absolute magnitude and phase angle are not given directly.

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of rotation are given. When a vector is described by means of its magnitude and angle it is said to be expressed in POLAR NOTATION or polar form. Figure 11-21 illustrates a vector with a magnitude of 5 units and an angle of 53 degrees.

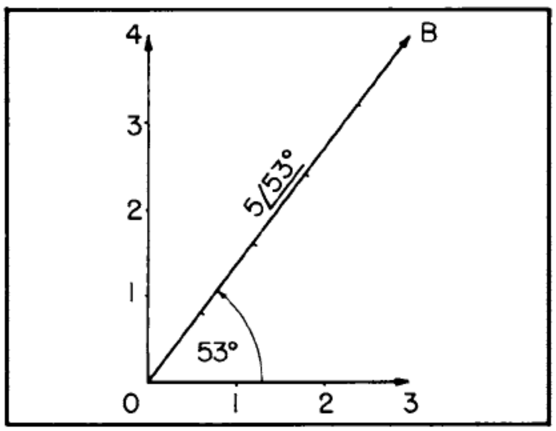

Figure 11-21 - Vector diagram, polar form.

A vector expressed in polar form may be graphed directly (without using the parallelogram method).

Example. Graph the two vectors described in polar notation as:

$$\text{Vector one } (V_1) = 25 \angle 40^\circ \text{ volts}$$

$$\text{Vector two } (V_2) = 35 \angle -25^\circ \text{ volts}$$

Notice that vector two has an angle of -25°. This means that the rotation of the vector is clockwise or 25 degrees below the reference line. Figure 11-22 illustrates the two vectors graphed from the information supplied in the polar notation form.

Note that the rectangular and polar notations are simply convenient methods of describing circuit conditions from mathematical and electrical viewpoints. An individual vector may be described in either form. For example, the vector OB(Figure 11-20) is:

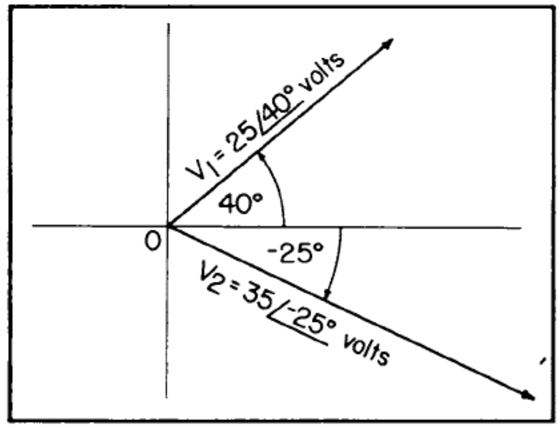

Figure 11-22 - Vector diagram, polar notation.

$$5\angle 53^\circ = 3 + j4$$

Circuit problems and discussions in this and following chapters will use these forms of notation.

Q21. What are the advantages of the polar form in describing a vector?

Q22. How would the vector $75\angle -90$ ohms be graphed?

11-14. Multiplication and Division of Vectors

Multiplication and division of vectors can be accomplished with more ease if the vectors are in polar form. The product of two vectors in polar form is found by multiplying the magnitudes of the vectors and algebraic addition of the angles.

Example. Determine the product of two vectors described as:

$$V_1 = 25 \angle 30^\circ$$

$$V_2 = 20 \angle 40^\circ$$

In polar form: $V_3 = V_1 + V_2$

$$V_3 = 25\angle 30^\circ \times 20\angle 40^\circ$$

Solution: Multiply the magnitudes of the vectors.

$$25 \times 20 = 500$$

Add the angles algebraically (taking note of their signs):

$$\angle 30^\circ + \angle 40^\circ = \angle 70^\circ$$

ombine the resultant magnitude and angle to escribe the resultant vector (V_3):

$$V_3 = 500 \angle 70^\circ$$

herefore: $V_3 = V_1 \times V_2$

$$500 \angle 70^\circ = 25\angle 30^\circ \times 20\angle 40^\circ$$

Division of two vectors in polar form is ccomplished by dividing the magnitude of the umerator by the magnitude of the denominator nd algebraic subtraction of the angles.

Example. Determine the quotient of two ectors in polar form described as:

$$V_1 = 30\angle 50^\circ$$

$$V_2 = 10\angle 25^\circ$$

n problem form:

$$V_3 = \frac{V_1}{V_2}$$

$$V_3 = \frac{30 \angle 50^\circ}{10 \angle 25^\circ}$$

olution: Divide the numerator by the denominator:

$$\frac{30}{10} = 3$$

lgebraically subtract the angles (taking note of heir signs):

$$(\angle 50^\circ) - (\angle 25^\circ) = \angle 25^\circ$$

ombine the resultant magnitude and angle to escribe the resultant vector (V_3)

$$V_3 = 3\angle 25^\circ$$

herefore: $V_3 = \frac{V_1}{V_2}$

$$3\angle 25^\circ = \frac{30\angle 50^\circ}{10\angle 25^\circ}$$

23. Determine the resultant of: $20\angle 45^\circ \times 2\angle 15^\circ$?

1-15. Trigonometric Functions

There are distinct relationships between the ides and angles of a right triangle. These elationships are discussed in depth in Volume and are summarized here for convenience. igure 11-23 illustrates the trigonometric relationships.

The relationships of the sides of a right riangle are proportionate to the magnitude of e included angles. These relationships define specific ratios and are referred to as the TRIGONOMETRIC FUNCTIONS.

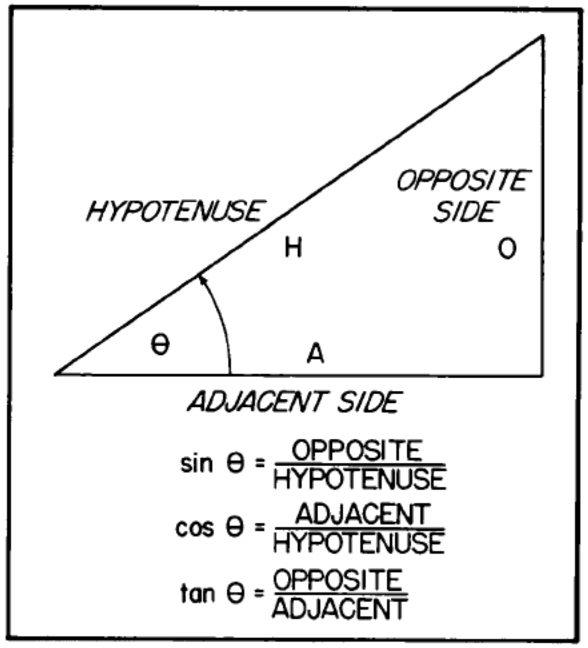

Figure 11-23 - Right triangle relationships.

It was shown previously (section 11-10) that the quantities impedance, inductive reactance, and resistance correspond to the sides of the right triangle. Figure 11-24 shows these relationships.

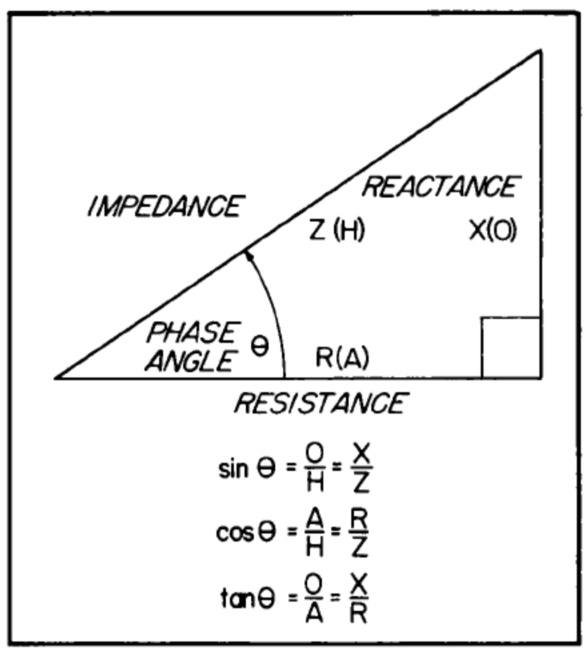

Figure 11-24 - Trigonometric and electrical relationships.

A21. The magnitude and phase angle are given.

A22. Downward along the Y-axis.

A23. $40\underline{/60^{\circ}}$

A few important equations may be determined from Figure 11-24.

Since: $$\sin \theta = \frac{X}{Z} \qquad (11\text{-}8)$$

then: $$\theta = \text{arc sin } \frac{X}{Z}$$

where: θ = angle of a vector with respect to the reference line, in degrees

arc sin = a form of notation which is read, "θ is equal to the angle whose sine is the ratio of $\frac{X}{Z}$".

X = reactance in ohms

Z = impedance in ohms

NOTE: The quantity X is representative of the total reactance in a circuit. $X = X_L$, $X = X_C$, $X = X_L - X_C$, depending on the type of circuit.

Since: $$\cos \theta = \frac{R}{Z} \qquad (11\text{-}9)$$

then: $$\theta = \text{arc cos } \frac{R}{Z}$$

since: $$\tan \theta = \frac{X}{R} \qquad (11\text{-}10)$$

then: $$\theta = \text{arc tan } \frac{X}{R}$$

The use of trigonometric functions as applied to the solution of ac circuits can best be shown through the use of example problems.

Example. In Figure 11-25, 50 volts are applied to a series ac circuit consisting of a coil with an X_L of 50 ohms and a 20 ohm resistor. Determine the total impedance and phase angle.

Solution: Determine phase angle

$$\theta = \text{arc tan } \frac{X_L}{R} \qquad (11\text{-}10)$$

$$\theta = \text{arc tan } \frac{50}{20}$$

$$\theta = \text{arc tan } 2.5$$

From the tables of natural trigonometric functions for angles in degrees and decimals, it is found that the arc tan (tangent) of 2.5 is 68.2 degrees.

Therefore: $\theta = 68.2$ degrees

Determine impedance:

$$\sin \theta = \frac{X_L}{Z} \qquad (11\text{-}8)$$

Transposing: $$Z = \frac{X_L}{\sin \theta}$$

$$Z = \frac{50}{\sin 68.2^{\circ}}$$

From the tables the sin of 68.2 degrees is found to be 0.9285.

Therefore: $$Z = \frac{50}{0.9285}$$

$$Z = 53.8 \text{ ohms}$$

The resultant vector (Z) can be described in polar notation as:

$$Z = 53.8\underline{/68.2^{\circ}}$$

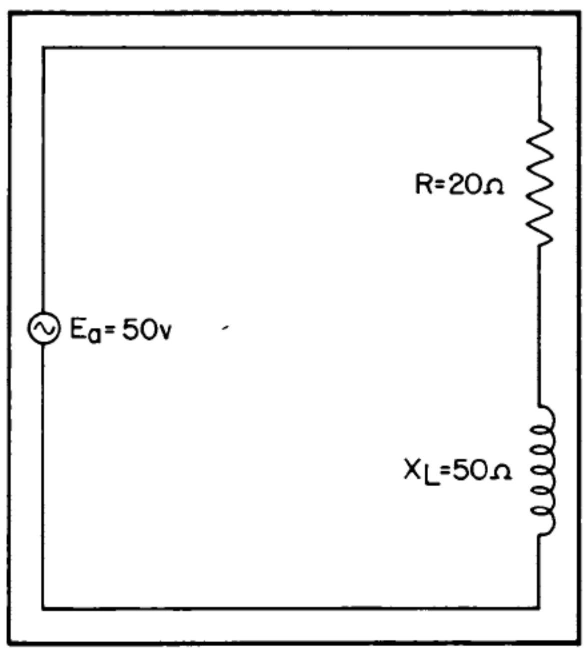

Figure 11-25 - Series ac RL circuit.

Example. For the circuit of Figure 11-25 the impedance of 53.8 ohms and the resistance of 20 ohms is given. Determine the phase angle and X_L:

Solution: Determine phase angle:

$$\theta = \text{arc cos } \frac{R}{Z} \qquad (11\text{-}9)$$

$$\theta = \text{arc cos } \frac{20}{53.8}$$

$$\theta = \text{arc cos } 0.371$$

From the tables the arc cos (cosine) of 0.371 is:

$$\theta = 68.2 \text{ degrees}$$

Determine X_L:

$$\sin \theta = \frac{X_L}{Z} \qquad (11\text{-}8)$$

Transposing:

$$X_L = \sin \theta \, Z$$

$$X_L = \sin 68.2^{\circ} \times 53.8$$

$$X_L = 0.9285 \times 53.8$$

$$X_L = 49.95 \text{ ohms}$$

$$X_L \cong 50 \text{ ohms}$$

As more advanced ac circuits are encountered, there will be many cases where it will be necessary to convert from rectangular form to polar form and vice versa.

Q24. What is the phase angle of a circuit whose impedance is 40 ohms and whose inductive reactance is 20 ohms?

11-16. Conversion from Rectangular to Polar Form

The process of converting a vector described in rectangular form to polar form involves the vector addition of the rectangular components and determination of the phase angle. Up to this point, the addition of quantities at right angles to each other required the use of a vector diagram or the Pythagorean theorem. Like the carpenter using a power saw instead of a hand saw, the trigonometric functions are used to simplify vector addition. The use of trigonometric function for converting from rectangular form is shown by example:

The total impedance of the circuit in Figure 11-25 is described in rectangular form.

$$Z = 20 + j\,50 \text{ ohms}$$

Determine Z and state in polar form:

Solution: Determine phase angle:

$$\theta = \text{arc tan } \frac{X_L}{R} \qquad (11\text{-}10)$$

Insert values from rectangular notation:

$$\theta = \text{arc tan } \frac{50}{20}$$

$$\theta = \text{arc tan } 2.5$$

$$\theta = 68.2 \text{ degrees}$$

Determine total impedance:

$$\sin \theta = \frac{X}{Z}$$

Transpose:

$$Z = \frac{X}{\sin \theta}$$

$$Z = \frac{50}{\sin 68.2^{\circ}}$$

$$Z = \frac{50}{0.9285}$$

$$Z = 53.8 \text{ ohms}$$

Therefore, $Z = 20 + j\,50$ ohms written in polar form is:

$$Z = 53.8 \angle 68.2^{\circ}$$

11-17. Conversion from Polar to Rectangular Form

The conversion of a quantity described in polar form involves the determination of its rectangular quantities through use of the trigonometric functions. Given the circuit impedance in polar form ($Z \angle n^{\circ}$), an equation can be derived for conversion into rectangular form ($Z = R \pm jX$) as follows:

As a trigonometric function:

$$\cos \theta = \frac{R}{Z} \qquad (11\text{-}9)$$

Transposed: $R = Z \cos \theta$

As a trigonometric function:

$$\sin \theta = \frac{X}{Z} \qquad (11\text{-}8)$$

Transposed: $X = Z \sin \theta$

Also: $Z = R \pm jX \qquad (11\text{-}7)$

Substituting the above trigonometric equations for R and X into equation (11-7) yields:

A24. 30 degrees. Equation (11-8).

$$Z = R \pm jX = \mathbf{Z} \cos\theta \pm \mathbf{Z} \sin\theta$$

where $\mathbf{Z}$ is a special symbol explained below.

Factoring the $\mathbf{Z}$ from the right side:

$$Z = \mathbf{Z}(\cos\theta \pm j\sin\theta) \qquad (11\text{-}11)$$

where: Z = the circuit impedance in rectangular form in ohms
$\mathbf{Z}$ = the absolute magnitude of the polar impedance in ohms
θ = the phase angle of the impedance in degrees

Example. It is desired to determine the value of the resistive and reactive quantities in a series circuit whose impedance is described in polar form as $2411\underline{/51.3^\circ}$ ohms

Given: $Z = 2411\underline{/51.3^\circ}$ ohms

Solution: Convert to rectangular form:

$$Z = \mathbf{Z}(\cos\theta + j\sin\theta) \qquad (11\text{-}11)$$

$$Z = 2411(\cos 51.3^\circ + j\sin 51.3^\circ)$$

$$Z = 2411(0.6252 + j0.7804)$$

$$Z = (2411 \times 0.6252) + (2411 \times j0.7804)$$

$$Z = 1507 + j1881.5 \text{ ohms}$$

Therefore, the resistive quantity of the circuit is equal to 1507 ohms and the reactive quantity is inductive (+j) and is equal to 1881.5 ohms.

Q25. Given a circuit impedance of $Z = 12\underline{/45^\circ}$ ohms, determine the value of inductive reactance.

11-18. Series RL Circuit Analysis

In this section, various methods of ac circuit analysis will be applied to a series RL circuit.

The student is reminded that while only one approach is given to the solution of an example problem, there are many cases where two or more approaches exist. The student is encouraged to try various approaches in an effort to develop a definite procedure best suited to himself.

Example. The series RL circuit in Figure 11-26 is to be analyzed. The following information is given:

Given: $E_a = 120$ volts

$f = 60$ cps

$R = 100$ ohms

$X_L = 145$ ohms

Find: $\theta = ?$ $E_L = ?$ $Z_t = ?$

$I_t = ?$ $E_R = ?$

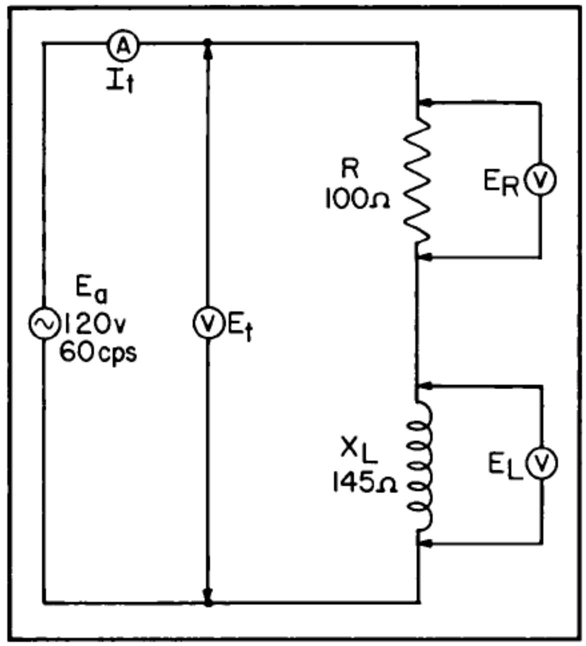

Figure 11-26 - Series RL circuit.

Solution: Determine the phase angle of the circuit.

Since X_L and R are given the phase angle is most easily determined by equation (11-10).

$$\tan\theta = \frac{X}{R} \qquad (11\text{-}10)$$

$$\theta = \arctan\frac{X_L}{R}$$

Inserting known terms:

$$\theta = \arctan\frac{145}{100}$$

$$\theta = \arctan 1.45$$

From the trigonometric tables the arc tan ("the angle whose tangent is") of 1.45 is found to be 55.4 degrees.

Thus: $\theta = 55.4$ degrees

Determine the total impedance of the circuit:

Since X_L, R, and θ are known, the impedance

can be found by application of either equation (11-8) or (11-9):

$$\cos \theta = \frac{R}{Z} \qquad (11\text{-}9)$$

transposed: $Z_t = \frac{R}{\cos \theta}$

insert values: $Z_t = \frac{100}{\cos 55.4^{\circ}}$

From the trigonometric tables the cosine of 55.4 degrees is found and inserted:

$$Z_t = \frac{100}{0.5678}$$

$$Z_t = 176.1 \text{ ohms}$$

In this case the above method is by far the easiest. Other methods that could have been employed are; the graphical method (triangle method using X_L and R vectors) and the Pythagorean theorem. The student is encouraged to use the alternate methods as a check for accuracy and a comparison between methods as to simplicity.

Determine total current of the circuit:

Since E_a, Z_t and θ are known, the total current (I_t) can be determined by application of Ohm's law for ac circuits.

$$I_t = \frac{E_a}{Z_t}$$

Writing E_a in polar form, $E_a = 120\underline{/0^{\circ}}$ volts.

NOTE: Assigning a quantity a zero degree phase denotes that quantity as being the reference or being in phase with the reference quantity. As such, it is graphed in standard position.

Writing Z_t in polar form, $Z_t = 176.1\underline{/55.4^{\circ}}$ ohms. Substituting polar forms in the equation:

$$I_t = \frac{120\underline{/0^{\circ}}}{176.1\underline{/55.4^{\circ}}}$$

Division of vector quantities is accomplished by dividing the magnitudes and algebraically subtracting the angles:

$$I_t = 0.681\underline{/-55.4^{\circ}} \text{ amps}$$

Notice that algebraic subtraction of the angles results in a negative sign thus indicating the current lagging the applied voltage.

Determine resistor voltage drop (E_R):

Since the value of resistance and the current through the resistance are known, E_R can be determined by application of Ohm's law:

$$I = \frac{E}{R}$$

transposed: $E_R = I_t \times R$

Total current is used as the current through the resistor because in a series circuit the current is common to all components.

Inserting values: $E_R = 0.681 \times 100$

$$E_R = 68.1 \text{ volts}$$

Current, being common in a series circuit, will be used as the reference. Since voltage and current are in phase in a resistor, E_R will be in phase with I_t. Writing E_R in rectangular form yields:

$$E_R = 68.1 + j0 \text{ volts}$$

Determine the inductor voltage drop (E_L):

$$E_L = I_t \times X_L$$

$$E_L = 0.681 \times 145$$

$$E_L = 98.75 \text{ volts}$$

E_L leads the current I_t through the coil by 90 degrees. Therefore, E_L is written in rectangular form as:

$$E_L = 0 + j98.75 \text{ volts}$$

Applying Kirchhoff's voltage law, the sum of the voltage drops around the circuit must equal the applied voltage. Mathematically this would appear to be:

$$E_a = E_R + E_L$$

Arithmetical addition or Arithmetically adding E_R and E_L would, of course, give the wrong results because quantities not in phase cannot be added arithmetically. Out of phase quantities may be combined directly when in rectangular form, therefore, substituting the rectangular notations for E_R and E_L will allow them to be added in accordance with the rules for addition of quantities in rectangular form.

$$\begin{array}{l} E_R = 68.1 + j0 \\ \underline{E_L = 0 \quad\; + j98.75} \\ E_t = 68.1 + j98.75 \text{ volts} \end{array}$$

A25. X_L = 8.48 ohms. From equation (11-11).
$Z = 8.48 + j\,8.48$ ohms.

Thus: $E_t = E_R + j\,E_L$

$E_t = 68.1 + j\,98.75$ volts

Since the circuit phase angle is known to be 55.4 degrees, the resultant vector (E_t) may be determined by insertion of values in one of the trigonometric equations. In substituting voltage for the quantities Z, X_L, and R the same right triangle relationships will apply. Figure 11-27 illustrates these relationships:

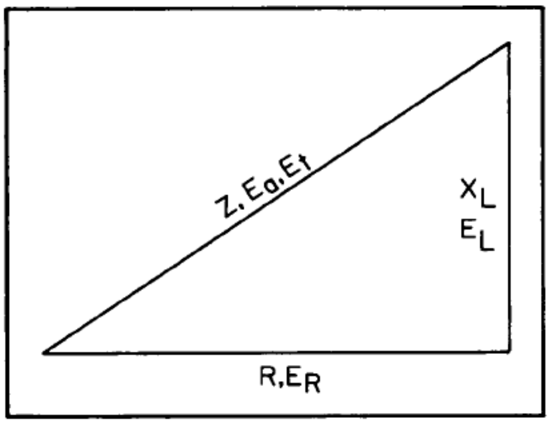

Figure 11-27 - Right triangle voltage relationships.

$$\cos \theta = \frac{R}{Z} \qquad (11\text{-}9)$$

Transposed: $Z = \dfrac{R}{\cos \theta}$

Substitute voltages:

$$E_t = \frac{E_R}{\cos \theta}$$

Insert values: $E_t = \dfrac{68.1}{\cos 55.4^{\circ}}$

$$E_t = \frac{68.1}{0.5678}$$

$$E_t = 119.9 \text{ volts}$$

$$E_t = 120 \text{ volts}$$

Thus, in Figure 11-26: $E_a = E_t$, thereby satisfying Kirchhoff's voltage law.

Q26. What form of notation is preferred if vectors are to be added?

Q27. What form of notation is used when vectors are to be multiplied or divided?

VARIATIONAL ANALYSIS FOR SERIES RL CIRCUITS

Figure 11-28A shows the series RL circuit that will be used during the following variational analysis. The conditions established in Figure 11-28A and B are understood to be the normal operating conditions of the circuit. The purpose of a variational analysis is to show the effect on overall circuit operation when one quantity is varied while all others are held constant. In the following analysis, the quantities of frequency, resistance, and inductance will be individually increased and decreased from their normal operating values.

11-19. Normal Operating Conditions

The circuit of Figure 11-28A is a series combination of a 12.73 millihenry coil and an 80 ohm resistor connected across an ac source of 100 volts RMS operating at a frequency of 1000 cycles per second. There is an ammeter connected in series with the circuit elements, to allow measurement of total circuit current flow. Voltmeters are connected in parallel with the individual components to measure resistor voltage, and inductor voltage (E_L). A voltmeter is also connected in parallel with the whole series circuit to allow measurement of the total voltage (E_t).

In the analysis of any problem the first step in a logical procedure is the listing of all known facts. From the circuit and meter readings of Figure 11-28A, the following facts are known:

E_a = 100 volts	E_L = 70.7 volts
f = 1 kc	E_R = 70.7 volts
R = 80 ohms	E_t = 100 volts
L = 12.73 millihenry	I_t = 0.884 amps

The only quantities of importance as yet undetermined are the inductive reactance, circuit impedance and circuit phase angle.

Determination of X_L: From the information available X_L can be found by application of Ohm's law for ac circuits.

$$I = \frac{E}{X}$$

Transposed: $X_L = \dfrac{E_L}{I_L}$

Since the total current in a series circuit is

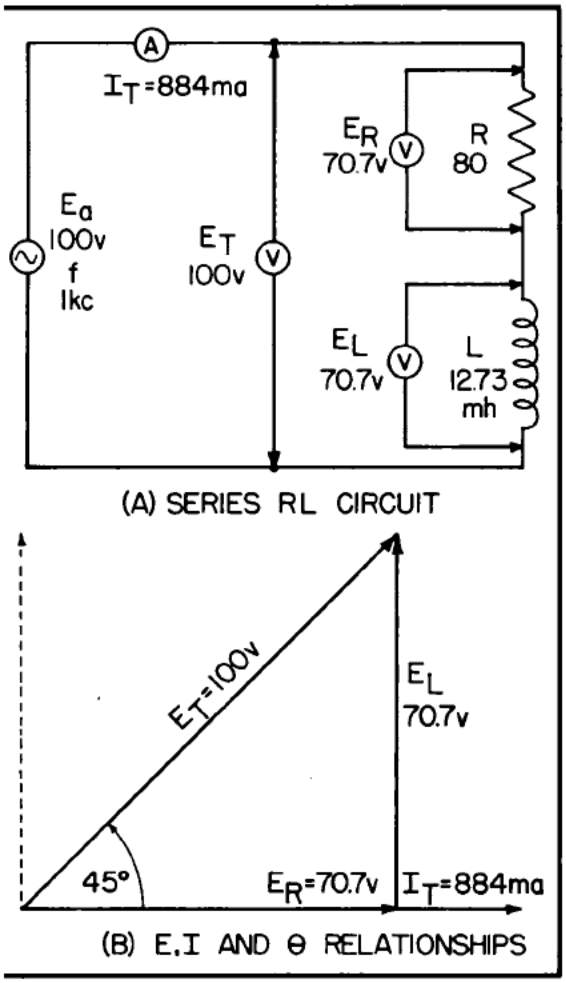

Figure 11-28 - RL circuit for variational analysis (normal operation)

lso the coil current:

$$X_L = \frac{70.7}{0.884}$$

$$X_L = 80 \text{ ohms}$$

$_L$ could also be found by application of the quation:

$$X_L = 2\pi f L$$

etermination of phase angle(θ):

$$\tan \theta = \frac{X}{R} \qquad (11\text{-}10)$$

ransposed: $$\theta = \arc\tan \frac{X_L}{R}$$

$$\theta = \arc\tan \frac{80}{80}$$

$$\theta = 45 \text{ degrees}$$

Determination of total impedance: (by Ohm's law)

$$Z_t = \frac{E_t}{I_t}$$

$$Z_t = \frac{100}{0.884}$$

$$Z_t = 113 \text{ ohms}$$

Z_t could also be determined by application of the trigonometric equations (11-8) or (11-9). Therefore, items to be added to the list of known facts about the circuit, under normal operating conditions are:

$$X_L = 80 \text{ ohms}$$

$$\theta = 45 \text{ degrees}$$

$$Z_t = 113\underline{/45^\circ} \text{ ohms}$$

The vector diagram (Figure 11-28B) illustrates the E, I, and θ relationships of the circuit. Current, being the common quantity, is graphed in standard position. E_R is seen to be in phase with I_t while E_L is leading I_t by 90 degrees. E_t, which is the same as E_a, is the vector sum of the component voltages and is leading I_t by 45 degrees.

11-20. Effect of Applied Frequency Variations

Frequency increased: Assume E_a, R, and L are held constant and frequency is doubled from 1 kc to 2 kc. Frequency variations will have no effect on resistance or inductance but will effect inductive reactance. Since X_L is a direct function of frequency, doubling the frequency will double the X_L. Thus, X_L will equal 160 ohms.

With 2 kc applied to the circuit the meter readings and known facts about the circuit (Figure 11-28) are as follows:

E_a = 100 volts	E_L = 89.44 volts
f = 2 kc	E_R = 44.72 volts
R = 80 ohms	E_t = 100 volts
L = 12.73 millihenry	I_t = 0.559 amps
X_L = 160 ohms	

It is noticed that with an increase in frequency and the accompanying increase in X_L, the total current decreases. This is an expected result. Since resistance remained constant, and X_L increased, Z_t (circuit opposition to current flow) must increase. Determination of the known quantities Z, and θ is carried out in the same manner as shown under normal operating conditions. They are found to be:

A26. Rectangular form.

A27. Polar notation.

$\theta = 63.5^{\circ}$

$Z_t = 178.7$ ohms

As was expected impedance is increased and since X_L is twice as large as resistance, it is natural to find an increase in the phase angle. Figure 11-29B is a vector diagram showing the E, I and θ relationships for the circuit under the increased frequency conditions. Figure 11-29B may be compared with Figure 11-29A (normal condition) to note effects.

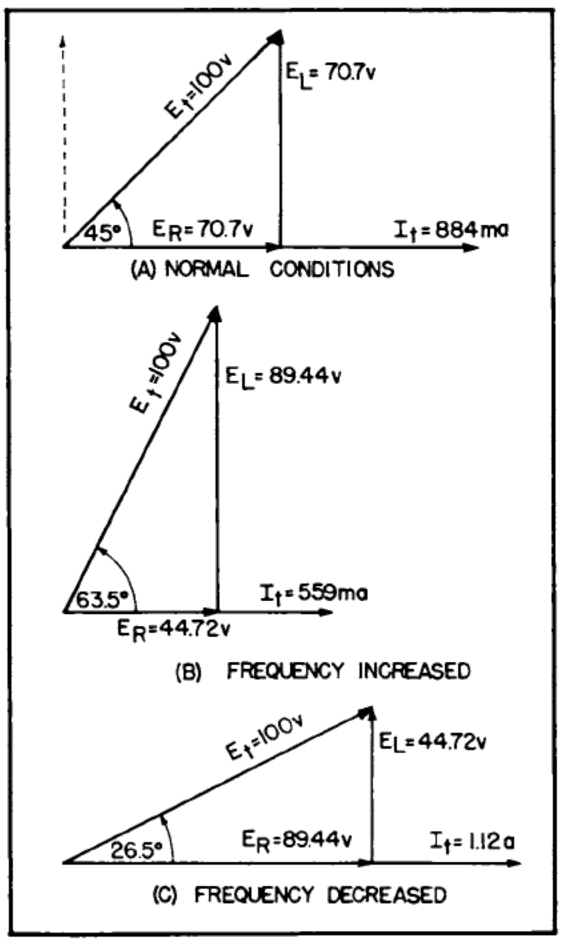

Figure 11-29 - Vector diagrams for effect of frequency variations on RL circuits.

Frequency Decreased: With the applied frequency decreased to 500 cps (half normal operating value) the meter indications and known facts are as follows:

E_a = 100 volts	E_L = 44.72 volts
f = 500 cps	E_R = 89.44 volts
R = 80 ohms	E_t = 100 volts
L = 12.73 mh	I_t = 1.12 amps
X_L = 40 ohms	

Computations show the following:

$\theta = 26.5$ degrees $\qquad Z_t = 89.6$ ohms

The above shows that decreasing frequency has decreased the X_L. Since X_L has decreased the impedance must also decrease. Lowering the opposition to current flow has the predictable effect of increasing current flow, as verified by the increased ammeter reading. By Ohm's law if current flow is increased through an unchanged resistor the voltage drop will increase. Comparison of Figure 11-29C with the normal vectors of Figure 11-29A will illustrate these effects.

Q28. Does the current lead or lag the voltage in a series RL circuit?

Q29. Will increasing the frequency increase or decrease the phase difference between circuit current and voltage?

11-21. Effect of Resistance Variations

To analyze the effect of an increase or decrease in the resistance of a series RL circuit E_a, f, and L will be held constant while resistance is varied.

Resistance Increased: With frequency constant X_L will not change. In a series circuit when one of the oppositions increase the total opposition (Z_t) will increase. Therefore, the operation of the circuit when the resistance is doubled will be similar (with a few exceptions) to the action when X_L was increased. With voltage held constant and impedance increased the current will decrease.

The circuit conditions and meter readings are listed below. Figure 11-30A shows a vector diagram of the circuit conditions when resistance is doubled. Comparison of Figure 11-30A and 11-29A will show the effect on circuit values.

E_a = 100V	E_R = 89.44V
f = 1 kc	E_t = 100V
R = 160 ohms	I_t = 559 ma
L = 12.37 mh	Z_t = 178.7 ohms
X_L = 80 ohms	θ = 26.5 degrees
E_L = 44.72V	

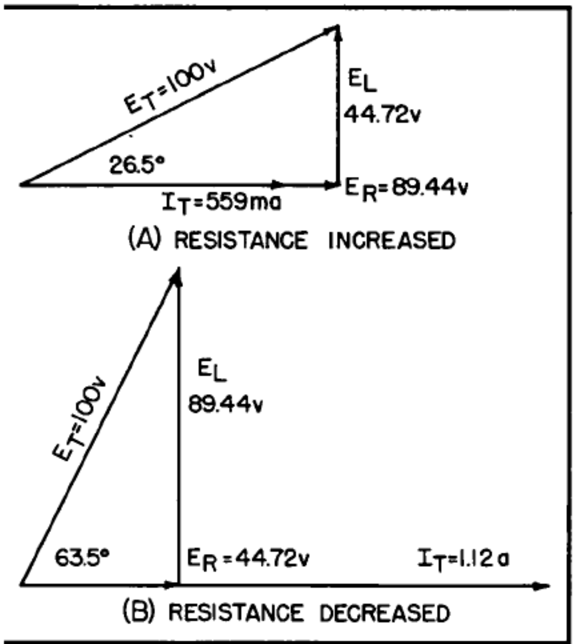

Figure 11-30 - Vector diagrams for effect of resistance variations on RL circuit.

Resistance Decreased: The effects with resistance decreased by half will be opposite to those noted when resistance was increased. Figure 11-30B illustrates the circuit under these conditions.

Q30. Will the power factor of the circuit increase or decrease with an increase of resistance?

11-22. Effect of Inductance Variations

Inductive reactance is a direct function of inductance. This is shown by the equation:

$$X_L = 2\pi f L \qquad (9\text{-}26)$$

From the equation it is noted that X_L is also a direct function of frequency. In section 11-20 it was shown that an increase in applied frequency caused the X_L, Z_t, θ, and E_L to increase while I_t and E_R decreased.

Due to the fact that X_L is related to inductance and frequency in the same manner, then a variation in inductance will have the same effect on a series circuit as a corresponding variation in frequency. In other words an increase in inductance will have the same effect as an increase in frequency and a decrease in inductance will produce the same effects as a decrease in frequency.

Q31. Explain why E_R decreases when inductance is increased.

11-23. Reactance, Resistance, and Phase Angle Relationships

If the inductive reactance of the circuit is large in respect to the resistance the phase angle will approach very close to 90 degrees. For practical considerations the impedance of the circuit is taken to be equal to the reactance of the coil if the X_L is ten times (or more) greater than the resistance. An example of this condition is given:

Example. Determine the total impedance and phase angle of a series RL combination consisting of a coil with an X_L of 100 ohms and a 10 ohm resistor.

Solution: Determine θ

$$\tan\theta = \frac{X}{R} \qquad (11\text{-}10)$$

$$\theta = \arctan\frac{X_L}{R}$$

$$\theta = \arctan\frac{100}{10}$$

$$\theta = \arctan 10$$

$$\theta = 84.3 \text{ degrees}$$

Determine Z_t:

$$\sin\theta = \frac{X}{Z} \qquad (11\text{-}8)$$

Transposed: $$Z_t = \frac{X_L}{\sin\theta}$$

$$Z_t = \frac{100}{0.9951}$$

$$Z_t = 100.4 \text{ ohms}$$

Thus is is seen that when the ratio of X_L to R is ten to one or greater, or if the phase angle is 84.3 degrees or greater the circuit is considered to consist of only an inductance.

The opposite effect is considered to be true when the resistance is ten (or more) times as great as the X_L. In other words the circuit is considered to consist of only a resistance if the ratio of resistance to X_L is ten to one or greater, or the phase angle is 5.73 degrees or less.

Q32. If X_L equals 15K ohms and resistance equals 110 ohms, how does the circuit appear to the generator?

A28. Lag.

A29. Increase

A30. Increase. P. F. is equal to the cosine of the angle. Cosine of the angle increases as the angle decreases.

A31. Increasing the inductance causes X_L and hence Z_t to increase. I_t is thereby decreased causing a decrease in resistor voltage.

A32. Inductive. X_L is more than ten times R.

POWER

Previously in this chapter power was discussed in relation to its action in ac circuits containing only pure components. True power was defined as the power actually dissipated by the resistance of the circuit. True power is expressed mathematically as:

$$P_t = P_a \times P.F. \qquad (11\text{-}3)$$

since $$P_a = E \times I$$

and $$P.F. = \cos\theta$$

then $$P_t = E \times I \times \cos\theta \qquad (11\text{-}12)$$

where: P_t = true power in watts
E = voltage in volts (RMS)
I = current in amps (RMS)
$\cos\theta$ = cos (phase angle between E & I)

11-24. Power in a Series RL Circuit

Power in a circuit containing both resistance and reactance will be a combination of resistive power and reactive power. Thus, for the circuit in Figure 11-31 the true power will be:

$$P_t = EI\cos\theta$$

$$P_t = 100 \times 7.07 \times 0.707$$

$$P_t = 500 \text{ watts}$$

The power curve is partly above the X-axis (Figure 11-31B) and partly below it. The axis of the power curve is displaced above the X-axis an amount proportional to the true power. The apparent power in this circuit is:

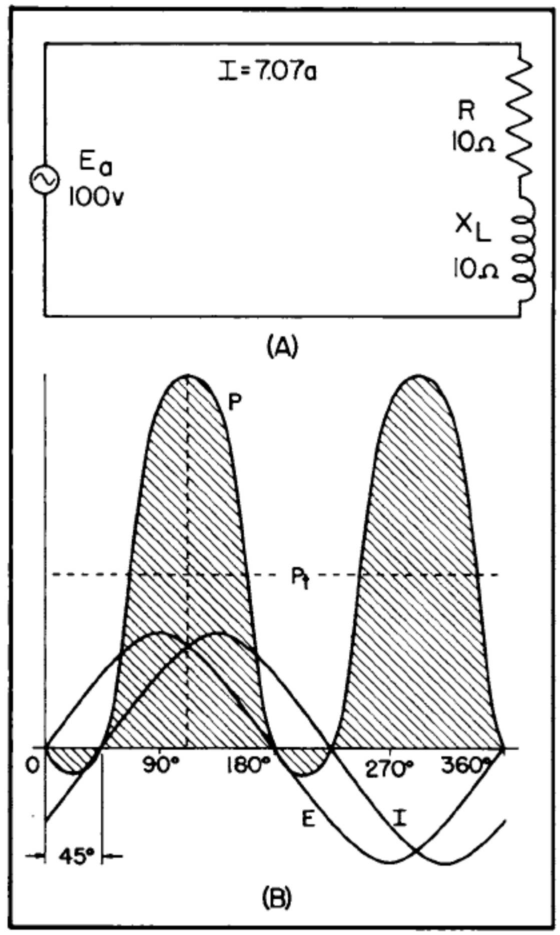

Figure 11-31 - Power in a circuit containing L and R in series.

$$P_a = E \times I$$

$$P_a = 100 \times 7.07$$

$$P_a = 707 \text{ volt-amperes}$$

The power factor is:

$$P.F. = \frac{\text{true power}}{\text{apparent power}}$$

$$P.F. = \frac{500}{707}$$

$$P.F. = 0.707 \text{ or } 70.7 \text{ percent}$$

The power absorbed by the reactive component will be returned to the circuit. The power absorbed by the resistor will be dissipated in the form of heat.

The power factor is the ratio of true power to apparent power and is indicated as a decimal

value between zero and one. Since the number one is the cosine of zero degrees, a power factor of one indicates a phase angle of zero degrees between the current and voltage in the circuit. In-phase current and voltage occur in a resistive circuit. Therefore, a power factor of one indicates a resistive circuit.

A power factor of zero (zero is the cosine of 90 degrees) indicates a purely reactive circuit. For good efficiency in an ac circuit the power factor should be close to unity.

Q33. What do the shaded areas of a power curve that extend below the axis represent?

FREQUENCY DISCRIMINATION

Any circuit which contains reactance will not respond equally to all frequencies. During variational analysis of a series RL circuit the overall action was found to be different for high frequencies than for low frequencies. In the process of analysis only a single frequency at a time was applied to the circuit. However, if a signal containing a range of frequencies is applied to a series RL circuit, the reaction of the circuit will be different for each individual frequency within this signal. For instance, as frequency increases total current decreases, and as frequency decreases, total current increases. There would be more current flowing for the low frequencies than for the high frequencies contained within the applied signal. Therefore, as far as current is concerned, this circuit is discriminating against the high frequencies.

11-25. Cut-Off Frequency

The value of resistance is not effected by a variation in frequency, but X_L is a direct function of frequency. Therefore, at a frequency of zero cycles per second (dc) the opposition of the coil is negligible and the circuit is considered to be resistive, the phase angle is zero and the true power will be at its maximum value.

Example. Consider a series circuit consisting of an 80 ohm resistor and a 12.73 mh coil with a 100 volts dc applied. Since the phase angle is zero and the ratio of resistance to reactance is greater than ten to one, the impedance of the circuit will equal 80 ohms. The current will be:

$$I_t = \frac{E_t}{Z_t}$$

$$I_t = \frac{100}{80}$$

$$I_t = 1.25 \text{ amps}$$

The true power of the circuit will be at its maximum value:

$$P_t = E \times I \times \cos\theta \qquad (11\text{-}12)$$

$$P_t = 100 \times 1.25 \times 1$$

$$P_t = 125 \text{ watts}$$

The dc source is replaced with an ac source (of 100 V RMS) and the frequency is increased.

The X_L of the coil will increase while the value of the resistor will remain at 80 ohms. When the frequency reaches 500 cycles per second, the X_L will have increased to 40 ohms. The phase angle of the circuit will be:

$$\theta = \text{arc tan}\ \frac{X_L}{R}$$

$$\theta = \text{arc tan}\ \frac{40}{80}$$

$$\theta = 26.5 \text{ degrees}$$

A computation of values will yield the following

$$Z_t = 89.6 \text{ ohms}$$

$$I_t = 1.12 \text{ amps}$$

Using the above values the true power of the circuit is seen to be decreasing as frequency increases:

$$P_t = E \times I \times \cos\theta \qquad (11\text{-}12)$$

$$P_t = 100 \times 1.12 \times 0.8949$$

$$P_t = 100.2 \text{ watts}$$

As frequency is increased further the current will continue to decrease and X_L will continue to increase. Eventually a frequency will be reached at which the X_L equals the resistance. For instance, at 1 kc:

$$X_L = 2\pi f L$$

$$X_L = (6.28) \times (1 \times 10^3) \times (12.73 \times 10^{-3})$$

$$X_L = 79.94 \text{ ohms}$$

Therefore, at 1 kc, the X_L is equal to the resistance. The phase angle of the circuit is 45 degrees and the total impedance is 113 ohms. Since $X_L = R$, then the voltage drops must be equal ($E_L = E_R$). The total current is decreased to:

$$I_t = \frac{E_t}{Z_t}$$

$$I_t = \frac{100}{113} = 0.884 \text{ amps}$$

A33. Reactive power which is returned to the source.

The true power of the circuit is:

$$P_t = E \times I \times \cos \theta \qquad (11\text{-}12)$$

$$P_t = 100 \times 0.884 \times 0.707 = 62.5 \text{ watts}$$

It is noted that the true power has decreased to half of its maximum value of 125 watts. The frequency at which $X_L = R$, $E_L = E_R$, and the true power has decreased to half of its maximum values is called the CUT-OFF-FREQUENCY, or HALF-POWER POINT. The term cut-off frequency is used because for frequencies beyond the cut-off point the response of the circuit is considered (in most cases) to drop below a usable value.

When the frequency is zero the coil will have no X_L. Therefore, all the voltage will be dropped across the resistance.

$$E_R = E_t$$

$$E_R = 100 \text{ V (when } f = 0)$$

When the cut-off frequency (for this particular circuit) of 1 kc is reached the resistive voltage drop will decrease to:

$$E_R = I_t \times R$$

$$E_R = 0.884 \times 80$$

$$E_R = 70.7 \text{ V (when } f = 1 \text{ kc)}$$

The resistor voltage has decreased to 70.7% of its maximum value. Thus, it can be said that at cut-off frequency E_R will equal 70.7% of E_t. Mathematically this is expressed as:

$$E_R = E_t \times 0.707 \text{ (at cut-off frequency)}$$

For this reason the cut-off, or half-power point is also called the "0.707 point".

A formula can be developed for determining the cut-off frequency in the following manner:

At the cut-off frequency (f_{co})

$$R = X_L$$

since $X_L = 2\pi f L$

then $R = 2\pi f L$

substitute f_{co} for f: $R = 2\pi f_{co} L$

Transposing to solve for f_{co} yields:

$$f_{co} = \frac{R}{2\pi L} \qquad (11\text{-}13)$$

where: f_{co} = cut-off frequency in cps
R = resistance in ohms
L = inductance in **henries**
2π = a constant (6.28)

In many electronic circuit applications it is necessary to pass the low frequencies and not pass the high frequencies or vice versa. A network designed to pass some frequencies while eliminating others is called a FILTER CIRCUIT. The frequencies that are removed and do not appear in the output are said to be ATTENUATED or discriminated against. The manner in which the output is taken from a series circuit determines whether the circuit will discriminate against high or low frequencies.

Q34. Why is the X_L of the coil negligible at a frequency of zero cps?

Q35. Define the term cut-off frequency.

Q36. Does the voltage on a component decrease to half of its maximum value at the half-power point?

11-26. Low Pass Filter (RL)

It was shown in section 11-25 that at frequencies below the cut-off frequency most of the voltage is developed across the resistor. Thus, a series RL circuit with the output taken across the resistor would be termed a LOW PASS FILTER because the largest output would be at frequencies below the f_{co}. Figure 11-32 illustrates a low pass filter network. The output (E_{out}) is taken across the resistor. The voltmeter connected across the circuit measures the input voltage (E_{in}). The generator (E_a) supplies a 300 volt RMS signal containing all frequencies.

Application of equation (11-13) will yield the f_{co} of this circuit.

$$f_{co} = \frac{R}{2\pi L} \qquad (11\text{-}13)$$

$$f_{co} = \frac{188.4}{6.28 \times 3 \times 10^{-3}}$$

$$f_{co} = 10 \text{ kc}$$

Thus, the low pass RL circuit just described will produce a usable output for frequencies from zero to approximately 10 kc. The input

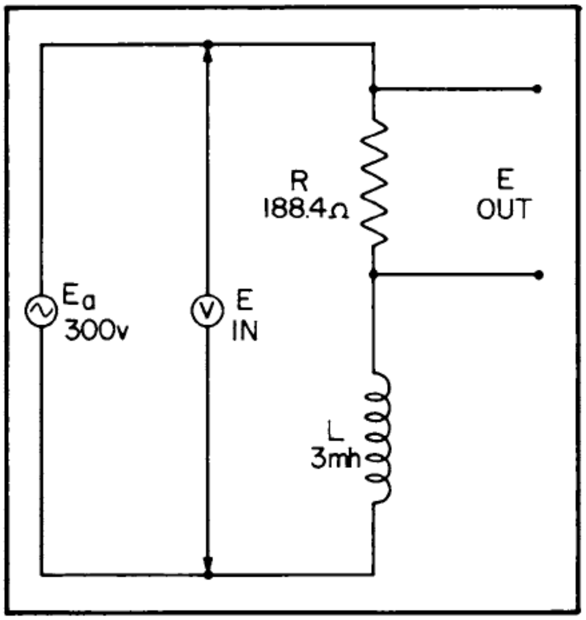

Figure 11-32 - Low pass filter (RL)

voltage will remain at 300 V, but the output voltage will drop off as f_{co} is approached. It was stated that at f_{co} the resistor voltage drop (E_{out}) will equal 70.7% of the total voltage (E_{in}). Therefore, at f_{co} the output voltage will have decreased to:

$$E_{out} = E_{in} \times 0.707$$

$$E_{out} = 300 \times 0.707$$

$$E_{out} = 212 \text{ volts}$$

A circuit which passes low frequencies and attenuates high frequencies is also called a HIGH FREQUENCY DISCRIMINATOR.

11-27. High Pass Filter (RL)

Taking the output of a series RL circuit from across the inductor instead of the resistor will produce results opposite to those of a low pass filter. Figure 11-33 shows an RL circuit connected as a high pass filter. Since the f_{co} for a high pass filter is computed in the same manner as for the low pass the f_{co} for the circuit of Figure 11-33 will also be 10 kc.

The high pass filter will pass only frequencies above the cut-off point and attenuate frequencies below the cut-off point. A high pass filter is also called a LOW FREQUENCY DISCRIMINATOR.

Q37. Will a high frequency discriminator pass high or low frequencies?

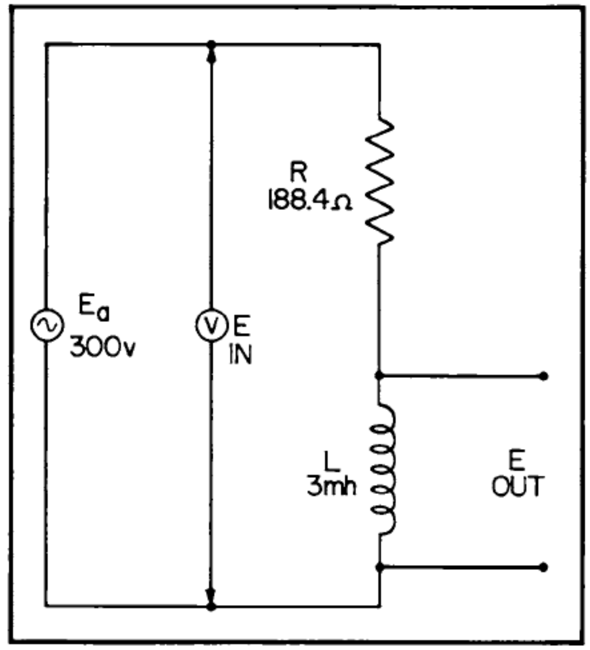

Figure 11-33 - High pass filer (RL)

Q38. In a high pass filter, frequencies below f_{co} will receive more or less attenuation than the frequencies above f_{co}?

EFFECTIVE RESISTANCE—NONUSEFUL ENERGY LOSSES IN AC CIRCUITS

11-28. Energy Concept of Resistance

When current flows through a conductor having appreciable resistance the flow is accompanied by the generation of heat. Work is done in moving the electrons through the conductors resistance. The energy converted into heat is not returned to the source when the current falls, but is expended rather than stored. Thus, energy is stored periodically in an inductance but always expended in a resistance.

Because resistance is the only circuit quality capable of expending electrical energy, all energy expended in any circuit can be identified in electrical terms, one factor of which is effective resistance. The effective resistance, R_{ac}, of any circuit may be defined as the ratio of the true power absorbed by the circuit to the square of the effective current flowing in the circuit, or

$$R_{ac} = \frac{P_t}{I^2}$$

When the power is expressed in watts and the current is expressed in amperes, the effective resistance will be in ohms. The dc circuit

A34. A coil exhibits reactance only when the current is changing.

A35. The point at which the true power of the circuit has decreased to half of its maximum value.

A36. No. Voltage decreases to 70.7% of maximum.

A37. Low frequencies.

A38. More.

---

resistance as measured by an ohmmeter, or dc bridge, may be considerably lower than the effective ac resistance as calculated from the readings on a wattmeter and an ammeter.

Example. Assume that an inductor (with a lumped resistance representing all losses) is drawing 10 amperes of current from a source and dissipating one kilowatt of power. The effective resistance (ac resistance) as calculated from wattmeter and ammeter readings is:

$$R_{ac} = \frac{P_t}{I^2}$$

$$R_{ac} = \frac{1000}{(10)^2}$$

$$R_{ac} = 10 \text{ ohms}$$

The dc resistance measured between the input terminals, with an ohmmeter for example, is 0.5 ohms. Thus, in this example the effective ac resistance is 10 divided by 0.5 or 20 times the dc resistance.

11-29. Effective Resistance of Conductors

The effective (ac resistance of electrical conductors) is frequently higher than their dc resistance especially when they are being used in high-frequency circuits, as in radio transmitters and receivers.

Direct current is distributed uniformly throughout the cross-sectional area of a homogeneous conductor. For example, if a conductor having a cross-sectional area of 1000 circular mils is carrying one ampere of direct current then one-thousandth of an ampere (one milliampere) is flowing in each circular mil of cross-sectional area. However, when the current in the conductor varies in amplitude, this uniform distribution throughout the conductors cross-section is no longer obtained. The accompanying magnetic field is strongest near the center of the conductor and weaker at the circumference. The varying field induces a voltage in the conductor that opposes the change in current. The voltage induced in that portion of the conductor near the center is greater than the voltage induced in the outer surface of the conductor. The total opposition to the current flow includes the effect of this induced EMF and is greater near the center of the conductor than at the surface. Therefore, the current divides inversely with the opposition (more of the current flowing near the circumference and less near the center of the conductor).

The overall result of this action is a decrease in the available area of cross-section to conduct the current and an increase in conductor resistance. The decrease in area and increase in resistance become pronounced at high frequencies, at high current densities, and at high magnetic flux densities. This action is called SKIN EFFECT. It represents the tendency of ac conductors to carry the circuit current on the surface, or skin, of the conductors rather than uniformly throughout their cross-section. As a result of this tendency, many electrical conductors are made of hollow tubing in order to save the added weight and expense of the unused central portion of the solid conductor. The ac-resistance of a conductor is approximately proportional to the frequency and length of the conductor, and inversely proportional to its diameter

Q39. Give the main difference between the dc resistance and the effective resistance of a conductor.

11-30. Effective Resistance of Inductors

When a conductor is wound in the form of a coil, the current is concentrated in the inner sides of the turns and into an area much smaller than would be the case in an isolated straight conductor. This action results in a large increase in effective resistance. The area in which the current is concentrated decreases as the frequency increases hence effective resistance will increase with frequency. When two or more conductors carrying alternating current are so placed that the magnetic field of one reacts with the field of the other, the resultant field around each conductor is no longer uniform. The change in current distribution in a conductor due to the action of an alternating current in a nearby conductor is called PROXIMITY EFFECT.

The proximity effect decreases as the separation between conductors increases. Thus, to lower the effective resistance of radio frequency inductance coils, it is common practice to space the turns a distance equal to the diameter of the

conductor. This decreases the reaction between magnetic field of adjacent turns and permits the current to distribute itself over a large area in the cross-section of each turn.

QUALITY OR Q

11-31. The Quality of an Inductor

The ratio of the energy stored in an inductor during the time the magnetic field is being established to the losses in the inductor during the same time is called the QUALITY, or Q of the inductor, it is also called the FIGURE OF MERIT of the inductor. This ratio is:

$$Q = \frac{I^2 X_L t}{I^2 Rt}$$

cancellation yields:

$$Q = \frac{X_L}{R} \qquad (11\text{-}14)$$

where: Q = a number representing the quality of the inductor.
X_L = inductive reactance of the coil in ohms
R = combined dc and ac resistances of the coil in ohms

The Q of the inductor is therefore equal to the ratio of the inductive reactance to the effective resistance in series with it, and it approaches a high value as R approaches a low value. Thus, the more efficient the inductor, the lower the losses in it and the higher is the Q.

In terms of the impedance triangle (Figure 11-34):

$$Q = \frac{X_L}{R}$$

$$Q = \tan \theta$$

where θ is the phase angle between the hypotenuse, Z, and the base, R. As θ approaches 90^o, tan θ approaches infinity, and the coil losses approach zero.

The Q of a coil does not vary extensively within the operating limits of a circuit. It would seem, from equation (11-14), that since X_L is a direct function of frequency that Q also must be a direct function of frequency. Such is not the case. It is true that as frequency increases the X_L would increase, but as frequency increases the effective resistance of the coil also increases. Since Q is an inverse function of the effective resistance the net effect of a frequency increase is to leave Q relatively unchanged.

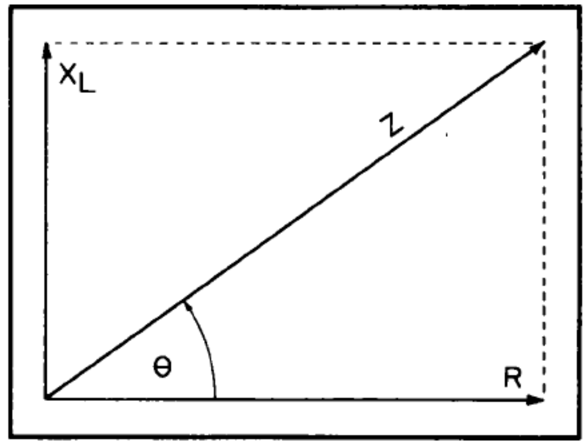

Figure 11-34 - Impedance triangle.

Q40. Decreasing the spacing between the turns of a coil will have what effect on the Q?

SERIES RC CIRCUITS

It has been shown that when a reactive component and a resistive component are combined in the same circuit the opposition of the circuit to current flow is termed impedance and is measured in ohms. The impedance of an RC circuit is determined in the same manner as was done in the study of RL circuits. An impedance triangle can be drawn for an RC circuit also, the only difference being that the capacitive reactance vector is drawn downward along the Y-axis and is 180 degrees displaced from the position occupied by the X_L vector.

11-32. E, I and Z Relationships

Since the operational analysis of an RC series circuit follows very closely the procedure used previously in RL circuits an example problem will be used to explain the various relationships.

Example. Figure 11-35 illustrates a series RC circuit consisting of a capacitor exhibiting 40 ohms of capacitive reactance and a resistor of 20 ohms connected across a 134 volt source. The source is operating at 100 cycles per second and is delivering 3 amps of current to the circuit.

The impedance of the circuit can be determined by the impedance trianble method of Figure 11-35 or by use of the trigonometric equations (11-8), (11-9), or (11-10).

A39. The dc resistance if the opposition offered to direct current flow only whereas effective resistance is the combination of dc and ac resistance and takes the effect of frequency into account.

A40. Decrease the Q. Proximity effect will increase the effective resistance which in turn will decrease the Q.

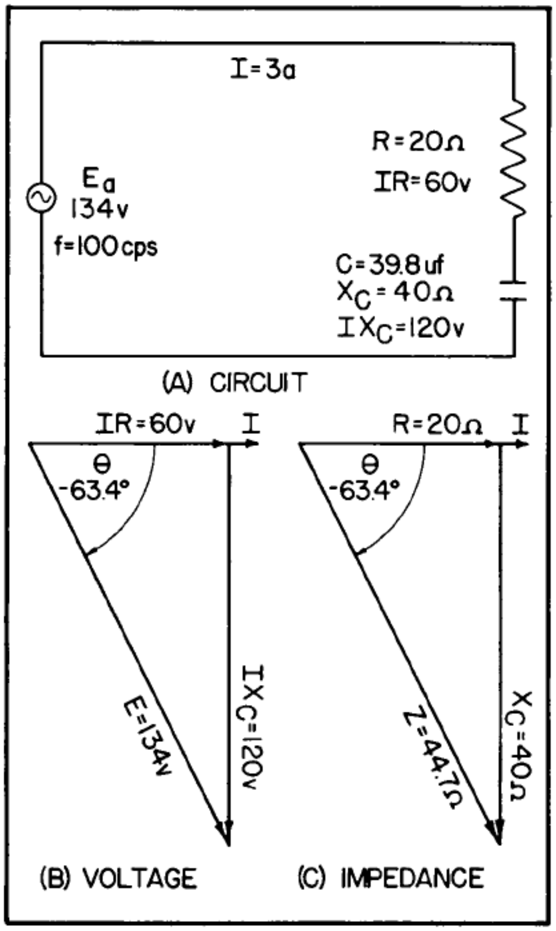

Figure 11-35 - Capacitive reactance and resistance in series.

Determine phase angle.

$$\tan \theta = \frac{X}{R} \quad (11\text{-}10)$$

$$\theta = \arc\tan \frac{X_C}{R}$$

$$\theta = \arc\tan \frac{40}{20}$$

$$\theta = -63.4 \text{ degrees}$$

Notice that the phase angle is minus 63.4 degrees due to the capacitive reactance causing a lagging impedance.

Determine the impedance:

$$\sin \theta = \frac{X}{Z} \quad (11\text{-}8)$$

Transposed $Z = \dfrac{X}{\sin \theta}$

$$Z = \frac{40}{0.8942}$$

$$Z = 44.7 \text{ ohms}$$

The impedance of the RC circuit described in rectangular form is:

$$Z = R \pm jX \quad (11\text{-}7)$$

$$Z = 20 - j\,40 \text{ ohms}$$

The impedance of the RC circuit described in polar form is:

$$Z = 44.7\underline{/-63.4^\circ} \text{ ohms}$$

The voltage across R and C are 90 degrees out of phase and equal to 60 volts and 120 volts respectively, as shown in the vector diagram in Figure 11-35B. The voltage across C is represented as I x X_C and is plotted vertically downward from the horizontal axis in order to indicate that the current leads the voltage across the capacitor by 90°. Angle θ between the voltage across the capacitor and the circuit current is represented as -90° because it is measured clockwise from the horizontal reference vector.

The total voltage is equal to the vector sum of IR and IX_C and is represented in the Figure 11-35B as the hypotenuse of a right triangle the base of which represents the voltage across R, having an effective value of 60 volts and the voltage drop across C, having an effective value of 120 volts.

The total voltage (E_t) is:

$$E_t = E_R - jE_{X_C}$$

$$E_t = 60 - j\,120 \text{ volts}$$

The true power of the circuit is:

$$P_t = E \times i \times \cos\theta$$

$$P_t = 134 \times 3 \times 0.4478$$

$$P_t = 180 \text{ watts}$$

The apparent power of the circuit is:

$$P_a = E \times I$$

$$P_a = 134 \times 3$$

$$P_a = 402 \text{ volt-amperes}$$

The power factor is equal to the cosine of the angle. Therefore, P. F. is:

$$P.F. = 0.4478$$

Q41. Will the total current in a series RC circuit lag or lead the total voltage?

VARIATIONAL ANALYSIS FOR SERIES RC CIRCUITS

Figure 11-36A shows the series RC circuit that will be used during the following variational analysis. The conditions established in Figure 11-36A and B will be the normal operating conditions of the circuit.

11-33. Normal Operating Conditions

The circuit of 11-36A is a series combination of a 1.99 microfarad capacitor and an 80 ohm resistor connected across an ac source of a 100 volts RMS operating at a frequency of 1000 cycles per second. Meters are connected at the appropriate places in order to monitor the circuit conditions. From Figure 11-36A the following facts are known about the circuit:

E_a = 100V	E_C = 70.7 V
f = 1 kc	E_R = 70.7 V
R = 80 ohms	E_t = 100 V
C = 1.99 uf	I_t = 0.844 amps

The vector diagram of Figure 11-36B shows the E, I, and phase relationships of the circuit. Current is used as the reference because the circuit is series connected. E_R is in phase with total current (I_t) while the capacitor voltage (E_C) is lagging the circuit current by 90 degrees. The voltmeter (E_t) reads the total voltage which is the vector sum of the two voltages (E_R and E_C). The total voltage is seen to be lagging the current by 45 degrees.

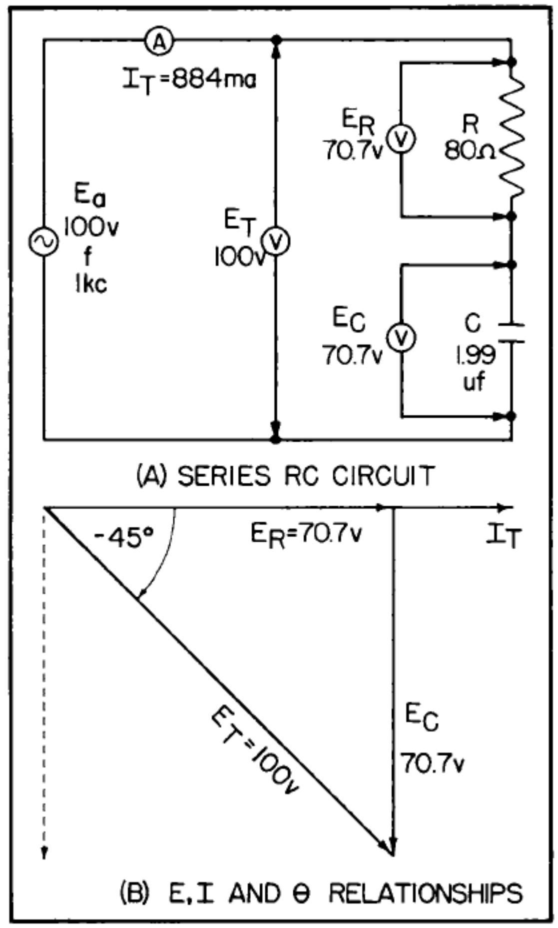

Figure 11-36 - RC circuit for variational analysis (normal operation).

From the information given the circuit phase angle, capacitor reactance, and total impedance can be found as follows:

$$X_C = \frac{1}{2\pi f C} \qquad (10\text{-}24)$$

$$X_C = 80 \text{ ohms}$$

Phase angle can be determined by trigonometric equations:

$$\theta = \arc\tan \frac{X_C}{R}$$

$$\theta = 45 \text{ degrees}$$

A41. Lead. Circuit phase angle is negative.

Total impedance can be determined by trigonometric equations or by Ohm's law:

$$Z_t = \frac{E_t}{I_t}$$

$$Z_t = 133 \text{ ohms}$$

$$Z_t = 133\underline{/-45^{\circ}} \text{ ohms}$$

11-34. Effect of Applied Frequency Variation

FREQUENCY INCREASED: Assume E_a, R, and C to be held constant and frequency is doubled from 1 kc to 2 kc. Frequency variations will have no effect on the resistance or the capacitance, but will effect the capacitive reactance. X_C is an inverse function of frequency so doubling the frequency will decrease the X_C to half of its normal operating value. Thus, X_C will equal 40 ohms. In reactangular form Z_t is described as:

$$Z_t = R - jX_C$$

$$Z_t = 80 - j\,40 \text{ ohms}$$

Conversion of total impedance to polar form by use of the trigonometric equations yields:

$$Z_t = 89.6\underline{/-26.5^{\circ}} \text{ ohms}$$

(conversion procedure covered in section 11-16)

Since the impedance has decreased the total current will increase. The meter readings are noted as follows:

I_t = 1.12 amps	E_R = 89.44 V
E_C = 44.72 V	E_t = 100 V

The true power of the circuit is found by the equation:

$$P_t = E \times I \times \cos\theta$$

$$P_t = 100 \times 1.12 \times 0.8949$$

$$P_t = 100.2 \text{ watts}$$

Figure 11-37A shows circuit conditions with frequency increased.

FREQUENCY DECREASED: With the applied frequency decreased to 500 cps (half normal) operating value) the circuit conditions are as follows:

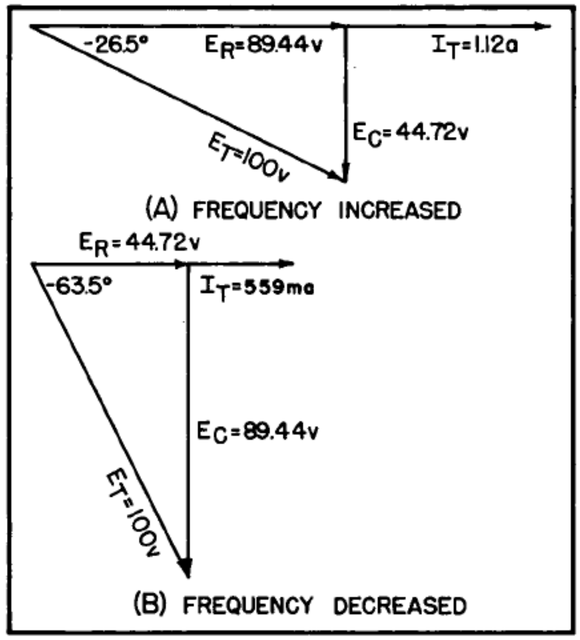

Figure 11-37 - Vector diagram for effect of frequency variations on RC circuit.

E_a = 100V	E_C = 89.44V
f = 500 cps	E_R = 44.72V
R = 80 ohms	I_t = 559 ma
X_C = 160 ohms	E_t = 100V

Calculation of Z_t by normal procedure yields:

$$Z_t = 178.7\underline{/-63.5^{\circ}} \text{ ohms}$$

Figure 11-37B shows circuit conditions with frequency decreased.

11-35. Effects of Varying R and C

Varying the resistance in a series RC circuit will have the same effect on circuit values as varying the resistance in a series RL circuit. A variation in the capacitance of a series RC circuit will produce effects exactly opposite to those produced by the variation of inductance in a series RL circuit.

The effect of the variation of one circuit quantity on the value of all other circuit quantities for RL and RC series circuits is tabulated in Table 11-2.

During the analysis of the RL circuit it was shown (section 11-23) that when the resistance is ten times (or more) larger than the reactance, or vice verse, the impedance of the circuit is taken to be equal to the larger component. The same considerations apply to RC

VARIABLE QUANTITIES		SERIES RL CIRCUITS									SERIES RC CIRCUITS								
		X_L	E_L	E_R	I_T	Z_T	P_a	P_t(TRUE)	P.F.	θ	X_C	E_C	E_R	I_T	Z_T	P_a	P_t(TRUE)	P.F.	θ
f	↑	↑	↑	↓	↓	↑	↓	↓	↓	↑	↓	↓	↑	↑	↓	↑	↑	↑	↓
	↓	↓	↓	↑	↑	↓	↑	↑	↑	↓	↑	↑	↓	↓	↑	↓	↓	↓	↑
L	↑	↑	↑	↓	↓	↑	↓	↓	↓	↑									
	↓	↓	↓	↑	↑	↓	↑	↑	↑	↓									
C	↑										↓	↓	↑	↑	↓	↑	↑	↑	↓
	↓										↑	↑	↓	↓	↑	↓	↓	↓	↑
R	↑	→	↓	↑	↓	↑	↓	↓	↑	↓	→	↓	↑	↓	↑	↓	↓	↑	↓
	↓	→	↑	↓	↑	↓	↑	↑	↓	↑	→	↑	↓	↑	↓	↑	↑	↓	↑
E_A	↑	→	↑	↑	↑	→	↑	↑	→	→	→	↑	↑	↑	→	↑	↑	→	→
	↓	→	↓	↓	↓	→	↓	↓	→	→	→	↓	↓	↓	→	↓	↓	→	→

↑ INCREASE

↓ DECREASE

→ REMAIN CONSTANT

TABLE 11-2
Comparison of RL and RC series circuits.

series circuits. If X_C is equal to, or greater than, ten times the resistance then the total impedance is equal to the X_C. Conversely, if resistance is ten or more times larger than X_C then Z_t is equal to the resistance. If the phase angle is greater than 84. 3 degrees, then $Z_t = X_C$. If the phase angle is less than 5. 73 degrees, then the circuit is considered to be resistive and $Z_t = R$.

The effect of a varying quantity on the operation of series RL and series RC circuits is summed up in Table 11-2.

The chart is divided into five major horizontal lines. Each of these major lines contains a quantity that can vary (frequency, inductance, etc.). Each major line is subdivided into two lines of which the top subdivision indicates an increase in the quantity and the bottom subdivision indicates a decrease in the quantity. The chart is divided vertically into two major sections, one for series RL and one for series RC.

The first column at the extreme left indicates the quantity to be varied and contains frequency (f), inductance (L), capacitance (C), resistance (R), and applied voltage (E_a). The second column contains arrows indicating an increase or decrease in the quantity. The remaining vertical columns contain the effected circuit quantities.

The use of Table 11-2 is best explained by an example.

Example. Determine the effect on the capacitor voltage (E_C) in a series RC circuit when the resistance is increased.

Solution: Look down the first column to resistance. Take the subdivision with the arrow pointing up to indicate increased resistance. Follow this subdivision across to the vertical

column (E_C) under series RC circuits. The arrow, in the block at the junction of the row and column, is pointing down. Therefore, when resistance is increased the capacitor voltage will decrease.

An arrow pointing to the right indicates there is no change.

Q42. What effect will an increase in frequency have on capacitance?

FREQUENCY DISCRIMINATION

An RC series circuit, since it contains reactance, will not respond equally to all frequencies. Therefore, an RC circuit will exhibit frequency discrimination similar in many respects to that encountered in the RL circuits.

The terms cut-off frequency, half-power point, and critical frequency have the same meanings as previously defined. Series circuits containing reactances are ac voltage dividers. The voltage developed across a reactive component depends on the reactance of the component which in turn depends on the frequency. X_C is an inverse function of frequency. Therefore, at a frequency of zero cycles per second (dc) the opposition of the capacitor will be maximum, and all of the applied voltage will be dropped across the capacitor. As frequency is increased the reactance of the capacitor will decrease and the voltage will divide between the resistor and capacitor. The cut-off frequency is reached when the voltage divides equally between the R and C.

11-36. Cut-Off Frequency of an RC Circuit

A formula can be developed for determining the cut-off frequency (f_{co}) of an RC series circuit in the following manner:

Since f_{co} occurs when:

$$R = X_C$$

Substituting the equations for X_C:

$$R = \frac{1}{2\pi f C}$$

Substituting f_{co} for f and transposing yields:

$$f_{co} = \frac{1}{2\pi RC} \qquad (11\text{-}15)$$

Example. Determine the cut-off frequency of a series RC circuit consisting of an 80 ohm resistor and a 1.99 microfarad capacitor.

Solution:
$$f_{co} = \frac{1}{2\pi \times RC} \qquad (11\text{-}15)$$

$$f_{co} = \frac{1}{6.28 \times 80 \times 1.99 \times 10^{-6}}$$

$$f_{co} = 1 \text{ kc}$$

Therefore, at 1 kc the resistance will be equal to the X_C and E_R will equal E_C.

11-37. Low Pass Filter (RC)

An RC series circuit connected as a low pass filter will pass frequencies below the f_{co} and attenuate the frequencies above the f_{co}. Since the capacitor develops the most voltage at the low frequencies, the output of an RC low pass filter (high frequency discriminator) will be taken across the capacitor as shown in Figure 11-38.

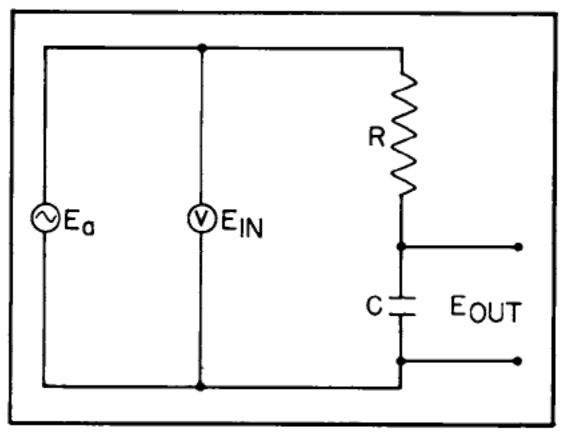

Figure 11-38 - Low pass filter (RC)

11-38. High Pass Filter (RC)

An RC series circuit connected as a high pass filter will pass frequencies above the f_{co} and attenuate the frequencies below the f_{co}. Since the resistor develops the most voltage at the high frequencies the output of an RC high pass filter (low frequency discriminator) will be taken across the resistor as shown in Figure 11-39.

Q43. Across what component is the output of a low frequency (RC) discriminator taken? Why?

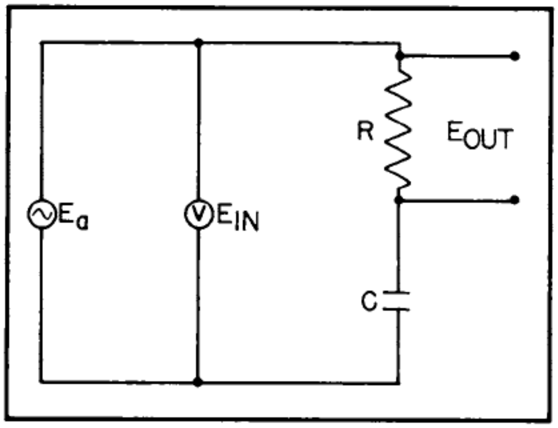

Figure 11-39 - High pass filter (RC)

RESISTANCE, INDUCTANCE, AND CAPACITANCE IN SERIES

In the preceding sections of this chapter, terms were clarified, ac reactance was explained, and the effects of individual inductors and capacitors were described. To do this, only the simplest two-element RL and RC circuits were employed. In this section, the subject coverage will be expanded to include more complex reactive circuits.

11-39. Relation of Voltages and Current in an RLC Series Circuits

When resistive, inductive, and capacitive elements are connected in series, their INDIVIDUAL characteristics are unchanged. That is, the current through and the voltage drop across the resistor are in phase while the voltage drops across the reactive components (assuming pure reactances) and the current through them are 90 degrees out of phase. However, a new relation must be recognized with the introduction of the three-element circuit. This pertains to the effect on total line voltage and current when connecting reactive elements in series, whose individual characteristics are opposite in nature, such as inductance and capacitance. Such a circuit is shown in Figure 11-40A.

In the figure, note first that CURRENT is the common reference for all three element voltages, because there is only one current in a series circuit, and it is common to all elements. The common series current is represented by the dashed line in Figure 11-40A. The voltage vector for each element, showing its individual relation to the common current, is drawn above each respective element. The total voltage E_t is the vector sum of the individual voltages of IR, IX_L, and IX_C.

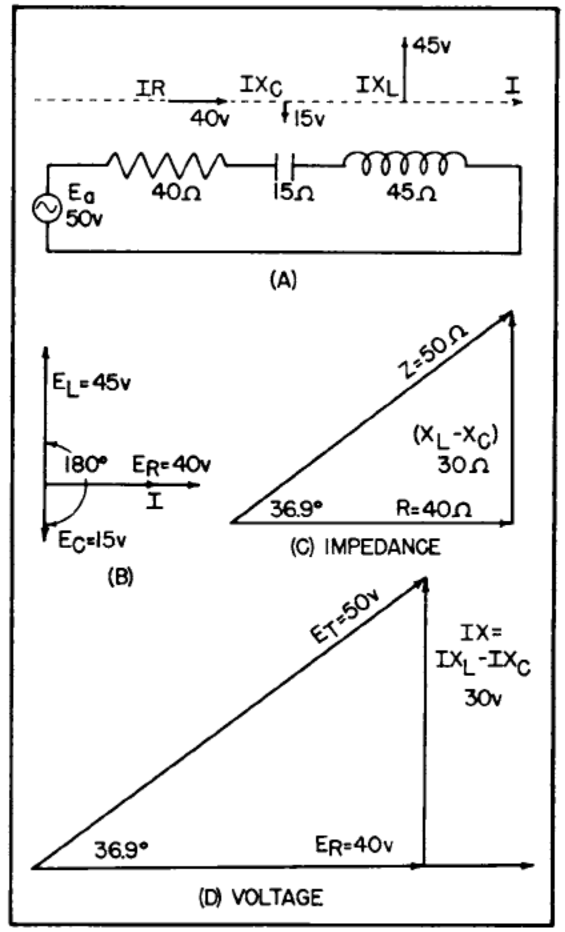

Figure 11-40 - Resistance, inductance, and capacitance connected in series.

The three element voltages are arranged for summation in part(B) of Figure 11-40. Since IX_L and IX_C are each 90 degrees away from I, they are, therefore, 180 degrees from each other. Vectors in direct opposition (180 degrees out of phase) may be subtracted directly. The total reactive voltage E_X is the difference of IX_L and IX_C.

or: $E_X = IX_L - IX_C = 45 - 15 = 30$ volts

The final relationship of line voltage and current, as seen from the source, is shown in part D. Had X_C been larger than X_L, the voltage would lag, rather than lead. When X_C and X_L are of equal value, line voltage and current will be in phase.

Q44. In a series RLC circuit, what is the relation of the current through the capacitor to the current through the coil?

A42. None. Capacitance is determined mainly by physical construction. Frequency will effect capacitive REACTANCE.

A43. The resistor. The resistor develops the most voltage at high frequencies because the reactance of the capacitor is too low to develop a usable voltage.

A44. They are in phase because the same current flows through all parts of a series circuit.

11-40. Impedance of RLC Series Circuits

The impedance of an RLC (three element) series circuit is computed in exactly the same manner described earlier for the two-element circuits. However, there is one additional operation to be performed. That is, the DIFFERENCE of X_L and X_C must be determined prior to computing total impedance. When employing the Pythagorean theorem-based formula for determining series impedance, the net reactance of the circuit is represented by the quantity in parenthesis $(X_L - X_C)$. Application of this formula to a series RLC circuit where $X_L = 45$ ohms, $X_C = 15$ ohms and $R = 40$ ohms, yields an impedance of:

$$Z = \sqrt{R^2 + X^2} \qquad (11\text{-}6)$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{40^2 + (45-15)^2}$$

$$Z = \sqrt{1600 + 900}$$

$$Z = \sqrt{2500} = 50 \text{ ohms}$$

Series impedance can also be determined by use of the vector triangle method. In an impedance triangle for a series circuit, the base always represents the series resistance, the altitude represents the NET reactance $(X_L - X_C)$, and the hypotenuse represents total impedance.

It should be noted that as the DIFFERENCE of X_L and X_C becomes greater, total impedance also increases. Conversely, when X_L and X_C are equal, their effects cancel eath other, and impedance is minimum, equal only to the series resistance. When X_L and X_C are equal their INDIVIDUAL voltages are 90 degrees out of phase with current, but their COLLECTIVE effect is zero because they are equal and opposite in nature. Therefore, when X_L and X_C are equal, line voltage and current are in phase. This condition is the same as if there were only resistance and no reactances in the circuit.

SERIES CIRCUIT RESONANCE

When the reactances in a series circuit cancel and, as far as the source is concerned, the circuit appears to contain only resistance, the circuit is said to be in a condition of RESONANCE. When resonance is established in a series circuit certain conditions will prevail.

1. The inductive reactance will be equal to the capacitive reactance.
2. The circuit impedance will be minimum.
3. The circuit current will be maximum.

11-41. Resonant Frequency

The reactance of capacitors and inductors is determined by their physical construction and the applied frequency. X_L varies directly with frequency and X_C varies inversely with frequency. Due to this relationship any combination of inductance and capacitance will have a specific frequency at which the reactances will be equal. The relationship of X_L, X_C, frequency, and the resonant frequency point (f_o) is shown in Figure 11-41. In a series RLC circuit the largest reactance value determines the appearance and phase angle of the circuit. It can be seen (from Figure 11-41) that below the resonant frequency point X_C is the larger reactance and above f_o the X_L is the larger reactance. Therefore, the circuit will appear capacitive below f_o and inductive above f_o.

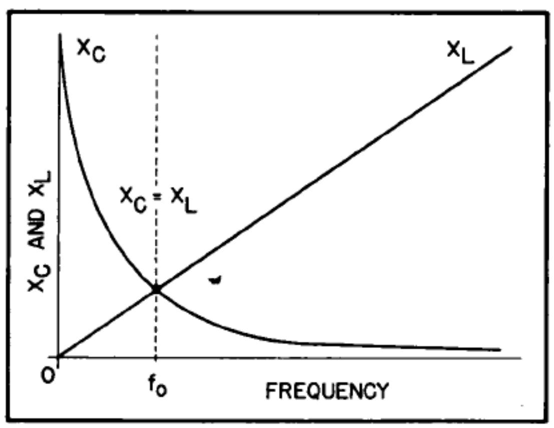

Figure 11-41 - Reactance curves for series RLC circuit.

An equation for determination of the resonant frequency can be developed in the following

manner:

At resonance $X_L = X_C$

Substituting the equation for X_L and X_C:

$$2\pi f L = \frac{1}{2\pi f C}$$

Transposing: $$f^2 = \frac{1}{4\pi^2 LC}$$

Solving for f: $$f_o = \frac{1}{2\pi\sqrt{LC}} \qquad (11\text{-}16)$$

where: f_o = resonant frequency in cycles per second.
L = inductance in henrys
C = capacitance in farads

Example. Determine the resonant frequency of a series RLC circuit consisting of an 80 microfarad capacitor, 10 microhenry coil, and a 10 ohm resistor.

Solution: $$f_o = \frac{1}{2\pi\sqrt{LC}} \qquad (11\text{-}16)$$

$$f_o = \frac{1}{6.28\text{x}\sqrt{80\text{x}10\text{x}10^{-12}}}$$

$$f_o = \frac{1\text{x}10^6}{177.5}$$

$$f_o = 5.63\ \text{kc}$$

Thus, when a frequency of 5.63 kc is applied to the circuit in the example the capacitive and inductive reactances will be equal.

Q45. Give two conditions of resonance for a series RLC circuit.

Q46. How does a series RLC circuit appear to the source when operating below the resonant frequency?

11-42. Resonant Series Circuit Analysis

In order for the series circuit (Figure 11-42) to be in resonance, the frequency of the applied voltage must be such that $X_L = X_C$.

When a series circuit contains resistance, inductive reactance and capacitive reactance, the total impedance for any frequency can be computed by the use of formula 11-7 modified to read:

$$Z = R + j(X_L - X_C)$$

Because X_L increases and X_C decreases with an increase in frequency, at a certain frequency (the resonant frequency) X_L will equal X_C, they will cancel, the j term will drop out, and Z will equal R. Therefore, at the resonant frequency, the power factor is unity. Furthermore, because the total impedance is now only the resistance, R, of the circuit, the circuit current is maximum. In other words, at resonance the generator is looking into a pure resistance.

At frequencies below resonance, X_C is greater than X_L and the circuit contains resistance and capacitive reactance; at frequencies above resonance, X_L is greater than X_C and the circuit contains resistance and inductive reactance. At resonance, the current is limited only by the relatively low value of resistance.

Because the circuit is a series circuit (Figure 11-42A), the same current flows in all parts of the circuit, and therefore the voltage across the capacitor is equal to the voltage across the inductor, because X_L is equal to X_C. These

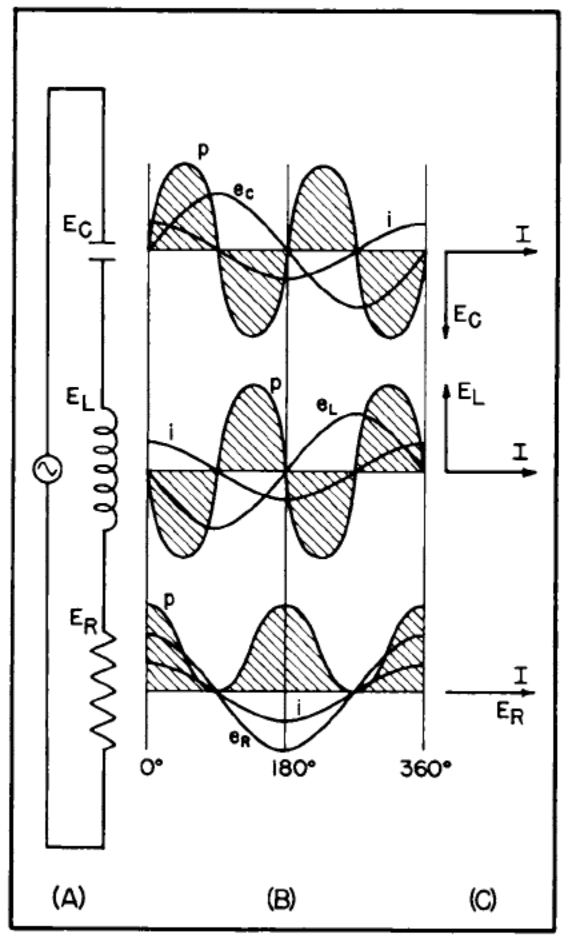

Figure 11-42 - Series resonance.

A45. X_L equal to X_C, circuit current maximum, and circuit impedance minimum.

A46. Capacitive. X_C is the largest reactance.

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voltages (Figure 11-42C), however are 180° out of phase, since the voltage across a capacitor lags the current through it by approximately 90° and the voltage across the inductor leads the current through it by approximately 90°. The total value of the input voltage, E_t, then appears across R and is shown as E_R in phase with the current, I (Figure 11-42C).

Assume that at a given instant corresponding to angle 0°, the current through the circuit is a maximum as indicated in Figure 11-42B. During the first quarter cycle (from 0° to 90° the circuit current falls from maximum to zero. The capacitor is receiving a charge, as is indicated by the rising voltage, e_c, across it. The product of the instantaneous values of e_c and i for this interval indicates a positive power curve. The shaded area under this curve represents the energy stored in the capacitor during this time it is receiving a charge.

During the first quarter of a cycle (0° to 90°), when the capacitor is receiving a charge, the magnetic field about the inductor is collapsing because the circuit current is falling and the inductor acts like a source of power that supplies the charging energy to the capacitor. The voltage, e_L, across the coil, is opposite in phase to the voltage building up across the capacitor and is shown below the line. Therefore, the product of the instantaneous values of the current and voltage across the inductor indicates a negative power curve for the coil between 0° and 90°.

During the second quarter cycle (90° to 180°) the capacitor discharges from maximum to zero, as indicated by the capacitor voltage curve, e_c, and the coil reverses its function and acts like a load on the capacitor. Thus, the capacitor now acts as a source of power. The product of a negative current and a positive voltage (e_c) indicates a negative power curve for the capacitor for this interval. During the same quarter cycle the current is rising through the inductor (in the opposite direction) and energy is being stored in the magnetic field. The product of the negative current and negative voltage, e_L, for the second quarter cycle indicates a positive power curve for the inductor.

A similar interchange of energy between the capacitor and inductor takes place in the third and fourth quarter cycles. Therefore, the average power supplied to the inductor and capacitor by an external source is essentially zero. All circuit losses are assumed to be in the resistor, R. The voltage across the resistor and the current through it are in phase. The product of the voltage and current curves associated with the resistor indicates a power curve that has its axis displaced above the X-axis. The displacement is proportional to the true average power which is equal to the product, EI (where E and I are effective values). Whatever power is dissipated in R is supplied by the source.

Q47. Why is the impedance of a series circuit small at resonance?

Q48. During what portions of the current waveform does the capacitor receive its charge?

11-43. Circuit Q

Determination of the Q of an inductor was discussed in section 11-31.

Similarly, in a capacitor the Q is a measure of the ratio of the energy stored to the energy dissipated in heat within the capacitor for equal intervals of time. This ratio is reduced by algebraic manipulation to the equation expressed below:

$$Q = \frac{X_C}{R} \qquad (11\text{-}17)$$

where: R = the effective resistance of the capacitor dielectric (losses).
Q = a number representing the quality of the capacitor.
X_C = capacitive reactance.

The effective resistance is low with respect to the capacitive reactance, and is such that when multiplied by the square of the effective capacitor current equals the true power dissipated in heat within the capacitor.

Since most of the losses in a solid-dielectric capacitor occur within the dielectric rather than in the plates, the Q of low-dielectric-loss capacitor are negligible, and thus the Q of such a capacitor may have a very high value.

The Q of a series-resonant circuit is the ratio of the energy stored to the energy lost in equal intervals of time. The expression becomes:

$$Q = \frac{X_L}{R} = \frac{X_C}{R}$$

where R represents the total effective series resistance of the entire circuit. Since the capacitor has negligible losses, the circuit Q

becomes equivalent to the Q of the coil. The circuit Q may be maintained satisfactorily high by keeping the circuit resistance to a minimum. This may be expressed mathematically in the following manner. The inductive voltage drop is:

$$E_L = IX_L$$

Since $$I = \frac{E}{R}$$

then $$E_L = \frac{EX_L}{R}$$

and $$Q = \frac{X_L}{R}$$

then $$E_L = QE$$

transposed $$Q = \frac{E_L}{E_R} \quad (11\text{-}18)$$

where: Q = a number representing the quality of the circuit.
E_L = the voltage across the inductor (can be E_C, voltage across the capacitor).
E_R = the voltage across the effective series resistance.

Therefore, the Q of the circuit is the ratio of the voltage across either the inductor or capacitor to that across the effective series resistance. In other words, the voltage gain of the series-resonant circuit depends on the circuit Q. Expressed mathematically:

$$V.G. = \frac{E_L}{E} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{IX_C}{IR} = \frac{X_C}{R} = Q \quad (11\text{-}19)$$

Q49. Why is circuit Q considered to be approximately equal to the Q of the coil?

RESONANT CIRCUIT ANALYSIS

11-44. Series RLC Circuit Analysis

Figure 11-43 shows the relation between the effective current and frequency in the vicinity of resonance for a series circuit containing a 159 uh coil, a 159 pf capacitor, and an effective series resistance of either 10 ohms, or 20 ohms.

The resonant frequency, f_o, is:

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$f_o = \frac{1}{6.28\sqrt{159\text{x}10^{-6}\text{x}159\text{x}10^{-12}}}$$

$$f_o = 1\text{x}10^6 \text{ cycles or } 1,000 \text{ kc.}$$

The reactances and impedance at resonance may likewise be determined. Thus:

$$X_{Lo} = 2\pi f L = 6.28\text{x}10^6\text{x}159\text{x}10^{-6}$$

$$X_{Lo} = 1000\underline{/+90^\circ}$$

where X_{Lo} is the inductive reactance at resonance. The +90° angle indicates that the IX_{Lo} and X_{Lo} vectors are plotted vertically upward

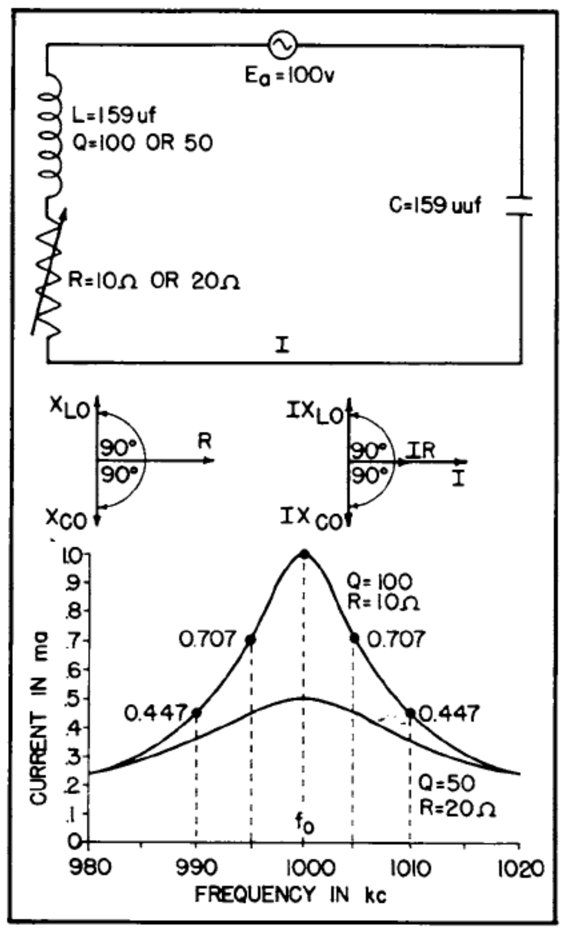

Figure 11-43 - Resonance curves of a series RLC circuit.

A47. Because the reactances cancel, leaving the small resistance of the circuit as the only opposition to current flow.

A48. When the current is decreasing toward zero.

A49. Because most of the effective resistance of the circuit is contained in the coil windings.

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because the current vector is horizontal and extends to the right. The current vector thus lags the voltage, IX_{Lo}, across the coil by 90° (counter-clockwise rotation is positive, Figure 11-43).

$$X_{Co} = \frac{1}{2\pi f C} = \frac{1}{6.28 \times 10^6 \times 159 \times 10^{-12}}$$

$$X_{Co} = 1000\angle{-90^\circ}$$

where X_{Co} is the capacitive reactance at resonance. The -90° angle indicates that the vectors X_{Co} and IX_{Co} are plotted vertically downward because the current vector is the horizontal reference vector extending to the right and leads the voltage drop, IX_{Co}, across the capacitor by 90° (Figure 11-43). Note that current is a common factor to both voltage and impedance vectors.

When the effective resistance of the circuit is equal to 10 ohms the impedance at resonance (Z_o) will be:

$$Z_o = R + j(X_L - X_C)$$

$$Z_o = 10 + j(1000 - 1000)$$

$$Z_o = 10 + j0 \text{ ohms}$$

$$Z_o = 10\angle{0^\circ} \text{ohms}$$

If the applied voltage is assumed to be 10 millivolts (mv) at a frequency of 1000 kc, the circuit current is:

$$I = \frac{E}{Z}$$

$$I = \frac{0.01}{10}$$

$$I = 1 \text{ ma}$$

At the resonant frequency, the voltage across the inductor is:

$$E_L = IX_L$$

$$E_L = 0.001 \times 1000$$

$$E_L = 1 \text{ volt}$$

The voltage across the capacitor is the same, except it is 180° out of phase with the voltage across the coil. The losses in the coil and capacitor are assumed to be lumped in the effective series resistance. The circuit Q is:

$$Q = \frac{X_L}{R}$$

$$Q = \frac{1000}{10}$$

$$Q = 100$$

The voltage gain at resonance is:

$$V.G. = \frac{E_L}{E}$$

$$V.G. = \frac{1.00}{0.01}$$

$$V.G. = 100$$

The resonance curves of current vs frequency are symmetrical about a vertical line (f_o) extending through the point of maximum current, (Figure 11-43).

The shape of the resonance curve may be approximated in the vicinity of resonance by applying the following rules that can be derived from the resonant circuit equations. (The derivation is not given because of its length).

RULE 1. If the frequency of the applied voltage is decreased by an amount $\frac{1}{2Q}$ times the resonant frequency, f_o, the current in the tuned circuit decreases to 0.707 of its value at the resonant frequency and leads the applied voltage by 45°.

Example. The input frequency of the circuit (Figure 11-43) is decreased from the resonant frequency (f_o) of 1000 kc by an amount equal to:

$$f_{decrease} = \frac{1}{2Q} \times f_o$$

$$f_{decrease} = \frac{1}{2 \times 100} \times 1000$$

$$f_{decrease} = 5 \text{ kc}$$

The new applied frequency is then:

$$f_{new} = f_o - f_{decrease}$$

$$f_{new} = 1000 - 5$$

$$f_{new} = 995 \text{ kc}$$

The X_L at the new applied frequency is:

$$X_L = 2\pi f L$$

$$X_L = 6.28 \text{x} 0.995 \text{x} 10^6 \text{x} 159 \text{x} 10^{-6}$$

$$X_L = 995\underline{/+90^\circ} \text{ ohms}$$

The X_C at the new applied frequency is:

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{6.28 \text{x} 0.995 \text{x} 10^6 \text{x} 159 \text{x} 10^{-12}}$$

$$X_C = 1005\underline{/-90^\circ} \text{ ohms}$$

The circuit impedance at 995 kc is:

$$Z_t = R + j(X_L - X_C)$$

$$Z_t = 10 + j(995 - 1005)$$

$$Z_t = 10 - j10 \text{ ohms}$$

Converting to polar form:

$$Z_t = 14.14\underline{/-45^\circ}$$

The circuit current at 995 kc is:

$$I_t = \frac{E_t}{Z_t}$$

$$I_t = \frac{0.01\underline{/0^\circ}}{14.14\underline{/-45^\circ}}$$

$$I_t = 0.707\underline{/45^\circ} \text{ ma}$$

At this frequency (which is the cut-off frequency) the voltage across the coil, or the capacitor, is reduced to approximately 70 percent of its value at resonance.

The voltage across the coil is:

$$E_L = IX_L$$

$$E_L = 0.707 \times 995$$

$$E_L = 705 \text{ mv}$$

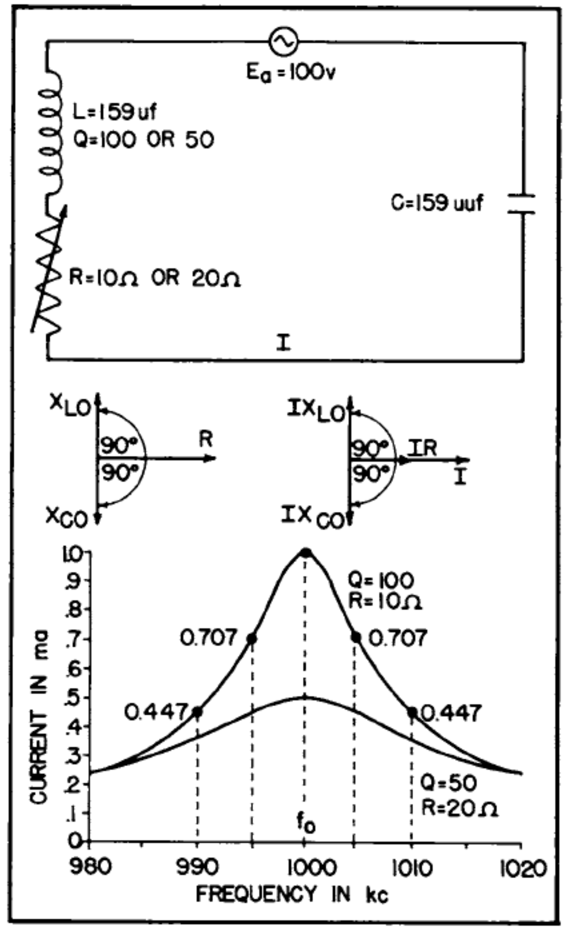

Figure 11-43 - Resonance curves of a series RLC circuit.

RULE 2. If the frequency of the applied voltage is decreased by an amount 1/Q times the resonant frequency, the current decreases to 0.447 of its value at resonance and leads the applied voltage by 63.4°.

Example. The input frequency of the circuit (Figure 11-43) is decreased from the resonant frequency at an amount equal to:

$$f_{decrease} = \frac{1}{Q} \times f_o$$

$$f_{decrease} = \frac{1}{100} \times 1000$$

$$f_{decrease} = 10 \text{ kc}$$

The new applied frequency is then:

$$f_{new} = f_o - f_{decrease}$$

$$f_{new} = 1000 - 10$$

$$f_{new} = 990 \text{ Kc}$$

The X_L at the new applied frequency of 990 Kc is:

$$X_L = 2\pi f L$$

$$X_L = 6.28 \times 0.990 \times 10^6 \times 159 \times 10^{-6}$$

$$X_L = 990\angle 90^\circ \text{ ohms}$$

The X_C at the new applied frequency of 990 Kc is:

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{6.28 \times 0.990 \times 10^6 \times 159 \times 10^{-12}}$$

$$X_C = 1010\angle -90^\circ \text{ ohms}$$

The impedance of the series circuit at 990 Kc is:

$$Z_t = R + j(X_L - X_C)$$

$$Z_t = 10 + j(990 - 1010)$$

$$Z_t = 10 - j20$$

Converting to polar form yields:

$$Z_t = 22.4\angle -63.4^\circ \text{ ohms}$$

At this frequency the circuit current is:

$$I_t = \frac{E_t}{Z_t}$$

$$I_t = \frac{0.10\angle 0^\circ}{22.4\angle -63.4^\circ}$$

$$I_t = 0.447\angle 63.4^\circ \text{ ma}$$

The voltage across the coil is:

$$E_L = IX_L$$

$$E_L = 0.447 \times 990$$

$$E_L = 444 \text{ mv}$$

Decreasing the applied frequency is seen (by the positive phase angles of I_t) to cause the total current to lead the applied voltage.

Corresponding increases in the frequency of the applied voltage about the resonant frequency will produce the same reductions in circuit current and voltage across the reactive portions of the circuit. In this case, however, the circuit lags the applied voltage instead of leading it. Thus, the resonance curve is symmetrical about the resonant frequency in the vicinity of resonance.

Q50. Will a series circuit have a higher voltage gain with a low Q coil or a high Q coil?

11-45. Influence of Q on Voltage Gain

If the circuit resistance is increased to 20 ohms, the Q is reduced to 50 and the resonance curve is flattened, as shown by the lower curve in Figure 11-44. The series-resonant circuit

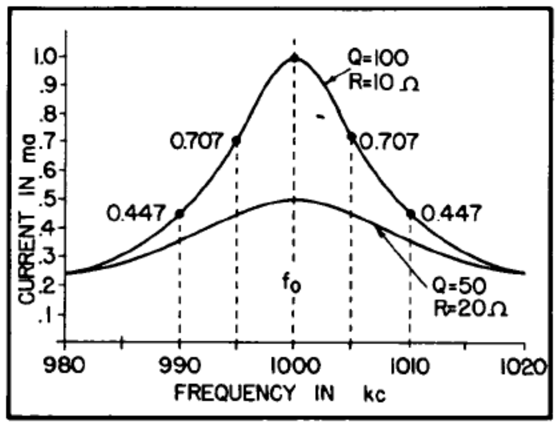

Figure 11-44 - Influence of Q on gain and bandwidth.

amplifies the applied voltage at the resonant frequency. If the circuit losses are low the circuit Q will be high and the voltage amplification relatively large. For resonant circuits involving iron-core coils the Q may range from 20 to 100. In practice, because nearly all of the resistance of a circuit is in the coil, the ratio of the inductive reactance to the resistance is especially important. The higher the Q of the coil, the better is the coil and the more effective is the series-resonant circuit that utilizes it.

11-46. Power in an RLC Series Circuit

Total true power in an RLC series circuit is the product of line voltage and current times the cosine of the angle between them. When X_L and X_C are equal, total impedance is at a minimum, and thus current is maximum. When maximum current flows, the series resistor

dissipates maximum power. When X_L and X_C are made unequal, total impedance increases, line current decreases, and moves out of phase with line voltage. Both the decrease in current and the creation of a phase difference cause a decrease in true power.

In the example above, decreasing the applied frequency caused a leading current, therefore a positive phase angle. A positive phase angle causes a leading power factor and vice versa. Thus, the P. F. is leading when the frequency is decreased and lagging when the frequency is increased (with respect to the resonant frequency).

11-47. Bandwidth

If the circuit Q is low, the amplification at resonance is relatively small and the circuit does not discriminate sharply between the resonant frequency and the frequencies on either side of resonance, as is shown by the lower curve in Figure 11-44. The range of frequencies included between the two frequencies at which the current drops to 70 percent of its value at resonance is called the BANDWIDTH for 70 percent response.

It was previously shown (section 11-24) that frequencies beyond the 70 percent or half-power point were considered to produce no useable output. The series resonant circuit is seen to have two half-power points, one above the resonant frequency point and one below. The two points are designated upper f_{co} and lower f_{co} or simply f_1 and f_2. The range of frequencies between these two points comprises the bandwidth. Figure 11-45 illustrates the bandwidths for high and low Q series resonant circuits. The bandwidth may be determined by the equation:

$$BW = \frac{f_o}{Q} \qquad (11\text{-}20)$$

$$BW = f_2 - f_1$$

where: BW = bandwidth of a series resonant circuit in units of frequency.
f_o = resonant frequency.
f_2 = the highest frequency the circuit will pass.
f_1 = the lowest frequency the circuit will pass.
Q = as defined previously.

Example. Determine the bandwidth for the curve shown in Figure 11-45B.

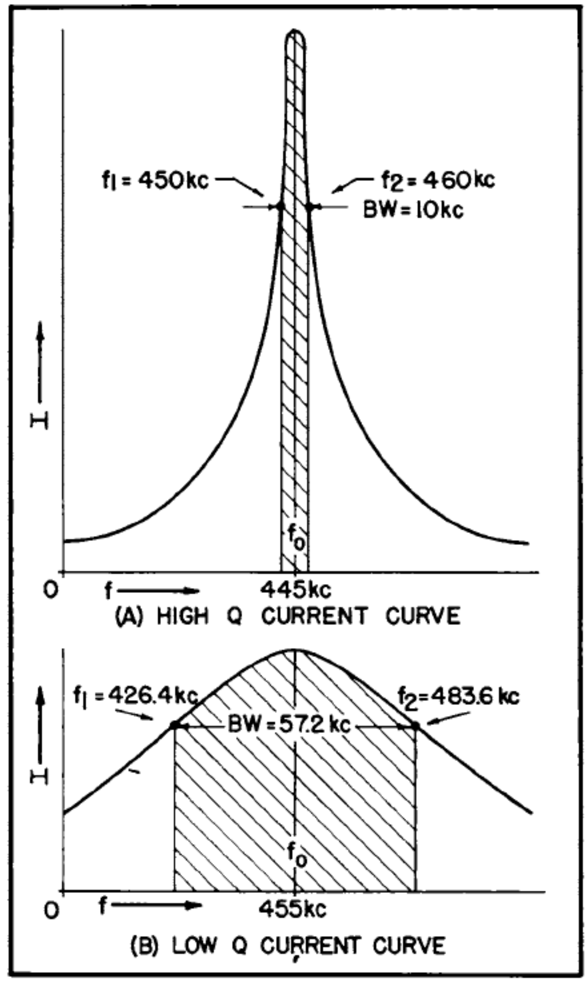

Figure 11-45 - Bandwidth for high and low Q series resonant circuits.

Solution: $$BW = f_2 - f_1 \qquad (11\text{-}21)$$

$$BW = 483.6 \text{ kc} - 426.4 \text{ kc}$$

$$BW = 57.2 \text{ kc}$$

The Q of the curve in part A is given as 45.5: determine the bandwidth.

$$BW = \frac{f_o}{Q}$$

$$BW = \frac{455 \text{ kc}}{45.5}$$

$$BW = 10 \text{ kc}$$

Q51. Define bandwidth.

A50. High Q coil.

A51. The frequency range of the circuit between the half-power points.

Q52. Which circuit will have better selectivity (sharper discrimination against frequencies above and below f_{co} points): a high or low Q circuit?

11-48. Applications of Series-Resonant Circuits

Series-resonant circuits are used largely as filters (to be treated later) for audio and radio frequencies. With proportionately larger component values the series circuit may be used as a power-supply filter. For example, assume that a dc generator has a ripple frequency of 500 cps. A series-resonant circuit tuned to 500 cps may be connected across the terminals of the generator and thus effectively short-circuit the ripple voltage. The coil and capacitor insulation must be able to withstand the relatively high ac voltages caused by the series-resonant action.

The series-tuned circuit may also be used to give an indication of frequency if the capacitor is calibrated for the appropriate frequency range. The capacitor and the inductor are connected in series with a current-indicating device across the source of the unknown frequency. At resonance the current as indicated by the device, will be maximum. Therefore, knowing the value of L and C, the value of the unknown frequency may be calculated using equation (11-16).

11-49. Resonant Conditions for Series RLC Circuits

Table 11-3 is used to sum up the various conditions occuring in a series RLC circuit at resonance.

QUANTITY	SERIES CIRCUIT
At resonance: Reactance $(X_L - X_C)$	Zero, because $X_L = X_C$
Resonant frequency	$\frac{1}{2\pi\sqrt{LC}}$
Impedance	Minimum; Z = R
I_{LINE}	Maximum value
I_L	I_{LINE}
I_C	I_{LINE}
E_L	$Q \times E_{LINE}$
E_C	$Q \times E_{LINE}$
Phase angle between E_{LINE} and I_{LINE}	0°
Angle between E_L&E_C	180°
Angle between I_L&I_C	0°
Desired value of Q	10 or more
Desired value of R	Low
Highest selectivity	High Q, low R, high $\frac{L}{C}$
When f is greater than f_o: Reactance	Inductive
Phase angle between I_{LINE} and E_{LINE}	Lagging current
When f is less than f_o: Reactance	Capacitive
Phase angle between I_{LINE} and E_{LINE}	Leading current

TABLE 11-3

EXERCISE 11

1. How much larger is the maximum voltage value than the effective voltage value?
2. If the average value of a sine wave is 50V what is the effective value?
3. A series circuit consisting of three resistors is connected across an ac source. The following information is given:
   E_a = 40V RMS R_2 = 40 ohms
   R_1 = 80 ohms R_3 = 35 ohms
   What is the peak voltage and power dissipation of R_2?
4. What value of voltage would a voltmeter read if it were connected across R_3 in the circuit of question number three?
5. What is the unit of measure for reactive power?
6. What would happen to current flow in a purely inductive circuit if the frequency were increased?
7. What is the power factor of a purely inductive circuit?
8. Define apparent power.
9. What is the frequency of a power curve for a 60 cycle sine wave?
10. Since a purely capacitive circuit contains no resistance, what quantity limits the flow of current when an ac signal is applied?
11. Define impedance.
12. In a 60 cycle ac circuit with an inductor of 0. 053 microhenry and a resistance of 50 ohms connected in series, determine the impedance.
13. Define the j operator.
14. Describe a vector with a magnitude of 30 volts that has been multiplied by $-j^{10}$. ($-j^{10} \times 30$ = ?)
15. Describe the impedance of an RC series circuit containing a 40 ohms resistor and a 50 microfarad capacitor in rectangular notation.
16. Describe the impedance of a series RC circuit containing a 70 ohm resistor and an 80 microfarad capacitor in polar notation.
17. Determine the resultant current vector if
    $E_a = 70\angle 0^\circ$ volts.
    $Z_t = 2\angle 36^\circ$ ohms
    I_t = ?
18. Determine the SUM of the following vectors
    $V_1 = 40\angle 20^\circ$ ohms
    $V_2 = 20\angle 30^\circ$ ohms
    V_3 = ?
19. A series RL circuit is described by the following information:
    E_a = 250 volts R = 100 ohms
    f = 400 cps X_L = 100 ohms
    Determine the phase angle and E_L.
20. Determine the inductance of the circuit in question 19 and describe the effect of doubling the inductance on circuit phase angle and current.
21. Determine the f_{co} and true power of the circuit in question 19.
22. If the output of the circuit in question 19 is taken across the resistor, what would E_{out} be at f_{co}? Is this a high or low pass circuit?
23. Define effective resistance.
24. Describe the term "skin effect".
25. Why does the Q of a coil change when the turns are pushed closer together?
26. What is the effective resistance of a coil with a Q of 50 and an X_L of 30 ohms?
27. The following facts are known concerning a series RC circuit:
    E_a = 500 volts R = 10 ohms
    f = 5 Kc C = 4 uf
    Determine: E_t, I_t, E_R, E_C, θ, P_a, P_t, P. F. and f_{co}.
28. Draw a schematic of the circuit of question 27 connected as a high pass filter. What is the current at f_{co}?
29. In a practical series resonant circuit, what is the total impedance equal to?
30. How does a series circuit appear below resonance? Explain why.
31. How does a variation in effective resistance effect the shape of the resonant current curve?
32. If the capacitance of a series circuit is increased will the resonant frequency be higher or lower?
33. How is the bandwidth of a series resonant circuit effected by an increase in resistance?
34. How can the resonant frequency be found if the value of the half-power points (f_1 and f_2) are known?
35. Define the term bandwidth.
36. What is the bandwidth of a series resonant circuit having an X_L of 2. 5 K ohms, an R of 30 ohms, an X_C of 150 ohms, and an f_o of 200 cps?
37. Give an instance where a bandpass network would be used to advantage.
38. Solve the following series circuit?
    Given: L = 600 mh E_a = 50 V
    C = 10 uf R = 15 ohms
    Find: f_o, Q, I_t, BW, f_1 and f_2.
39. Determine the phase angle, P_a, P_t and P. F. of the circuit in question 38.
40. Determine the voltage gain of the circuit in question 38.

A52. The high Q circuit.

CHAPTER 12

PARALLEL CIRCUITS AND RESONANCE

ı previous chapters the behavior of circuit roperties such as resistance, inductance, and apacitance has been considered on both an in-ividual basis, and when arranged into series ircuits. The purpose of this chapter is the study f parallel circuits to which an alternating volt-ge is applied. While the basic characteristics f each component remains unchanged, it will ecome apparent that connecting these various omponents together in a parallel configuration ·ill produce an effect that differs greatly from ircuit action when only one component is used.

The first circuits to be analyzed will consist f parallel arrangements of pure resistances, nd then pure inductances. Next, parallel cir-uits containing both resistance and inductance ·ill be studied. Following a similar treatment f resistance and capacitance in parallel circuits, ıe effects of parallel resonance will be exam-ıed. Both ideal and practical parallel resonant ircuit theory is presented. To fully comprehend ıe circuit operation, a familiarity with rectan-ular and polar notation, vector analysis, and :igonometric functions is required. These ubjects are discussed in detail in Volume 8.

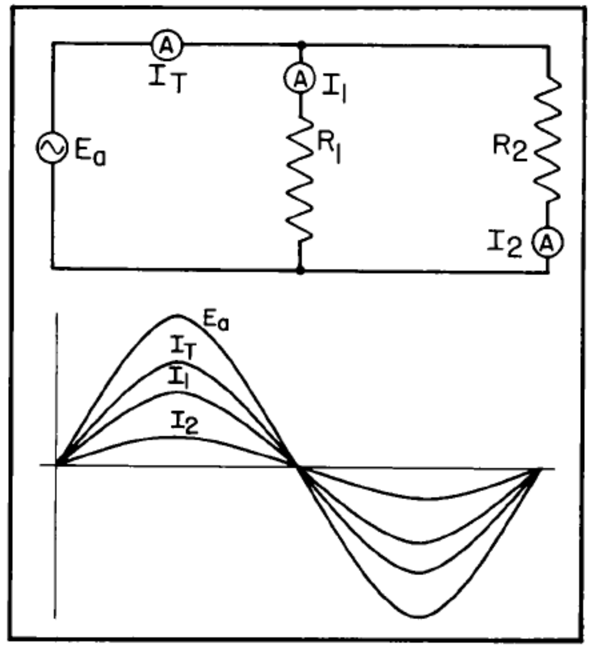

Figure 12-1 - Phase relationships in a purely resistive circuit.

2-1. Parallel Resistive Circuits

A parallel circuit is defined as one that pos-esses more than one path for current flow. /hen an alternating voltage is applied to two esistors which are connected in parallel, the oltage across each resistor is the same. The urrent flowing through each resistor depends n the value of resistance that it has. Because ıere is more than one path for current flow, the ›tal resistance offered by this type of network ; less than the resistance offered by any one of ıe individual resistances. The currents and oltages in a resistive circuit, be it series or arallel, are in phase. An increase in the ac oltage applied will cause a proportionate in-rease in the current but will not affect the phase elation between the current and the voltage.

No vector diagram is required to represent n in-phase relationship, but the current-voltage elationship may be pictured graphically as in igure 12-1.

12-2. Parallel Inductive Circuits

If pure inductors (no inherent resistance) are connected in parallel across an ac source, the total current flow is dependent on the total op-position of the circuit. The total inductance of inductors in parallel (with no mutual inductance present) can be computed by the equation:

$$L_t = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \cdots \frac{1}{L_n}} \qquad (9\text{-}10)$$

Equation (9-10) indicates that connecting addi-tional inductors in parallel will lower the total inductance. To determine the total opposition of a purely inductive circuit, insert L_t in place of L in equation (9-26):

$$X_L = 2\pi f L \qquad (9\text{-}26)$$

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Inserting L_t: $X_{L_t} = 2\pi f L_t$

If adding more parallel connected inductors lowers L_t, then by the above equation the total opposition (X_{L_t}) will also be lowered. According to equation (9-27)

$$I_L = \frac{E_a}{X_L} \qquad (9\text{-}27)$$

decreasing the opposition (reactance) will increase the current flow. Figure 12-2A illustrates a purely inductive parallel circuit. The reactances of each inductor have been determined by use of equation 9-26. Figure 12-2B illustrates the current voltage relationships in each inductor and in the circuit. The voltage leads the current in an inductor. Therefore, the voltage for the inductors (E_L) is shown at

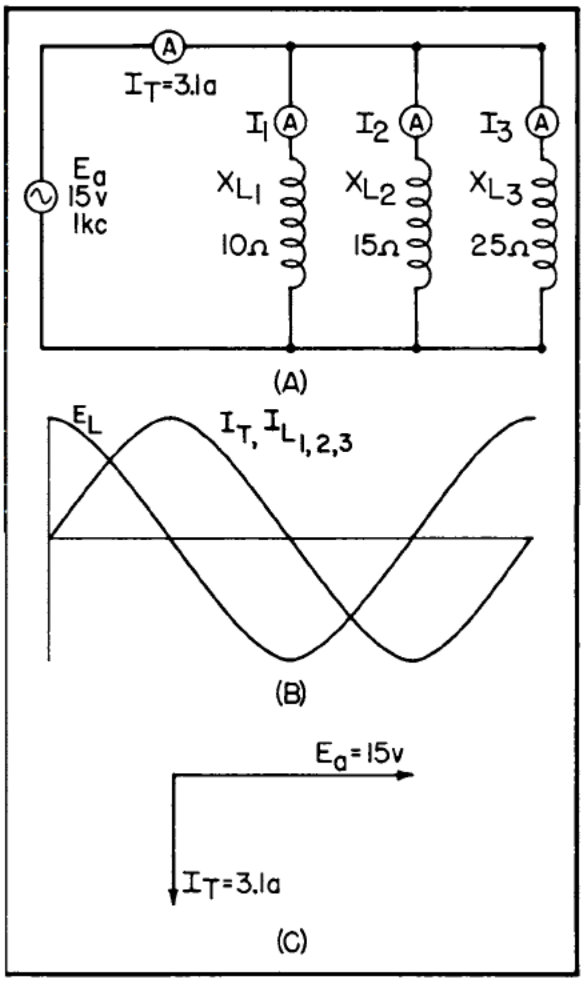

Figure 12-2 - E and I relationships in a purely inductive circuit.

maximum when the coil current (I_L) is at minimum. The waveform for E_L also describes the applied voltage (E_a) which is common to all the inductors. The current waveform (I_L) also describes the total circuit current (I_t) which is the sum of all the lagging branch currents. Figure 12-2C is a vector representation of the same information given in Figure 12-2B but in a much simpler form. It has been previously stated that a vector is in standard position when plotted along the horizontal axis to the right of the origin. Therefore, Figure 12-2C shows the applied voltage (common quantity) used as the reference and plotted in standard position with I_t lagging by 90°

Applying parallel circuit theory to Figure 12-2A, it can be seen that the largest current will flow through the branch with the lowest reactance. Branch one (XL_1) will have:

$$I_1 = \frac{E_a}{X_{L1}} \qquad (9\text{-}27)$$

$$I_1 = \frac{15}{10}$$

$$I_1 = 1.5 \text{ amps}$$

Branch two (X_{L2})

$$I_2 = \frac{E_a}{X_{L2}}$$

$$I_2 = \frac{15}{15}$$

$$I_2 = 1 \text{ amp}$$

Branch three (X_{L3})

$$I_3 = \frac{E_a}{X_{L3}}$$

$$I_3 = \frac{15}{25}$$

$$I_3 = 0.6 \text{ amp}$$

By Kirchhoff's current law the total current must be equal to the sum of the branch currents or:

$$I_t = I_1 + I_2 + I_3$$

$$I_t = 1.5 + 1 + 0.6$$

$$I_t = 3.1 \text{ amps}$$

The conditions in the circuit in Figure 12-2A may be represented by the vector diagram in Figure 12-2C which shows the total circuit voltage (E_a) of 15 volts leading the total circuit current of 3.1 amperes by 90°.

NOTE: The algebraic addition used above can only be used when all quantities being added differ in phase by 0° or 180°. For example, two quantities differing in phase by 90° cannot be added algebraically.

Since the vector diagram shows the currents through each inductor to have the same phase angle as I_t, algebraic addition is used. It should be pointed out that this example, because of the use of pure inductances, is a special case. Due to the presence of resistance in all practical circuits the branch currents of a parallel circuit containing reactances are seldom in phase, Therefore, VECTOR ADDITION MUST BE USED.

Q1. If one branch of a parallel inductive circuit opens, what will happen to the total reactance?

CHARACTERISTICS OF PARALLEL RL CIRCUITS

12-3. Vector and Waveform Diagrams

In some respects the circuit of Figure 12-3A is similar to the purely inductive parallel circuit just discussed.

For instance, applied voltage E_a is still the quantity which is common to both components and is therefore plotted in standard position in the vector diagram. Also the magnitude of the individual branch currents is determined by the opposition of the individual branches. Figure 12-3B shows a composite diagram of E_a, I_R and I_L. Since the vector diagram shows that the two branch currents are NOT in phase, it will be necessary to use vector addition in order to determine the total current.

Example. Determine the phase angle, I_t, P_t, Z_t, I_L and I_R in a parallel RL circuit containing a 1.4 millihenry coil, a 25 ohm resistor, and a 10 volt source operating at 4 kilocycles.

Solution: Determine X_L of coil:

$$X_L = 2\pi f L \qquad (9\text{-}26)$$

$$X_L = (6.28)(4 \times 10^3)(1.4 \times 10^{-3})$$

$$X_L = 35.17 \text{ ohms}$$

Determine current in branch one (I_1):

$$I_1 = \frac{E_a}{R}$$

$$I_1 = \frac{10}{25}$$

$$I_1 = 0.4 \text{ amp}$$

$$I_1 = 400 \text{ ma}$$

Determine current in branch two (I_2):

$$I_2 = \frac{E_a}{jX_L}$$

$$I_2 = \frac{10}{j35.17}$$

$$I_2 = -j284 \text{ ma}$$

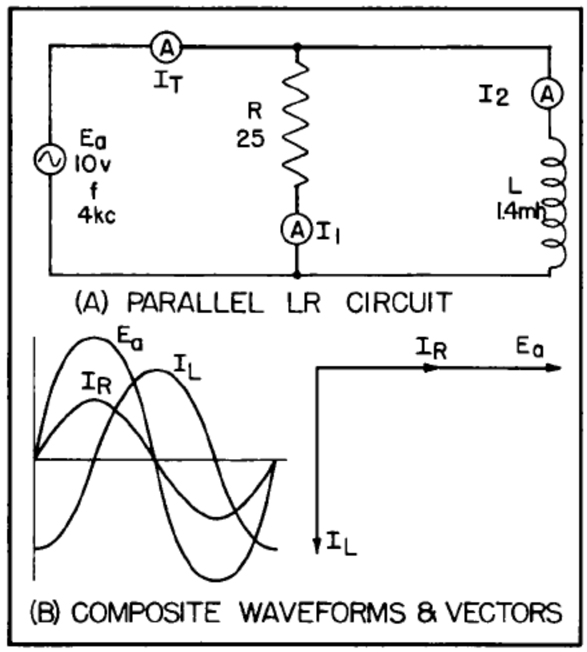

Figure 12-3 - Parallel circuit analysis.

Total current may now be determined either graphically or mathematically. If a vector diagram is constructed to scale, a very close approximation of values and angles may be obtained. To verify this I_t will be determined by both methods.

Figure 12-4 illustrates the graphical method. The results show that I_t is approximately 490 ma and lagging E_a by an angle of approximately 36°.

A1. Since total inductance will increase, the total reactance will increase.

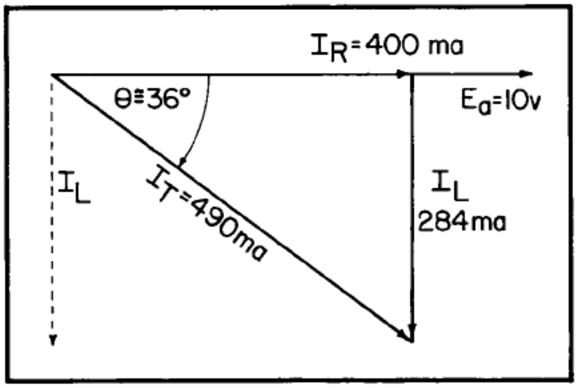

Figure 12-4 - Vector analysis of RL circuit.

The vector diagram approximation may be satisfactory for many applications. For applications requiring greater accuracy I_t can be determined by use of the following equations:

$$I_t = I_R - jI_L \qquad (12\text{-}1)$$

or:

$$I_t = \frac{I_R}{\cos\theta} \qquad (12\text{-}2)$$

or:

$$I_t = \frac{I_L}{\sin\theta} \qquad (12\text{-}3)$$

where: I_t = total current in amps
I_R = resistor current in amps
I_L = inductor current in amps
θ = phase angle

In order to use equation (12-2) or (12-3) it is first necessary to determine the phase angle. This is done by use of the equation expressed below:

$$\tan\theta = \frac{-jI_L}{I_R}$$

therefore:

$$\theta = \arctan\frac{-jI_L}{I_R} \qquad (12\text{-}4)$$

where: values are as previously defined.
arctan = "the angle whose tan is"

Determine θ: $\theta = \arctan\frac{-jI_L}{I_R}$

$$\theta = \arctan\frac{-j0.284}{0.400} = -0.71$$

$$\theta = -35.40^{\circ}$$

The magnitude and sign of the phase angle (θ) is determined by the value and type (resistor, inductor, etc.) of circuit components. If the total current is leading the total voltage then the phase angle is leading. If the total current is lagging the total voltage then the phase angle is lagging.

Determine I_t: $I_t = \frac{I_R}{\cos\theta}$ (12-2)

$$I_t = \frac{0.4}{\cos 35.4^{\circ}}$$

$$I_t = \frac{0.4}{0.815}$$

$$I_t = 491 \underline{/-35.4^{\circ}} \text{ ma}$$

This value agrees very closely with the approximation made from the vector.

Determine total impedance:

$$I = \frac{E}{Z} \qquad (11\text{-}4)$$

transposing: $Z_t = \frac{E_a}{I_t}$ (12-5)

$$Z_t = \frac{10 \underline{/0^{\circ}}}{0.491 \underline{/-35.4^{\circ}}}$$

$$Z_t = 20.36 \underline{/35.4^{\circ}} \text{ ohms}$$

Determine the true power dissipated by the circuit:

$$P_t = P_a \times P.F. \qquad (12\text{-}6)$$

where: P_t = true power in watts
P_a = apparent power = $E_a \times I_t$, in volt-amperes
P.F. = power factor = $\frac{I_R}{I_t}$ = $\cos\theta$

Changing equation (12-6) to fit known quantities:

$$P_t = P_a \times P.F. \qquad (12\text{-}6)$$

$$P_t = E_a \times I_t \times \cos\theta$$

$$P_t = 10 \times 0.491 \times 0.815$$

$$P_t = 4.002 \text{ watts}$$

$$P_t = 4 \text{ watts}$$

Since at four kilocycles the inductive reactance (35.17 ohms) is higher than the resistance (25 ohms), the current through the resistor will be greater than the current through the inductor. Thus, the total current will be more resistive than reactive. If the circuit is to be described as possessing a predominant characteristic, it would be considered as resistive.

The value of reactance in an RL circuit is a variable dependent on frequency. Therefore, the applied frequency is a factor in determining the magnitude and phase of total current.

For every RL circuit there is a frequency that will make the value of the reactance equal to the value of resistance. When the two values are equal, the phase angle is equal to 45 degrees. If a change in frequency causes the phase angle to increase, the inductive reactance must have decreased permitting a greater amount of current to flow in that branch. If the angle is less than 45 degrees, the greater percentage of the total current flows through the resistive branch.

Q2. What is the relationship between total current and inductive current?

Q3. If the inductance of an RL circuit increases, what happens to the phase angle and the power factor?

VARIATIONAL ANALYSIS

The behavior of a parallel RL circuit will vary considerably when circuit conditions are changed. The purpose of this section is to show the effect on circuit values and overall operation when either frequency, resistance, inductance, or applied voltage is varied.

12-4. Effect of Varying Frequency

To point out the effect that an increase or decrease in source frequency will have on circuit operation, the circuit of Figure 12-5A is analyzed by vector diagrams for three different values of applied frequency.

The applied voltage, resistance, and inductance, will be held constant at 100 volts, 200 ohms, and 2 millihenrys respectively.

Figure 12-5B shows the conditions of the circuit when a frequency of 20 kc is applied.

With a frequency of 20 kc applied the inductive reactance of the coil is 251.2 ohms. Since the opposition of the inductive branch is only slightly higher than the 200 ohms of the resistive branch, the division of current through the two branches will be nearly equal. The larger of the two currents will flow through the lower opposition which is the resistance. The opposition of a branch will be the determining factor for the magnitude of current flow through that particular branch, since the voltage applied to each branch is the same. Using Ohm's law the current in the resistive branch is found to be 500 ma. Applying equation (9-27) the current in the inductive branch is found to be 398 ma. To find the total current and phase angle, a vector diagram (Figure 12-5B) is constructed. I_R, being in phase with E_a is plotted in standard position. Since the current through a coil (having low dc resistance) lags the voltage across it by approximately 90°, I_L is plotted along the vertical axis (as shown by the dotted line). According to the procedure for the addition of vectors, as presented in section 8-21, the tail of the I_L vector is transferred to the head of the I_R vector.

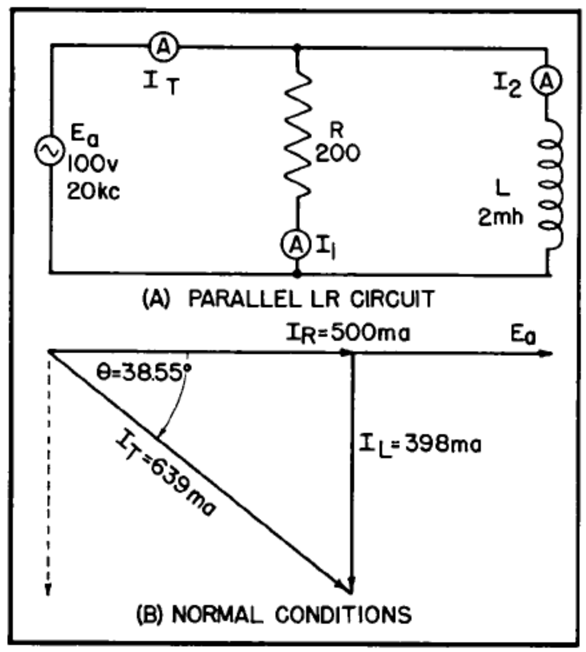

Figure 12-5 - Parallel RL circuit and vector diagram.

In following this procedure, it is essential that

A2. The total current leads the inductive current.

A3. Phase angle decreases, power factor increases.

the angle to the reference line (standard position) and the length of the original vector be reproduced accurately. To accomplish addition of these vectors, draw a final vector between the tail of the I_R vector and the head of the I_L vector. Measuring the length of this final vector and its angle with respect to the reference line will show that the total current is equal to $639\angle{-38.55^\circ}$ ma. Impedance and power may be calculated using the procedures set forth in section 12-3.

Increased Frequency: If the values of E_a, R, and L are maintained constant and the applied frequency is increased, significant changes will occur in a number of circuit quantities. For simplicity it will be assumed that the frequency is increased to 40 kc, twice its original value. Since X_L is a direct function of frequency, doubling the frequency will also double X_L. Therefore, at 40 kc X_L will be approximately 502 ohms.

As a result of the twofold increase in X_L the current in the inductive branch will decrease to 199 ma, one-half of its original value. The current through the resistive branch will remain at 500 ma, since changes in applied frequency have no effect on circuit resistance.

The circuit vector diagram for an applied frequency of 40 kc is shown in Figure 12-6A. If the magnitude and angle of the resultant vector are measured, the total current is found to be approximately $538\angle{-21.7^\circ}$ ma.

To summarize, an increase in the frequency applied to a simple parallel LR circuit will increase the inductive reactance, increase total impedance, decrease current flow in the inductive branch, decrease total current flow, decrease the phase angle and will not affect true power.

Q4. Doubling the frequency applied to a parallel RL circuit will double the X_L. Will the inductance also be doubled?

Decreased Frequency: To examine the effects of a reduction in applied frequency, assume that the source frequency is reduced to 10 kc. The applied frequency will now be one-half its original value, which in turn causes X_L to drop to 125.6 ohms. This reduction in X_L to one-half its original value permits the current in the inductive branch to double. Thus, the inductive current will increase to 796 ma, while the resistive current will remain at 500 ma. By constructing the vector diagram, as shown in Figure 12-6B, the total current is seen to have increased and now equals $942\angle{-57.83^\circ}$ ma.

To summarize, decreasing the frequency of a parallel LR circuit will increase current flow in the inductive branch, increase total current, increase the phase angle, decrease total impedance, and will not affect true power.

Q5. A variation in the frequency applied to the parallel RL circuit did not affect the true power dissipated by the circuit. Explain why.

12-5. Effect of Varying Resistance

In order to determine the effect of a variation in resistance on the operation of a parallel RL circuit, Figure 12-7 is used to establish normal operating conditions.

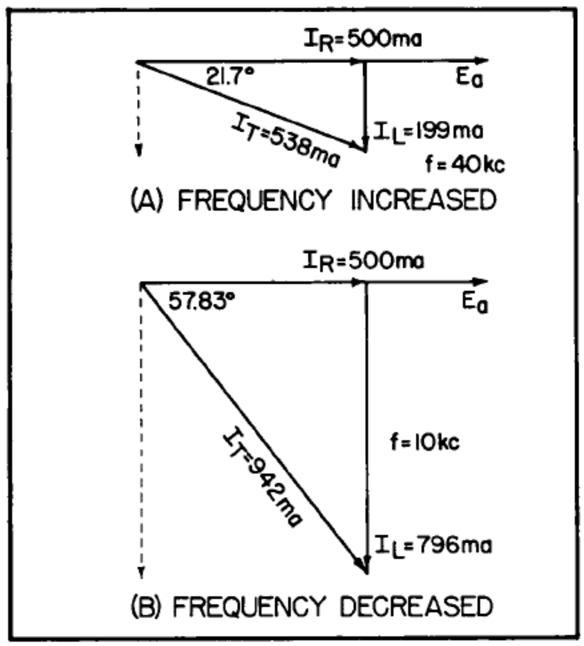

Figure 12-6 - Vector analysis of LR circuit under varied frequency conditions.

Increased Resistance: In Figure 12-8A the resistance has been doubled. The vector diagram indicates that the result of increased resistance is to decrease the current (I_R) through the resistive branch, decrease the total current, increase the phase angle, decrease apparent power, increase total impedance and decrease true power.

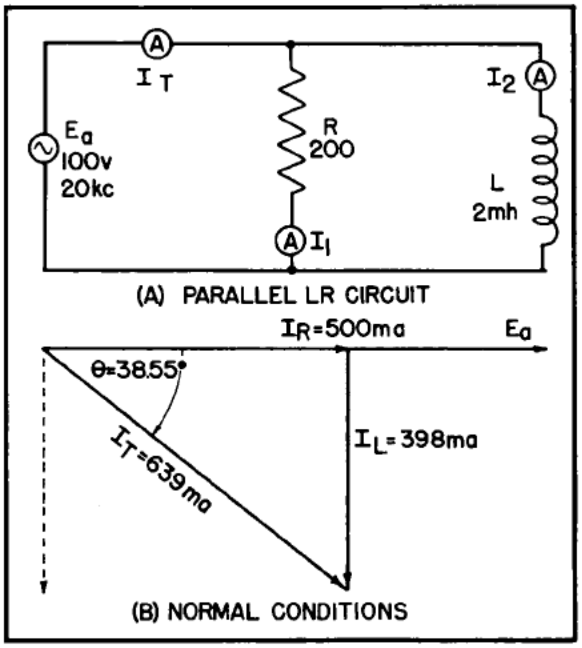

Figure 12-7 - Parallel RL circuit and vector diagram.

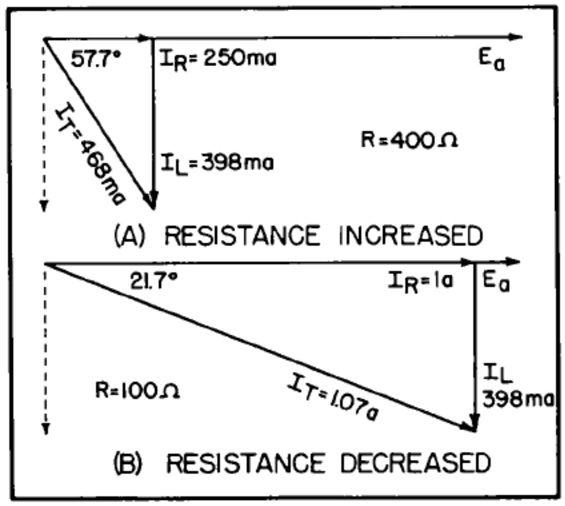

Figure 12-8 - Vector analysis of parallel LR circuit under varied resistance conditions.

Decreased Resistance: Figure 12-8B illustrates the condition of the circuit when the resistance is decreased to half of its original value. It can be seen from the vector diagram that the current through the inductive branch is unchanged while the current through the resistive branch has doubled. The overall result of decreased resistance will be increased total current, decreased phase angle, decreased total impedance and increased true power.

12-6. Effect of Varying Inductance and Voltage

A variation of inductance will produce the same effect on circuit values as a similar variation in frequency. The reason for this will be clear if it is remembered that X_L is a direct function of inductance as well as frequency. Therefore, if the frequency is held constant and the inductance doubled the same value of X_L will result as when the inductance was held constant and the frequency doubled. The above may be made clearer if it is stated that an increase in inductance will decrease current through the inductor, decrease total current, decrease phase angle, decrease apparent power and increase total impedance. A decrease in inductance will have the opposite effect on all the above values.

Current is a direct function of applied voltage and power varies as the square of applied voltage. Therefore, an increase of applied voltage (E_a) will merely increase all values of current and power. A decrease of E_a will decrease all values of current and power.

Q6. If the resistance of a parallel RL circuit is halved will the true power increase to double or four times the original value?

12-7. Equivalent Circuit

It has been shown on previous occasions that a seemingly complicated circuit may often be simplified by reducing it to an equivalent circuit. Any parallel circuit, no matter how complex, may be reduced to an equivalent series circuit, which will present the same characteristics (impedance and phase angle) to the source. The distinct advantage of being able to determine the equivalent series impedance of a parallel circuit, without knowing the applied voltage or current, can best be presented by use of a sample problem.

Example. Determine the approximate total current in the circuit of Figure 12-9.

Given:	$E_a = 10V$	$R_1 = 5$ ohms
	$X_L = 30$ ohms	$R_2 = 5$ ohms

Solution: In order to determine the approximate current in Figure 12-9A the impedance of the circuit must first be determined. This would be a relatively simple task if the impedance of the parallel circuit (within the dotted box) were known. This parallel circuit may be converted

A4. No. Inductance is mainly determined by the physical properties of the coil and not by applied voltage or frequency.

A5. True power is dissipated by resistance. Therefore, true power did not change because current and voltage of the resistive branch did not change.

A6. Double. $P = E^2/R$

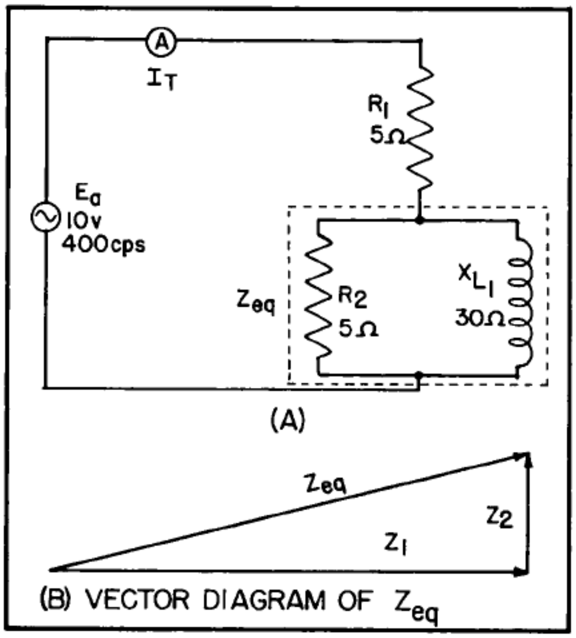

Figure 12-9 - Series equivalent circuit determination.

to an equivalent series impedance which may be substituted in place of the parallel combination. As a result of this action the original complex circuit will be transformed into a series circuit which may then be solved by application of standard series circuit methods.

In the determination of the equivalent series impedance (Z_{eq}), rectangular notation and a derivation of the "product over the sum" method used for parallel resistive circuits will be employed. The product over the sum equation for parallel impedances is expressed mathematically as:

$$Z_t = \frac{Z_1 Z_2}{Z_1 + Z_2} \qquad (12\text{-}7)$$

where: Z_t = total impedance in ohms
Z_1 = impedance of branch one in ohms
Z_2 = impedance of branch two in ohms

In the application of equation (12-7) to the circuit of Figure 12-9A, the resistive branch will be designated Z_1, and written in rectangular notation as $(5 + j\,0)$ while the inductive branch will be designated Z_2 and written $(0 + j\,30)$.

Insert values:

$$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$Z_{eq} = \frac{(5 + j\,0)(0 + j\,30)}{(5 + j\,0) + (0 + j\,30)}$$

Multiplication of the complex numerator and addition of the complex denominator yields:

$$Z_{eq} = \frac{0 + j\,150}{5 + j\,30}$$

To rationalize the complex denominator (remove the j factor), multiply both numerator and denominator by the conjugate of the denominator:

$$Z_{eq} = \frac{0 + j\,150}{5 + j\,30} \times \frac{5 - j\,30}{5 - j\,30}$$

$$Z_{eq} = \frac{4500 + j\,750}{925}$$

Dividing the denominator into BOTH the real and reactive parts of the numerator:

$$Z_{eq} = 4.86 + j\,0.81 \text{ ohms}$$

The complex answer indicates that the equivalent series impedance, Z_{eq}, consists of a resistive component $(4.86 + j\,0)$ and a reactive component $(0 + j\,0.81)$. The equivalent impedance is the vector sum of these two components (see Figure 12-9B). Since the resistive component is much larger than the reactive component, Z_{eq} can be considered to be predominately resistive. An approximation of Z_{eq} may be made by measuring the resultant vector. A more accurate determination may be made by use of the following equation:

$$Z = \sqrt{R^2 + X_L^2} \qquad (11\text{-}6)$$

Insert values:

$$Z_{eq} = \sqrt{(4.86)^2 + (0.81)^2}$$

$$Z_{eq} = 4.93 \text{ ohms}$$

Thus, Z_{eq} is approximately 5 ohms. Figure 12-10 shows the equivalent series circuit formed by the insertion of Z_{eq}.

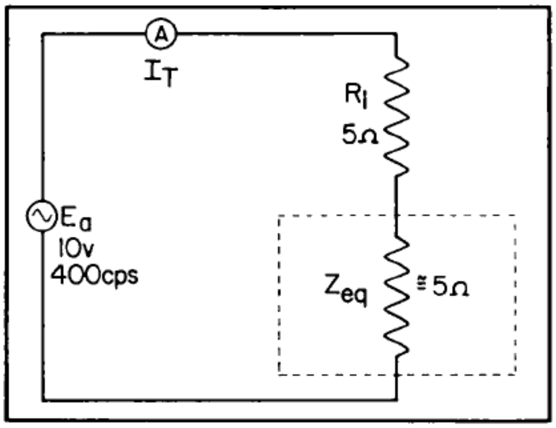

Figure 12-10 - Series equivalent circuit.

Figure 12-10 may now be solved by application of the equation:

$$I_t = \frac{E_a}{R_1 + Z_{eq}}$$

$$I_t = \frac{10}{10} = 1 \text{ amp}$$

NOTE: The series equivalent of a parallel circuit is valid only for the given operating frequency. If the frequency is varied then a new equivalent circuit must be determined for each new operating frequency.

Q7. Write the impedance equation, using rectangular form, for a parallel RL circuit if resistance is twelve ohms and the inductive reactance is thirty ohms.

Q8. Why is the equivalent circuit only valid at the given operating frequency?

"Q" CONSIDERATIONS

12-8. Effect of High Q Coils on Parallel RL Circuits.

The Q of a coil is defined mathematically as the ratio of inductive reactance to the inherent or effective resistance of the coil.

$$Q = \frac{X_L}{R} \qquad (11\text{-}14)$$

As was previously mentioned, the R in equation (11-14) represents the effective (all) losses of the coil. Since Q is an inverse function of R, it follows that a high Q indicates the existance of a low effective resistance in the coil. In previous examples of parallel RL circuits in this chapter the effective resistance, which is represented as a lumped resistance in series with the coil, has been neglected. The effective resistance of a coil will aid in limiting the current through the inductive branch and will also affect the phase angle of the branch current. If the Q of a coil is high, the amount of current limiting and the effect on phase angle due to the effective resistance of the coil is negligible.

To illustrate the effects of coil Q it is first necessary to establish ideal conditions (perfect inductor and resistor). Figure 12-11A shows a circuit in which branch one contains a pure resistance and branch two contains a pure inductance. Using the values in the diagram the current through each branch, the total current, and the circuit phase angle can be computed.

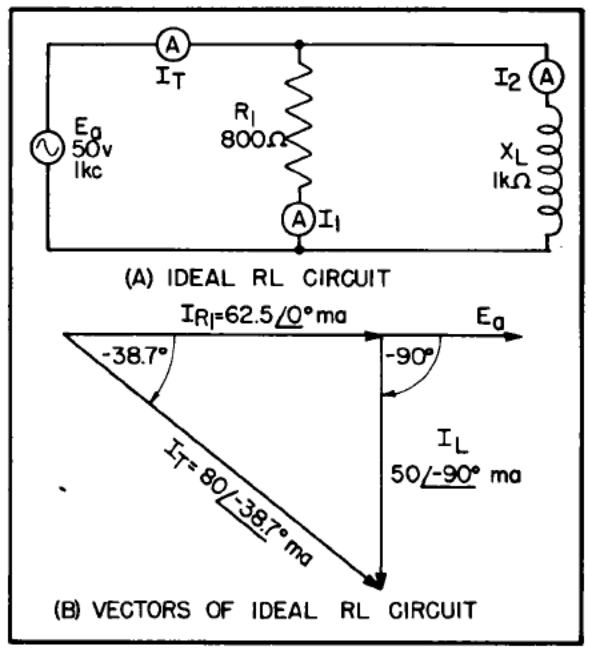

Figure 12-11 - Circuit conditions with perfect coil and resistor.

The current through branch one is:

$$I_1 = \frac{E_a}{R}$$

$$I_1 = \frac{50\angle 0^\circ}{800\angle 0^\circ}$$

$$I_1 = 62.5\angle 0^\circ \text{ ma}$$

A7. $Z = \frac{(12+j0)(0+j30)}{(12+j0)+(0+j30)}$

A8. Changing the frequency will change the X_L and thereby change the equivalent impedance.

The current through branch two is:

$$I_2 = \frac{Ea}{jX_L}$$

$$I_2 = \frac{50 \angle 0^\circ}{1000 \angle 90^\circ}$$

$$I_2 = 50 \angle -90^\circ \text{ ma}$$

The total current is equal to the sum of the branch currents, or:

$$I_t = I_1 + I_2$$

$$I_t = (62.5 + jo) + (0 - j50)$$

$$I_t = 62.5 - j50 \text{ ma}$$

Converting to polar form:

$$\theta = \arctan \frac{-j0.050}{0.0625}$$

$$\theta = \arctan 0.8$$

$$\theta = -38.7^\circ$$

$$I_t = \frac{I2}{\sin \theta}$$

$$I_t = \frac{0.050}{0.625}$$

$$I_t = 80 \angle -38.7^\circ \text{ ma}$$

These results are summarized by means of vectors in part B of Figure 12-11. Notice that the current in branch one (I_{R1}) is in phase with the applied voltage, and that the current in branch two (I_L) is EXACTLY -90° out of phase with the applied voltage. Since the total current in an RL circuit is neither entirely resistive nor entirely reactive, it must lag the applied voltage by some angle between 0° and -90°. For the values of resistance and reactance shown in Figure 12-11 the total current lags the applied voltage by -38.7°.

If the perfect coil in Figure 12-11 is replaced with a practical coil, power losses will be introduced into the heretofore lossless inductive branch. For purposes of analysis these losses can be represented by a resistor connected in series with the coil. This resistance is assumed to account for all of the losses of the coil, both ac and dc, and is called the effective resistance (R_e) of the coil.

Now that the characteristics of a perfect coil have been established, the operation of an RL circuit containing a high Q coil (10 or greater) can be examined. If the perfect coil is replaced with a practical one having a Q of 50, the RL circuit will appear as shown in Figure 12-12A.

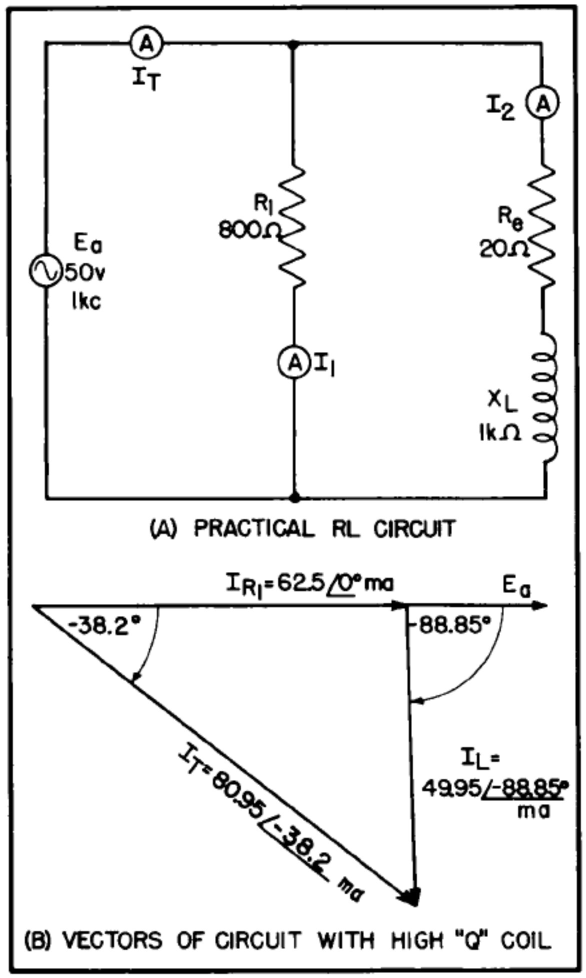

Figure 12-12 - Circuit conditions with high Q coil.

Solving for current I_1:

$$I_1 = \frac{Ea}{R_1}$$

$$I_1 = \frac{50}{800}$$

$$I_1 = 62.5 + j0 \text{ ma}$$

n order to find I_2 the impedance of branch two nust be computed:

$$Z_2 = 20 + j1000 \text{ ohms}$$

Converting Z_2 to polar form:

$$\theta = \arctan \frac{jX_L}{R_e}$$

$$\theta = \arctan \frac{j1000}{20}$$

$$\theta = \arctan 50$$

$$\theta = 88.85^\circ$$

$$Z_2 = \frac{j1000}{\sin \theta}$$

$$Z_2 = \frac{1000}{0.999}$$

$$Z_2 = 1001 \underline{/88.85^\circ} \text{ ohms}$$

Computing I_2:

$$I_2 = \frac{E_a}{Z_2}$$

$$I_2 = \frac{50 \underline{/0^\circ}}{1001 \underline{/88.85^\circ}}$$

$$I_2 = 49.95 \underline{/-88.85^\circ} \text{ ma}$$

Converting I_2 to rectangular form:

$$I_2 = 49.95 (\cos 88.85^\circ - j \sin 88.85^\circ)$$

$$I_2 = 49.95 (0.020 - j0.999)$$

$$I_2 = 0.999 - j49.95 \text{ ma}$$

Computing I_t:

$$I_t = I_1 + I_2$$

$$I_t = (62.5 + j0) + (0.999 - j49.95)$$

$$I_t = 63.49 - j49.95$$

Converting to polar form:

$$\theta = \arctan \frac{-j49.95}{63.49}$$

$$\theta = -38.2^\circ$$

$$I_t = \frac{49.95}{\sin 38.2^\circ} = \frac{49.95}{0.6184}$$

$$I_t = 80.95 \underline{/-38.2^\circ} \text{ ma}$$

The complete data for the circuit is illustrated by the vector diagram in Figure 12-12B. Notice that replacing the perfect coil with a practical high Q coil has very little effect on overall circuit conditions.

The vector diagram indicates that the effective resistance of a high Q coil has practically no effect on the phase angle of the inductive branch. In other words, the current (I_2) lags the applied voltage (E_a) by almost 90°. The computations of impedance and current for the inductive branch also indicate very little change between the conditions existing for an ideal inductor and one with very low effective resistance (high Q).

A comparison of the vector diagram in 12-11B (no loss coil) with vector diagram 12-12B (high Q coil) will show only a minor effect on circuit current and phase angle as a result of including the effective resistance of the high Q coil. Due to its negligible effect, the inherent resistance of a high Q coil is usually neglected during computations in a parallel circuit.

Q9. What will happen to the effective resistance of a coil if frequency is increased?

12-9. Effect of Low Q Coils on Parallel RL Circuits

In most applications when the Q of a coil is less than 10, the effect of coil resistance on circuit current and impedance can no longer be neglected. To show the effect of a low Q coil on a parallel RL circuit an example problem using the same circuit as was used in section 12-8 will be utilized, the only change being the substitution of a coil with a Q of two instead of 50. The low Q circuit is shown in Figure 12-13A.

Solving for the current in branch one:

$$I_1 = \frac{E_a}{R_1}$$

$$I_1 = 62.5 \underline{/0^\circ} \text{ ma}$$

Computing the impedance of branch two:

$$\theta = \arctan \frac{jX_L}{R_e}$$

$$\theta = \arctan \frac{j1000}{500}$$

A9. Due to skin effect and radiation losses the effective resistance will increase.

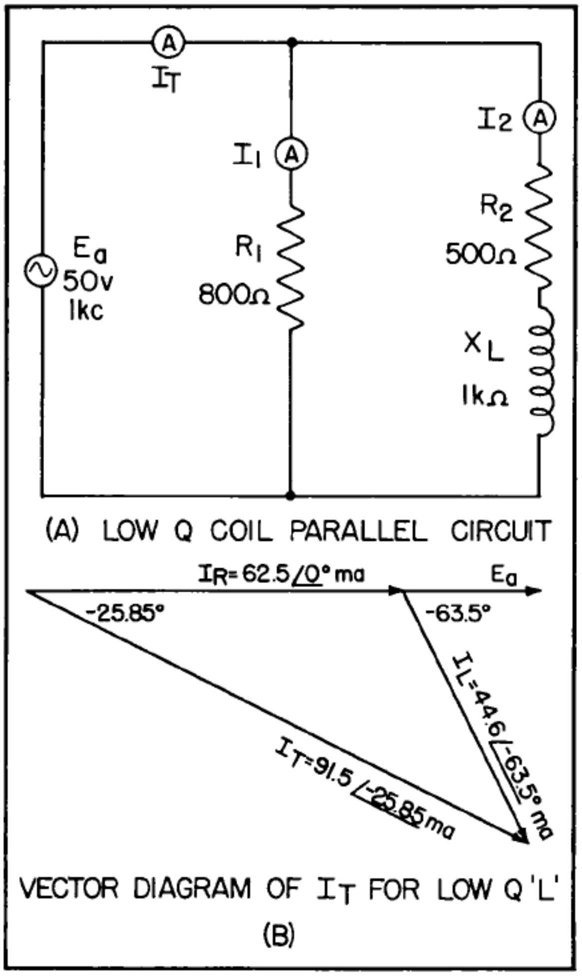

Figure 12-13 - Effect of low Q coil on parallel RL circuit.

$$\theta = 63.5^\circ$$

$$Z_2 = \frac{R_e}{\cos\theta}$$

$$Z_2 = \frac{500}{0.446}$$

$$Z_2 = 1121 \underline{/63.5^\circ} \text{ ohms}$$

Determine the current in branch two:

$$I_2 = \frac{E_a}{Z_2}$$

$$I_2 = \frac{50 \underline{/0^\circ}}{1121 \underline{/63.5^\circ}}$$

$$I_2 = 44.6 \underline{/-63.5^\circ} \text{ ma}$$

Converting I_2 to rectangular form:

$$I_2 = 44.6\,(\cos 63.5^\circ - j \sin 63.5^\circ)$$

$$I_2 = 44.6\,(0.446 - j\,0.895)$$

$$I_2 = 19.9 - j\,39.9 \text{ ma}$$

The total current is obtained by adding I_1 and I_2:

$$I_t = I_1 + I_2$$

$$I_t = (62.5 + j\,0) + (19.9 - j\,39.9)$$

$$I_t = 82.4 - j\,39.9 \text{ ma}$$

I_t in polar form is:

$$I_t = 91.5 \underline{/-25.85^\circ} \text{ ma}$$

It should be noted at this time that the large change in Q of the coil (from Q = 50 down to Q = 2) has decreased the current in the inductive branch by (5.35 ma), while the current flow in branch one (I_1) remains unchanged. A comparison of Figures 12-12B and 12-13B will show the current in the high Q coil (Figure 12-12B) lagging applied voltage by 88.85°, while the current in the low Q coil (Figure 12-13B) is lagging E_a by 63.5°. Therefore, insertion of the low Q coil has caused a decrease of 25.35° in the phase angle of the inductive branch current.

It should now be apparent that coil resistance must be taken into consideration if the Q of the coil is less than ten. Otherwise, the values calculated for the circuit will differ considerably from those existing in the actual circuit.

Q10. What is the advantage of converting quantities in polar form to rectangular form?

RC PARALLEL CIRCUITS

12-10. Purely Capacitive Parallel Circuits

A purely capacitive parallel circuit behaves in much the same manner as a purely inductive parallel circuit except for the following differ-

ences: (1) the capacitive reactance (X_C) is INVERSELY PROPORTIONAL to frequency and (2) current LEADS the voltage in a capacitive circuit.

Figure 12-14A illustrates a purely capacitive parallel circuit. Because this is a parallel circuit the voltage across each capacitor is equal to the source voltage (E_a). The current through each branch is determined by the opposition of the individual branch. Figure 12-14B shows the current for each capacitor (I_c) and the total current (I_t) as being in phase with each other and 90° ahead of the voltage. This condition (whereby all the branch currents are in phase

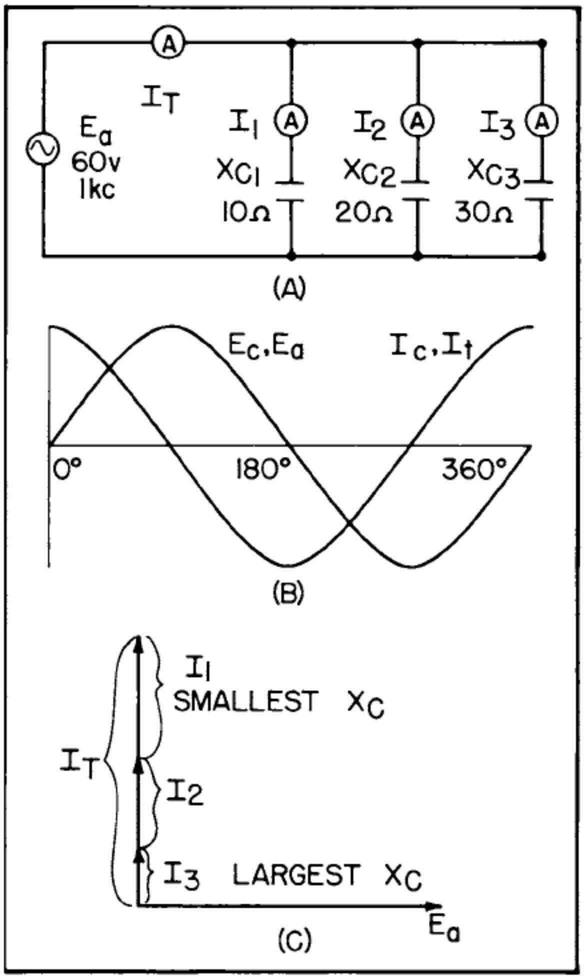

Figure 12-14 - E and I relationships in a purely capacitive circuit.

with each other and total current) will only apply if all components are perfect (no resistance). Figure 12-14C is the vector representation of the circuit. E_a, being common to all components, is shown plotted in standard position. Since capacitive current is leading voltage by 90° the current vector in a capacitive circuit is plotted upward on the vertical axis. The different lengths of the current vectors indicate their relative magnitudes. Since the reactance of branch one is the lowest (X_{C1} = 10 ohms) the current vector (I_1) is the longest. Total current in a PURELY capacitive circuit is the algebraic sum of the current vectors (branch currents). Expressed mathematically:

$$I_t = I_1 + I_2 + I_3$$

NOTE: As with pure inductors the algebraic addition is a special case. Vector addition will normally be used because in practice, all circuits contain some resistance.

CHARACTERISTICS OF PARALLEL RC CIRCUITS

12-11. Vector and Waveform Analysis

Parallel RC circuits may be resolved in much the same way as are parallel RL circuits. Figure 12-15A illustrates a parallel RC circuit.

Figure 12-15B shows a composite diagram of the circuit conditions. The current vectors I_R and I_C are out of phase, therefore, vector addition must be used to determine total current.

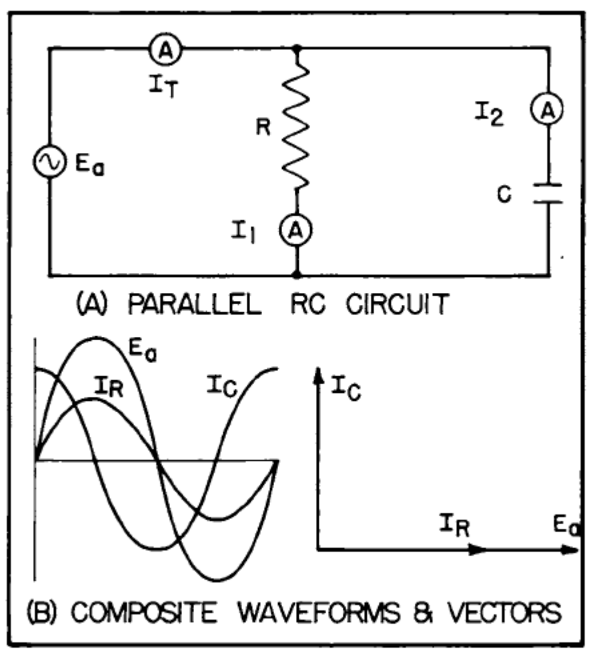

Figure 12-15 - Parallel RC circuit analysis.

A10. Complex numbers in rectangular form may be added or subtracted easily.

12-12. Assumed Voltage Method

It is not always necessary to know the voltage across a parallel circuit in order to solve for impedance. Since impedance is independent of voltage ANY assumed value of voltage may be applied and the correct value of impedance determined. To demonstrate this, the impedance of the circuit in Figure 12-16 will be computed by the assumed voltage method and checked by the product over the sum method.

Example. Determine total impedance of a 10 ohm resistor in parallel with a capacitor exhibiting 25 ohms of reactance. Solve by assumed voltage method.

Given: $R = 10$ ohms
$X_C = 25$ ohms
$E_a = ?$

Find: $Z_t = ?$

Solution: Assume an applied voltage of 50 volts.

Then:

$$I_1 = \frac{E_a}{R}$$

$$I_1 = \frac{50}{10}$$

$$I_1 = 5 \text{ amps}$$

$$I_2 = \frac{E_a}{X_C}$$

$$I_2 = \frac{50}{25}$$

$$I_2 = 2 \text{ amps}$$

Determine phase angle:

$$\theta = \arctan \frac{I_C}{I_R} = \frac{I_2}{I_1} \qquad (12\text{-}10)$$

$$\theta = \arctan \frac{2}{5}$$

$$\theta = 21.8^\circ$$

Determine total current:

$$I_t = \frac{I_R}{\cos\theta} \qquad (12\text{-}2)$$

$$I_t = \frac{5}{0.9285}$$

$$I_t = 5.39 \text{ amps}$$

To determine total impedance, divide assumed applied voltage by the calculated total current.

$$Z_t = \frac{E_a}{I_t}$$

$$Z_t = \frac{50}{5.39}$$

$$Z_t = 9.28 \text{ ohms}$$

Therefore, by the assumed voltage method the total impedance is described in polar notation as $9.28 \angle -21.8^\circ$ ohms.

Solution by product over the sum method:

For this solution Z_1 will be the resistive branch and be written: $Z_1 = 10 + j0$.

Z_2 will be the reactive branch and be written:

$$Z_2 = 0 - j25$$

Insert values in equation and solve:

$$Z_t = \frac{Z_1 Z_2}{Z_1 + Z_2} \qquad (12\text{-}7)$$

$$Z_t = \frac{(10 + j0)(0 - j25)}{(10 + j0) + (0 - j25)}$$

$$Z_t = \frac{(0 - j250)}{(10 - j25)} \times \frac{(10 + j25)}{(10 + j25)}$$

$$Z_t = \frac{6250 - j2500}{725}$$

$$Z_t = 8.62 - j3.45$$

Determine Z_t by equation:

$$Z_t = \sqrt{R^2 + X_C^2}$$

$$Z_t = \sqrt{(8.62)^2 + (3.45)^2}$$

$$Z_t = 9.28 \text{ ohms}$$

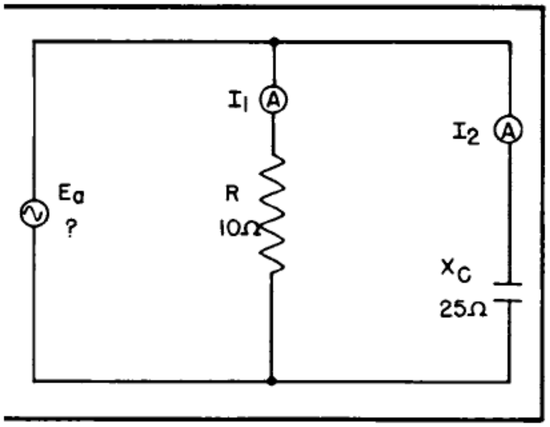

Figure 12-16 - Parallel RC circuit.

herefore, the value obtained for Z_t by the :oduct over the sum method is the same as ıat obtained by the assumed voltage method. he assumed voltage method is applicable to ıy parallel circuit.

2-13. RC Circuit Analysis

The solving of an RC circuit follows the method :eviously applied to LR circuits. In view of ıis, and to prevent needless repetition an ex- nple circuit will be solved without detailed ıstructions. The student experiencing diffi- ılty is directed to review section 12-3.

Example. The circuit of Figure 12-17A con- .ins a 36 ohm resistor in parallel with a 6 ıicrofarad capacitor. Both are connected :ross a 25 volt source operating at a frequency : 1.46 kilocycles. Solve for the following:

X_C = ?	I_t = ?	P_t = ?
I_R = ?	Z = ?	P_a = ?
I_C = ?	θ = ?	P.F. = ?

iven: E_a = 25 volts
f = 1.46 kc
R = 36 ohms
C = 6 microfarads

ɔlution:

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{(6.28)(1.46 \times 10^3)(6 \times 10^{-6})}$$

$$X_C = 18.2 \text{ ohms}$$

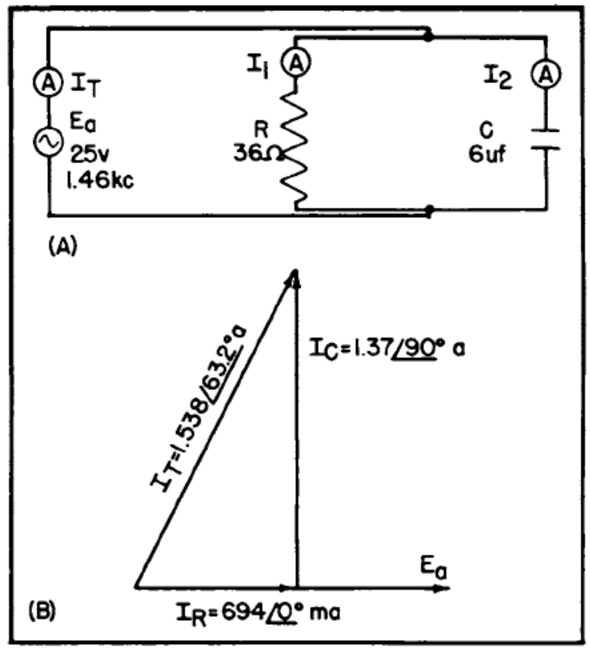

Figure 12-17 - Parallel RC circuit.

$$I_1 = \frac{E_a}{R}$$

$$I_1 = \frac{25}{36}$$

$$I_1 = 694 \text{ ma}$$

$$I_2 = \frac{E_a}{X_C}$$

$$I_2 = \frac{25}{18.2}$$

$$I_2 = 1.37 \angle 90^\circ \text{ amps}$$

$$\theta = \arctan \frac{I_C}{I_R}$$

$$\theta = \arctan \frac{1.37}{0.694}$$

$$\theta = 63.2^\circ$$

$$I_t = \frac{I_C}{\sin \theta}$$

$$I_t = \frac{1.37}{0.894}$$

$$I_t = 1.538 \angle 63.2^\circ \text{ amps}$$

$$Z_t = \frac{E_a}{I_t}$$

$$Z_t = \frac{25 \angle 0^\circ}{1.538 \angle 63.2^\circ}$$

$$Z_t = 16.25 \angle -63.2^\circ \text{ ohms}$$

$$P.F. = \cos \theta$$

$$P.F. = 0.451$$

$$P_a = E_a \times I_t$$

$$P_a = 25 \times 1.538$$

$$P_a = 38.4 \text{ volt-amperes}$$

$$P_t = P_a \times P.F.$$

$$P_t = 38.4 \times 0.451$$

$$P_t = 17.32 \text{ watts}$$

The equations and method of solving a parallel RC circuit can be seen to be similar in many respects to those used in RL parallel circuits. The vector diagram of Figure 12-17 sums up the circuit just described. The only major difference between the RC vector diagram and an RL vector diagram is the angles the reactive component vectors (I_C, I_L, etc) and the resultant vectors (Z_t, I_t, etc.) assume in respect to the standard position.

VARIATIONAL ANALYSIS

12-14. Effects of Varying Circuit Properties

The effects of a variation in one of the circuit properties such as capacitance, resistance, etc., on circuit action has been analyzed in detail for an RL circuit. Many of the results will be the same for the RC circuits while others will be directly opposite. For convenience a table has been provided at the end of this section comparing RL and RC parallel circuits under varying circuit conditions.

In order to analyze the effects of a change in some quantity there must be a standard for reference. The circuit and vector diagram of Figure 12-18 will be assumed to be the normal operating conditions.

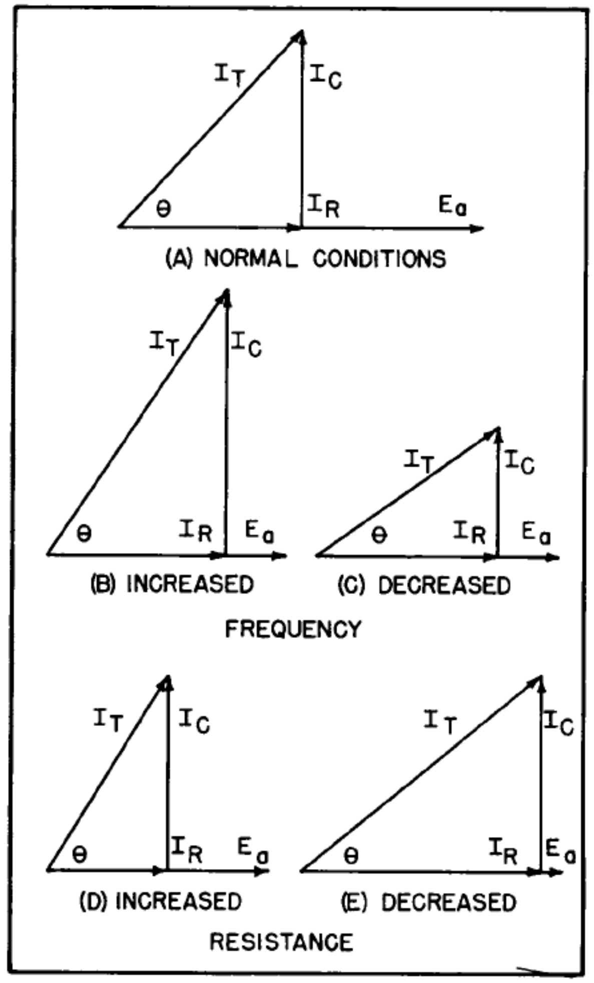

Figure 12-18 - Vector diagrams for variational analysis.

Variation of Frequency: Capacitive reactance is inversely proportional to frequency. Therefore, an INCREASE in frequency will decrease X_C. A decreased X_C will allow more current flow in the capacitive branch. Figure 12-18B shows I_C increased. An increase in I_C will also increase I_t. Total impedance will decrease. Apparent power is a direct function of current and will therefore increase. True power will remain unchanged. As can be seen in Figure 12-18B the phase angle of the circuit has also increased. A decrease in frequency will have the opposite effect on the above quantities. A vector diagram of the circuit with frequency decreased is shown in Figure 12-18C.

Variation of Resistance: A variation in the parallel resistance will cause basically the same results in both RL and RC circuits. The current

VARIABLE QUANTITIES		PARALLEL RL CIRCUIT									PARALLEL RC CIRCUIT								
		X_L	I_L	I_R	I_T	Z_T	$P_{apparent}$	P_{true}	Pwr. Fact	θ	X_C	I_C	I_R	I_T	Z_T	$P_{apparent}$	P_{true}	Pwr. Fact	θ
f	↑	↑	↓	→	↓	↑	↓	→	↑	↓	↓	↑	→	↑	↓	↑	→	↓	↑
	↓	↓	↑	→	↑	↓	↑	→	↓	↑	↑	↓	→	↓	↑	↓	→	↑	↓
L	↑	↑	↓	→	↓	↑	↓	→	↑	↓									
	↓	↓	↑	→	↑	↓	↑	→	↓	↑									
C	↑										↓	↑	→	↑	↓	↑	→	↓	↑
	↓										↑	↓	→	↓	↑	↓	→	↑	↓
R	↑	→	→	↓	↓	↑	↓	↓	↓	↑	→	→	↓	↓	↑	↓	↓	↓	↑
	↓	→	→	↑	↑	↓	↑	↑	↑	↓	→	→	↑	↑	↓	↑	↑	↑	↓
E_A	↑	→	↑	↑	↑	→	↑	↑	→	→	→	↑	↑	↑	→	↑	↑	→	→
	↓	→	↓	↓	↓	→	↓	↓	→	→	→	↓	↓	↓	→	↓	↓	→	→

↑ INCREASE
↓ DECREASE
→ REMAIN CONSTANT

TABLE 12-1
Comparison of RL and RC parallel circuits.

through the reactive branch will remain unchanged. An increase in resistance is illustrated in Figure 12-18D. The current in the resistive branch (I_R) has decreased. The result is a decrease in I_t and an increase in phase angle. Due to the inverse relationship of current and impedance the Z_t will increase. True power and apparent power will decrease. A decrease in resistance results in the opposite effect and is depicted in Figure 12-18E.

Variation of Applied Voltage: All currents in the parallel RC circuit vary directly as voltage varies. Power varies as the square of voltage. Resistance, capacitance and impedance are independent of voltage.

The use of Table 12-1 is best explained by an example.

Example. Determine the effect on the apparent power (P_a) in a parallel RC circuit when the capacitance is decreased.

Solution: Look down the first column to capacitance. Take the subdivision with the arrow pointing down to indicate decreased capacitance. Follow this subdivision across to the vertical column marked (P_a) under the parallel RC circuit. The arrow, in the block at the junction of the row and column, is pointing down. Therefore, when capacitance is decreased the apparent power will decrease.

(An arrow pointing to the right indicates there is no change.)

Q11. An increase in capacitance in a parallel RC circuit will have what effect on the phase angle of the circuit and the voltage drop across the resistor?

A11. Phase angle increases, the voltage drop across the resistor remains the same.

12-15. Parallel RCL Circuits (Non-Resonant)

A parallel RCL circuit may be analyzed in the same manner as the parallel RC and the parallel RL circuits. It is simply a three branch circuit consisting of a resistor, capacitor and an inductor connected in parallel. A parallel RCL circuit and its associated vector diagram is shown in Figure 12-19.

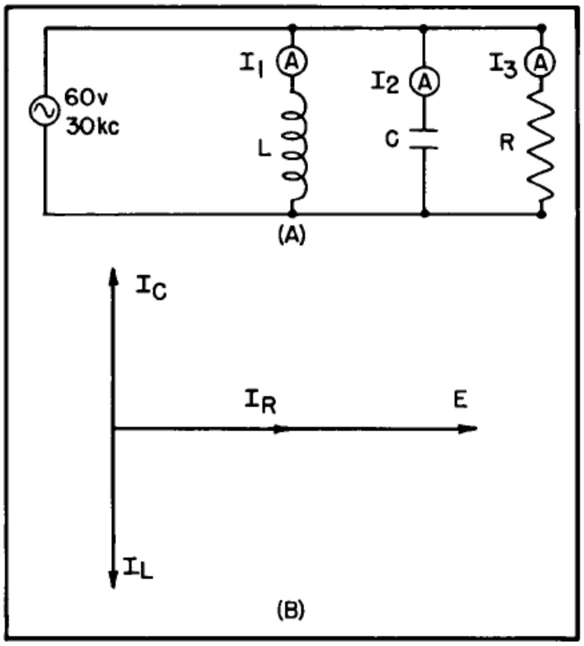

Figure 12-19 - Parallel RCL circuit and its associated vector diagram.

The vector diagram in (B) of Figure 12-19 has much in common with the vector diagram for a series RCL circuit because the two reactive components are one-hundred and eighty degrees out of phase with each other. Consider the solution of a three branch parallel RCL circuit (Figure 12-19). If the values for the resistance, capacitance, and inductance are as follows:

$R = 4.8\,k \qquad C = 6000\ pf$

$f = 30\,kc \qquad L = 6\ mh$

the circuit variables may be computed in the following manner:

$$X_L = 2\pi f L$$

$$X_L = (6.28)(30 \times 10^3)(6 \times 10^{-3})$$

$$X_L = j\,1130.4 \text{ ohms}$$

$$I_1 \text{ or } I_L = \frac{E_a}{j X_L}$$

$$I_L = \frac{60}{j\,1130.4}$$

$$I_L = 0 - j\,53.1 \text{ ma}$$

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{(6.28)(30 \times 10^3)(6 \times 10^{-9})}$$

$$X_C = -j\,884.9 \text{ ohms}$$

$$I_2 \text{ or } I_C = \frac{E_a}{-j X_C}$$

$$I_C = \frac{60}{-j\,884.9}$$

$$I_C = 0 + j\,67.8 \text{ ma}$$

$$I_3 \text{ or } I_R = \frac{E_a}{R}$$

$$I_R = \frac{60}{4.8 \times 10^3}$$

$$I_R = 12.5 + j\,0 \text{ ma}$$

Vector addition may be accomplished either by graphical means or by addition of the rectangular components:

$$\begin{aligned} I_L &= \ \ 0.0 - j\,53.1 \\ I_C &= \ \ 0.0 + j\,67.8 \\ I_R &= 12.5 + j\ \ 0.0 \\ \hline I_t &= 12.5 + j\,14.7 \end{aligned}$$

Since the branch currents are not in phase with one another it is not possible to algebraically combine the above terms to find total current. The phase angle must be determined trigonometrically and then the total current and power can be computed.

$$\theta = \arctan \frac{I_{XC} - I_{XL}}{I_R}$$

$$\theta = \frac{14.7}{12.5} = 1.175$$

$\theta = 49.6^\circ$ (angle of total current)

I_t is now calculated:

$$I_t = \frac{I_R}{\cos\theta}$$

$$I_t = \frac{12.5 \times 10^{-3}}{\cos 49.6^\circ}$$

$$I_t = 19.3 \text{ ma}$$

$$I_t = 19.3 \underline{/49.6^\circ} \text{ ma}$$

Determine Z_t:

$$Z_t = \frac{E_a}{I_t}$$

$$Z_t = \frac{60 \underline{/0^\circ}}{19.3 \times 10^{-3} \underline{/49.6^\circ}}$$

$$Z_t = 3108 \underline{/-49.6^\circ} \text{ ohms}$$

Determine true power:

$$P_t = P_a \times \cos\theta$$

$$P_t = E_a \times I_t \times \cos\theta$$

$$P_t = (60)(19.3 \times 10^{-3})(0.648)$$

$$P_t = 750 \text{ mw}$$

Using the value $Z = 3.1 \underline{/-49.6^\circ}$ k ohms, the series equivalent circuit will be:

$$Z_t = \mathbf{Z} (\cos\theta - j\sin\theta)$$

$$Z_t = 3.1 \times 10^3 (\cos 49.6^\circ - j\sin 49.6^\circ)$$

$$Z_t = 3.1 \times 10^3 (0.648 - j0.762)$$

$$Z_t = 2008 - j2362 \text{ ohms}$$

The series equivalent circuit is an RC circuit consisting of the values shown in Figure 12-20. Compute the value of capacity in the series equivalent as follows:

$$X_C = \frac{1}{2\pi f C}$$

transposing:

$$C = \frac{1}{(2\pi f)(X_{eq})}$$

$$C = \frac{1}{(6.28)(3 \times 10^4)(2.362 \times 10^3)}$$

$$C = 2247 \text{ picofarad}$$

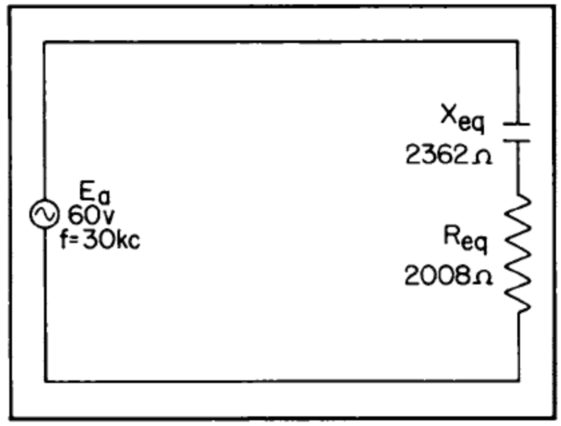

Figure 12-20 - RC series equivalent circuit.

Q12. Why is the impedance angle (mathematically speaking) and total current angle opposite in sign?

Q13. What indicates how a series equivalent circuit appears (inductive or capacitive) to the source?

12-16. Parallel LC Resonance

The ideal parallel resonant circuit is one that contains only inductance and capacitance. Resistance and its effects are not considered in an ideal parallel resonant circuit. One condition for parallel resonance is the application of that frequency which will cause the inductive reactance to equal the capacitive reactance. The formula used to determine the resonant frequency of a parallel LC circuit is the same as the one used for a series circuit.

$$f_o = \frac{1}{2\pi\sqrt{LC}} \qquad (11\text{-}16)$$

where: f_o = resonant frequency in cps
L = inductance in henrys
C = capacitance in farads

If the circuit values are those shown in Figure 12-21A, the resonant frequency may be computed as follows:

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

A12. Because when dividing polar vectors, the angle in the denominator is subtracted.

A13. The sign of the phase angle.

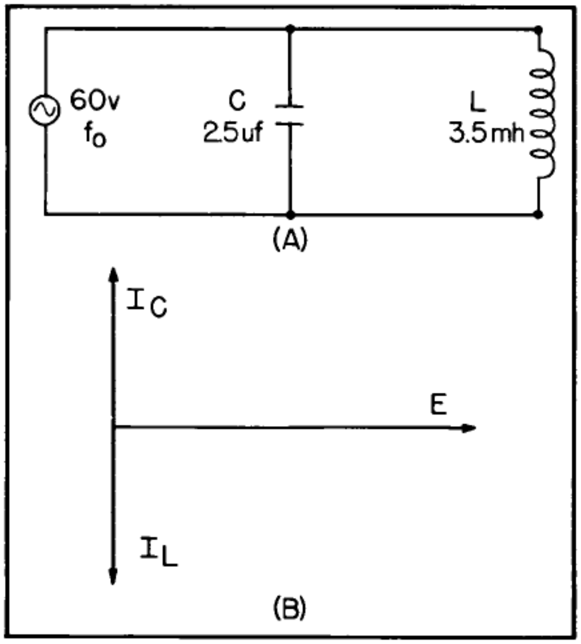

Figure 12-21 - Parallel LC circuit at resonance.

$$f_o = \frac{1}{6.28\sqrt{(3.5 \times 10^{-3})(2.5 \times 10^{-6})}}$$

f_o = 1700 cycles per second

At the resonant frequency:

Determine X_L:

$$X_L = 2\pi f L$$

$$X_L = (6.28)(1.7 \times 10^3)(3.5 \times 10^{-3})$$

$$X_L \approx 37.4 \text{ ohms}$$

Determine X_C:

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{(6.28)(1.7 \times 10^3)(2.5 \times 10^{-6})}$$

$$X_C \approx 37.4 \text{ ohms}$$

Determine IC:

$$I_C = \frac{E_a}{-jX_C}$$

$$I_C = \frac{60}{-j37.4}$$

$$I_C = 0 + j1.605 \text{ amps}$$

Determine I_L:

$$I_L = \frac{E_a}{jX_L}$$

$$I_L = \frac{60}{j37.4}$$

$$I_L = 0 - j1.605 \text{ amps}$$

The total current is determined by addition of the two currents in rectangular form:

$$\begin{array}{l} I_C = 0 + j1.605 \\ I_L = 0 - j1.605 \\ \hline I_t = 0 \end{array}$$

Therefore, in an ideal resonant parallel circuit the line current (I_t) is zero. If total current is zero then:

$$Z = \frac{E_{app}}{I_t} = \frac{60}{0} = \text{UNDEFINED}$$

or: it may be said that the impedance approaches infinity.

At frequencies other than the natural resonant frequency of the circuit, X_C will not be equal to X_L and some amount of current will be drawn from the source. If the applied frequency is lower than the resonant frequency of the circuit, X_L will be smaller than X_C and a lagging source current will result. When the applied frequency is above the resonant frequency, X_C is smaller than X_L and the source current leads the source voltage.

To obtain an overall view of the operation of a parallel LC circuit, a graph can be constructed in which impedance and current are plotted as a function of frequency. To obtain the information for the graph, the capacitive and inductive reactances, impedance, and total current are computed for a group of frequencies centered about the resonant frequency. These computations have been performed for a parallel circuit containing a 2.5 microfarad capacitor and a 3.5 millihenry inductor, and have been tabulated in TABLE 12-2.

f (cps)	X_L (ohms)	X_C (ohms)	Z (ohms)	I_t (amps)
700	15.39	90.95	18.53	3.238
800	17.59	79.58	22.59	2.657
900	19.79	70.74	27.48	2.183
1000	21.99	63.66	33.60	1.786
1100	24.19	57.88	41.56	1.444
1200	26.39	53.05	52.51	1.142
1300	28.59	48.97	68.69	0.874
1400	30.79	45.47	95.34	0.629
1500	32.99	42.44	148.08	0.405
1600	35.19	39.79	314.25	0.197
*1700	37.39	37.45	22221.00	Zero
1800	39.58	35.34	332.18	0.181
1900	41.78	33.51	169.17	0.355
2000	43.98	31.83	115.22	0.521
2100	46.18	30.32	88.23	0.680
2200	48.38	28.94	72.01	0.833
2300	50.58	27.68	61.13	0.982
2400	52.78	26.53	53.33	1.125
2500	54.98	25.47	47.44	1.265
2600	57.18	24.49	42.83	1.401
2700	59.38	23.58	39.11	1.534

*Resonant frequency

'ABLE 12-2 - Reactance, impedance, and cur-ent as a function of frequency.

At 700 cycles for example, the inductive and apacitive reactances are found to be 15.4 ohms nd 90.95 ohms respectively. The impedance of he circuit can be computed by the product over he sum method as follows:

$$Z = \frac{(j\,X_L)(-j\,X_C)}{(j\,X_L) + (-j\,X_C)}$$

$$Z = \frac{(0 + j\,15.4)(0 - j\,91)}{0 - j\,75.6}$$

$$Z = \frac{1400 \angle 0^\circ}{75.6 \angle -90^\circ}$$

$$Z = 18.53 \angle 90^\circ \text{ ohms}$$

f the applied voltage is 60 volts the total current s:

$$I_t = \frac{Ea}{Z}$$

$$I_t = \frac{60 \angle 0^\circ}{18.53 \angle 90^\circ}$$

$$I_t \approx 3.24 \angle -90^\circ \text{ amps}$$

Notice, that, at 700 cps the impedance and current have phase angles of 90 degrees indicating that the circuit appears as a pure reactance. At all frequencies below the resonant frequency of 1700 cps, the ideal circuit appears as a pure inductance and the source current lags the source voltage by exactly -90 degrees.

As the frequency is increased above 700 cps the impedance rises and the source current decreases. At 1700 cps the inductive branch current becomes equal to the capacitive branch current causing the total current to diminish to zero. Since no current is drawn from the source the impedance of the circuit is infinite.

To investigate the operation of the circuit for applied frequencies above the natural resonant frequency of the circuit, the impedance and current will be computed for a frequency of 2100 cps.

$$Z = \frac{(j\,X_L)(-j\,X_C)}{(j\,X_L) + (-j\,X_C)}$$

$$Z = \frac{(0 + j\,46.2)(0 - j\,30.3)}{0 + j\,15.9}$$

$$Z = \frac{1404 \angle 0^\circ}{15.9 \angle 90^\circ}$$

$$Z = 88.23 \angle -90^\circ$$

The current at 2100 cps is:

$$I_t = \frac{E_a}{Z}$$

$$I_t = \frac{60 \angle 0^\circ}{88.23 \angle -90^\circ}$$

$$I_t = 0.68 \angle 90^\circ \text{ amp}$$

The phase angles of the impedance and current at 2100 cps show that the LC circuit appears as a pure capacitive reactance to the source. This condition exists for all applied frequencies which are higher than the resonant frequency, since, for these frequencies I_C is greater than I_L.

At this point certain conclusions can be drawn concerning the characteristics of a parallel LC circuit. These are as follows:

1. For every possible parallel combination of inductance and capacitance, a frequency exists which will make X_C equal to X_L. This frequency is called the natural resonant frequency of the circuit.

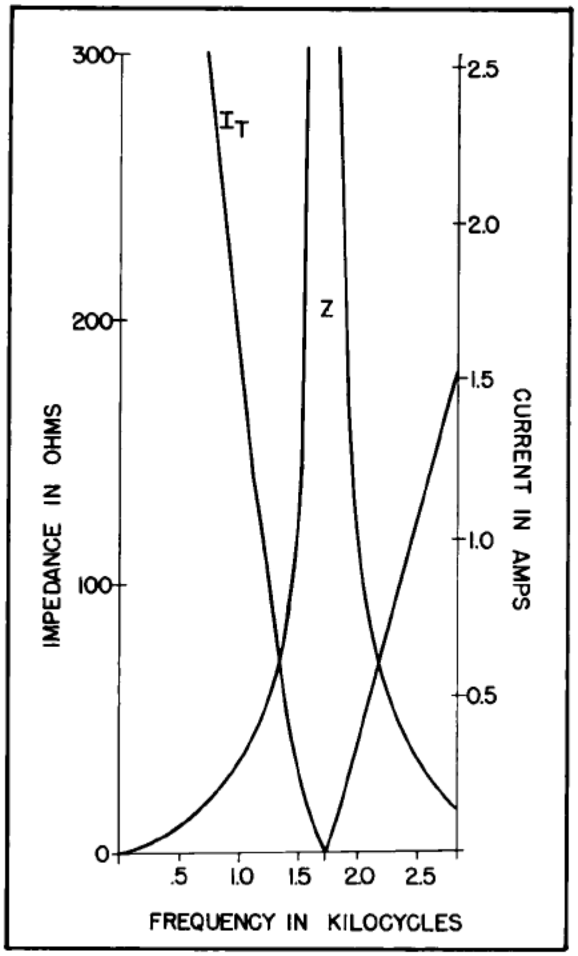

Figure 12-22 - Resonance characteristics.

2. When the circuit is operated at its resonant frequency I_C is equal to I_L.
3. At resonance the source current is minimum. (zero current in the ideal LC circuit)
4. At resonance the impedance of the circuit is maximum (infinite in the ideal circuit).
5. At resonance the circuit appears resistive to the source, the phase angle is zero, and the power factor is unity.
6. When the applied frequency is below the natural resonant frequency of the circuit, I_L is greater than I_C and the circuit appears inductive.
7. When the applied frequency is above the natural resonant frequency of the circuit, I_C is greater than I_L and the circuit appears capacitive.

The graph in Figure 12-22 was obtained by plotting the values of Z and I_t listed in TABLE 12-2. Since these curves show the magnitude of impedance and current as a function of frequency, they are called FREQUENCY RESPONSE CURVES. The usefulness of a response curve lies in the fact that the behavior of the circuit, over a band of frequencies, can be determined quickly and conveniently, by a brief glance at the curve. Notice, that due to the manner in which X_L and X_C vary with frequency, the current and impedance curves are not exactly symmetrical with respect to the resonant frequency.

Q14. Theoretically, how much power would be consumed by an ideal parallel LC circuit operating below its resonant frequency, if the source current and voltage are 50 ma and 100 volts respectively?

Q15. What is the power factor of an ideal parallel LC circuit at a frequency other than its resonant frequency?

Q16. What is the amplitude relationship that exists between line current at the resonant frequency, and the line current above the resonant frequency in an ideal resonant circuit?

12-17. Practical Resonant Circuits

The primary difference that exists between the ideal parallel resonant circuit and the practical parallel resonant circuit is that the practical parallel LC circuit contains resistance. This resistance exists throughout the circuit, however, most of it is located in the inductive branch of the circuit. For purposes of analysis all of the circuit resistance will be represented by a single resistor placed in series with the inductive branch. This resistance will be assumed to account for all of the circuit losses, both ac and dc.

The schematic diagram of a practical parallel LC circuit is shown in Figure 12-23. In this circuit the impedance of the capacitive branch is equal to X_C, while, the impedance of the inductive branch is equal to the vector sum of X_L and R. If the source is adjusted to the frequency at which X_L is equal to X_C the current through the inductive branch will be smaller than the current through the capacitive branch. The total current will lead the applied voltage by a small angle making the circuit appear slightly capacitive to the source.

If the applied frequency is reduced slightly the current through the capacitive branch decreases and the current through the inductive branch increases. The current through the inductive branch will cancel a greater percentage of the capacitive branch current and the total current will attain its minimum value.

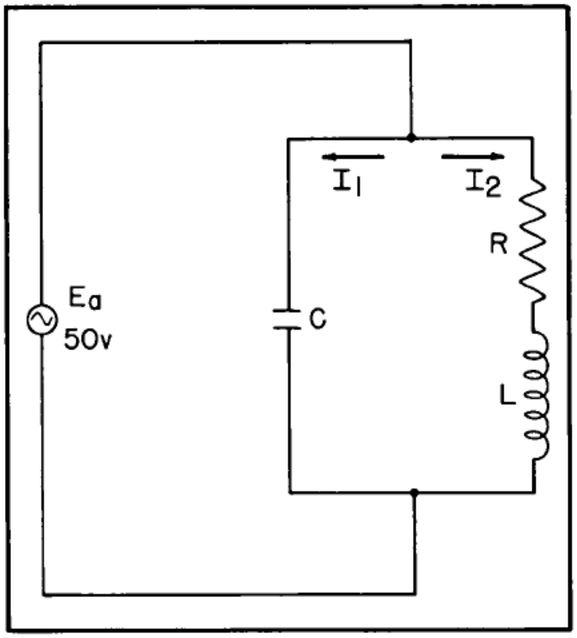

Figure 12-23 - Practical two branch parallel resonant circuit.

By decreasing the applied frequency somewhat further, the inductive component of the RL branch can be made to completely cancel the leading current through the capacitive branch and the circuit will appear purely resistive to the source. At this frequency the circuit phase angle will be zero and the power factor will be unity. Thus, when the coil contains resistance the previously stated conditions for resonance do not all occur at the same applied frequency.

When appreciable resistance exists in the coil, the circuit will have three resonant frequencies depending on how one chooses to define resonance. The resonant frequency can be defined as the frequency at which X_L is equal to X_C, OR the frequency at which the line current is minimum, OR the frequency at which the circuit appears purely resistive.

If the Q of the circuit is high the three resonant frequencies will be very close together and the circuit can be considered to have one resonant frequency. Since most of the tuned circuits encountered are of the high Q type, the remainder of this chapter will be restricted to circuits having a Q of ten or greater. These circuits will be considered to have a single resonant frequency at which X_L is equal to X_C.

Due to the energy losses which occur in a practical parallel LC circuit, some current must be drawn from the source. Because of this current the circuit will have a finite impedance at the resonant frequency. Formulas for computing the impedance are derived as follows:

The total impedance of a two branch circuit can be computed by the product over the sum method. If the capacitive branch is called Z_1 and the inductive branch is called Z_2, then:

$$Z_t = \frac{(-j X_C)(R + j X_L)}{(-j X_C) + (R + j X_L)}$$

rearranging:

$$Z_t = \frac{-j X_C(R + j X_L)}{R + (j X_L - j X_C)}$$

Since in a high Q coil X_L is at least ten times the value of R, the R can be dropped from the numerator. R cannot be dropped from the denominator since near resonance R could be large in comparison to the quantity $(j X_L - j X_C)$. Therefore:

$$Z_t = \frac{X_C X_L}{R + (j X_L - j X_C)}$$

At resonance X_L is equal to X_C and the quantity $(j X_L - j X_C)$ becomes zero. Thus:

$$Z_t = \frac{X_C X_L}{R}$$

Since the values of X_L and X_C are equal at resonance, X_L can be substituted for X_C.

$$Z_t = \frac{X_L}{R} \times \frac{X_L}{1}$$

Since: $$Q = \frac{X_L}{R}$$

then: $$Z_t = Q \times X_L \qquad (12\text{-}11)$$

Equation (12-11) shows that the total impedance of a parallel resonant LC circuit is directly proportional to Q and X_L and inversely proportional to coil resistance.

Another important equation for the impedance of a resonant circuit is derived in the following manner.

$$Z_t = \frac{X_L}{R} \times X_L$$

Since the circuit is resonant X_C can be substituted for X_L.

$$Z_t = \frac{X_L}{R} \times X_C$$

A14. No power will be consumed since the ideal circuit contains no resistance.

A15. Zero. The circuit will be purely reactive.

A16. The current above and below the resonant frequency will always be greater than the current at the resonant frequency.

Substituting for X_L and X_C:

$$Z_t = \frac{\omega L}{R} \times \frac{1}{\omega C}$$

or:

$$Z_t = \frac{L}{RC} \qquad (12\text{-}12)$$

where: Z_t = total impedance in ohms
L = inductance in henrys
C = capacitance in farads
R = resistance in ohms

It must be remembered that equations (12-11) and (12-12) were derived on the assumption that the circuit is operated at resonance. These equations should not be used for non-resonant circuits.

Example. Find the impedance and total current of a resonant circuit consisting of a 3.5 millihenry coil having a resistance of 2.5 ohms, connected in parallel with a 2.5 microfarad capacitor across a source of 50 volts.

Given: $L = 3.5$ mh
$C = 2.5$ uf
$R = 2.5$ ohms
$E_a = 50$ volts

Solution: First find the frequency at which the circuit is operating.

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$f_o = \frac{1}{(6.28)\sqrt{(3.5 \times 10^{-3})(2.5 \times 10^{-6})}}$$

$$f_o = 1.7 \text{ kc}$$

Solving for X_L:

$$X_L = 2\pi f L$$

$$X_L = (6.28)(1.7 \times 10^3)(3.5 \times 10^{-3})$$

$$X_L = 37.4 \text{ ohms}$$

Determine the Q:

$$Q = \frac{X_L}{R}$$

$$Q = \frac{37.4}{2.5}$$

$$Q = 14.95$$

The impedance is found using equation (12-11):

$$Z_t = QX_L \qquad (12\text{-}11)$$

$$Z_t = 14.95 \times 37.4$$

$$Z_t = 560 \text{ ohms}$$

As a check the impedance can be computed using equation (12-12).

$$Z_t = \frac{L}{RC} \qquad (12\text{-}12)$$

$$Z_t = \frac{3.5 \times 10^{-3}}{(2.5)(2.5 \times 10^{-6})}$$

$$Z_t = 560 \text{ ohms}$$

Thus, the two equations for impedance give identical results.

To find the line current, the source voltage is divided by the impedance of the circuit.

$$I_t = \frac{E_a}{Z_t}$$

$$I_t = \frac{50}{560}$$

$$I_t = 89.2 \text{ ma}$$

To show why a practical LC circuit must draw some current at resonance, the circuit in Figure 12-24 will be analyzed. In order to obtain values of current, reactance, and resistance that are easy to work with, the Q of the circuit has been made low.

The circuit consists of a capacitor connected in parallel with a coil having a resistance of 900 ohms. The frequency of the source voltage is such that the reactance of the capacitor is 3000 ohms and the reactance of the coil is 2700 ohms. The impedance (Z_L) of the inductive branch is the vector sum of X_L and R. This impedance is computed as follows:

$$\theta_L = \arctan \frac{jX_L}{R}$$

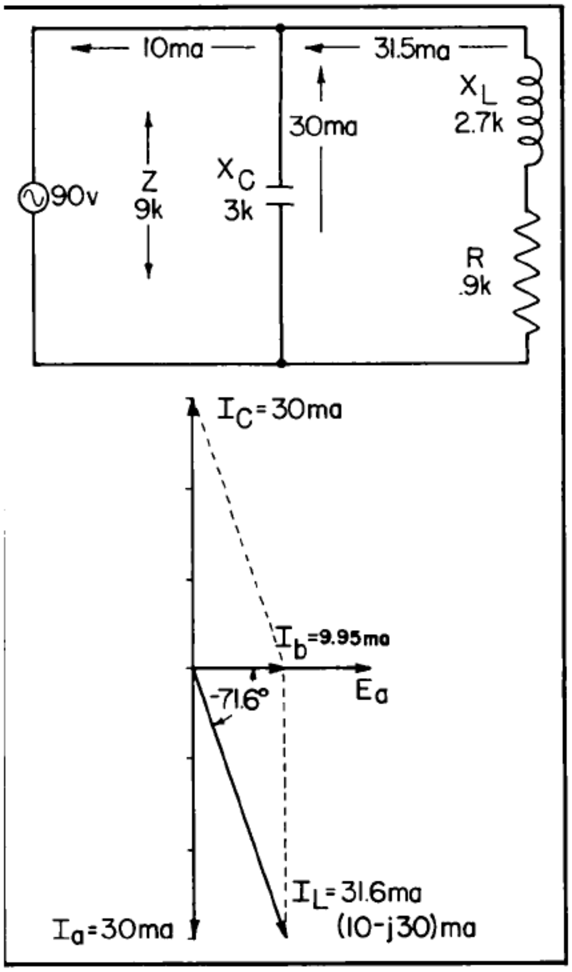

Figure 12-24 - Resonant circuit analysis.

$$\theta_L = \arctan \frac{2700}{900}$$

$$\theta_L = 71.6^\circ$$

$$Z_L = \frac{j X_L}{\sin \theta_L}$$

$$Z_L = \frac{2700}{0.949}$$

$$Z_L = 2845 \angle 71.6^\circ \text{ ohms}$$

The current through the capacitive branch is:

$$I_C = \frac{E_a}{X_C}$$

$$I_C = \frac{90 \angle 0^\circ}{3000 \angle -90^\circ}$$

$$I_C = 30 \angle 90^\circ \text{ ma}$$

The current through the inductive branch is:

$$I_L = \frac{90 \angle 0^\circ}{2845 \angle 71.6^\circ}$$

$$I_L = 31.6 \angle -71.6^\circ \text{ ma}$$

The total or line current (I_t) is equal to the sum of the branch currents I_C and I_L. These currents must be converted to rectangular form before they can be added.

$$I_C = 30 \angle 90^\circ \text{ ma}$$

$$I_C = 0 + j\,30 \text{ ma}$$

and:

$$I_L = 31.6 \angle -71.6^\circ \text{ ma}$$

$$I_L = 31.6 (\cos 71.6^\circ - j \sin 71.6^\circ)$$

$$I_L = 31.6 (0.315 - j0.949)$$

$$I_L = 9.95 - j\,30 \text{ ma}$$

Adding I_C and I_L:

$$\begin{array}{l} I_C = \quad\; 0 + j\,30 \text{ ma} \\ \underline{I_L = 9.95 - j\,30 \text{ ma}} \\ I_t = 9.95 + j\,0 \text{ ma} \end{array}$$

or:

$$I_t \cong 10 \angle 0^\circ \text{ ma}$$

Note, that in adding the two branch currents the reactive components cancel and the total current is purely resistive.

The impedance is found by dividing the source voltage by the total current.

$$Z_t = \frac{E_a}{I_t} = \frac{90 \angle 0^\circ \text{volts}}{9.95 \angle 0^\circ \text{ ma}}$$

$$Z_t = 9045 \quad 0^\circ \text{ ohms}$$

Notice, that the phase angle of the impedance is zero and that the circuit appears purely resistive to the source. Since the circuit has a low Q coil ($Q = 3$) the impedance is not exactly equal to Q times X_L, but is equal to Q times X_C.

The overall operation of the circuit can now be summarized with the aid of the vector diagram in Figure 12-24. In the example presented, the applied frequency was adjusted until the inductive

component of current exactly cancelled the capacitive current. This is shown in Figure 12-24 by resolving the inductive branch current (I_L) into a resistive component (I_b) of 9.95 ma, and an inductive component (I_a) of 30 ma. After cancellation of the inductive component (I_a) and the capacitive component (I_C), which are 180° out of phase, only the resistive component remains. It is this inphase current which the source must supply to make up for the power losses dissipated by the coil resistance.

Q17. Will decreasing the effective resistance of the coil increase or decrease the circuit impedance at resonance?

TANK CIRCUIT ACTION

Parallel LC circuits are called TANK CIRCUITS. A parallel LC circuit operates by a periodic transfer of energy from one component to the other during the first half cycle and reversing the procedure during the second half cycle of operation. In other words a complete cycle of operation has taken place when the energy of one component has been transferred to the other component and then back to the original component. The current that flows WITHIN the tank circuit is called CIRCULATING CURRENT (I_{cir}). The current that flows in the EXTERNAL circuit between the source and the tank circuit is called LINE CURRENT (I_{line}). Since the circulating current is common to both components the inductor and capacitor can be said to act as if they were connected in series as far as tank circuit action is concerned. In order to fully understand the circulating current and the action of energy transfer between components in a tank circuit, a step-by-step analysis will be made of an ideal tank circuit. Figure 12-25A depicts the circuit to be used in the following discussion.

It was previously shown that when a voltage is applied to a coil, during the first instant a maximum CEMF will be produced and virtually no current will flow through the coil. In other words the coil represents an open circuit, or very high impedance. A capacitor, on the other hand, represents a very low impedance circuit during the first instant because maximum current flows. It was also shown that if a capacitive circuit contains no series resistance, the charging time of the capacitor will be almost instantaneous.

Keeping the above in mind, it can be seen that if the switch in Figure 12-25A were closed the following sequence of events would occur.

The instant the switch is closed the coil will develop a CEMF almost equal to the applied voltage and virtually no current will flow through the inductive branch. Since the coil will appear as an open component at the first instant the source will only "see" a capacitor connected across its terminals. The capacitor, having no resistance in series with it, will charge to the source potential almost instantaneously. At the exact instant the capacitor has completed its charge the switch is opened and all circuit action is stopped, merely for the purpose of explanation. The action so far has taken only an instant. In fact, the capacitor has assumed its charge so rapidly that no significant current has started to flow through the inductive branch. Therefore, no field has been established around the coil.

The conditions now existing in the circuit are illustrated in Figure 12-25B. With E_a disconnected the capacitor will now act as the source. The voltage and current conditions are as illustrated in Figure 12-25F. At time zero, voltage across the tank (E_{tank}) is maximum and since the capacitor has not started to discharge the circulating current (I_{cir}) is zero.

The capacitor will now commence to discharge through the coil. The coil will produce a CEMF of a polarity opposite to the capacitor's polarity. This CEMF, the capacitor voltage and hence the circuit voltage (E_{tank}) will decrease toward zero as the capacitor discharges. Since CEMF across the coil (the opposition of the coil to current flow) is decreasing the current flow (I_{cir}) will increase. Figure 12-25C shows the action of the circuit after the capacitor has been discharging for a short time. The capacitor's charge has decreased while the increasing current flow has begun to establish a field around the coil. The voltage and current conditions correspond to time one (t_1) in Figure 12-25F. The above action will continue until the capacitor has discharged completely. Since the circuit is assumed to have no losses, all the energy that was contained in the capacitor has now been transferred to the inductor. Originally this energy was stored in the electrostatic field of the capacitor and it is now stored in the electromagnetic field of the inductor. Since E_{tank} leads the circulating current by 90°, I_{cir} will be maximum when E is minimum as at t_2 in Figure 12-25F. At the instant the capacitor is discharged completely the circulating current will ATTEMPT to cease flowing. Since inductance opposes any change in current, the field of the coil will begin collapsing in order to maintain current flow in the same direction. The inductor is now acting as the source for the circuit and, in accordance with inductive theory, the collapsing field will induce a voltage in the coil with the polarity shown in Figure

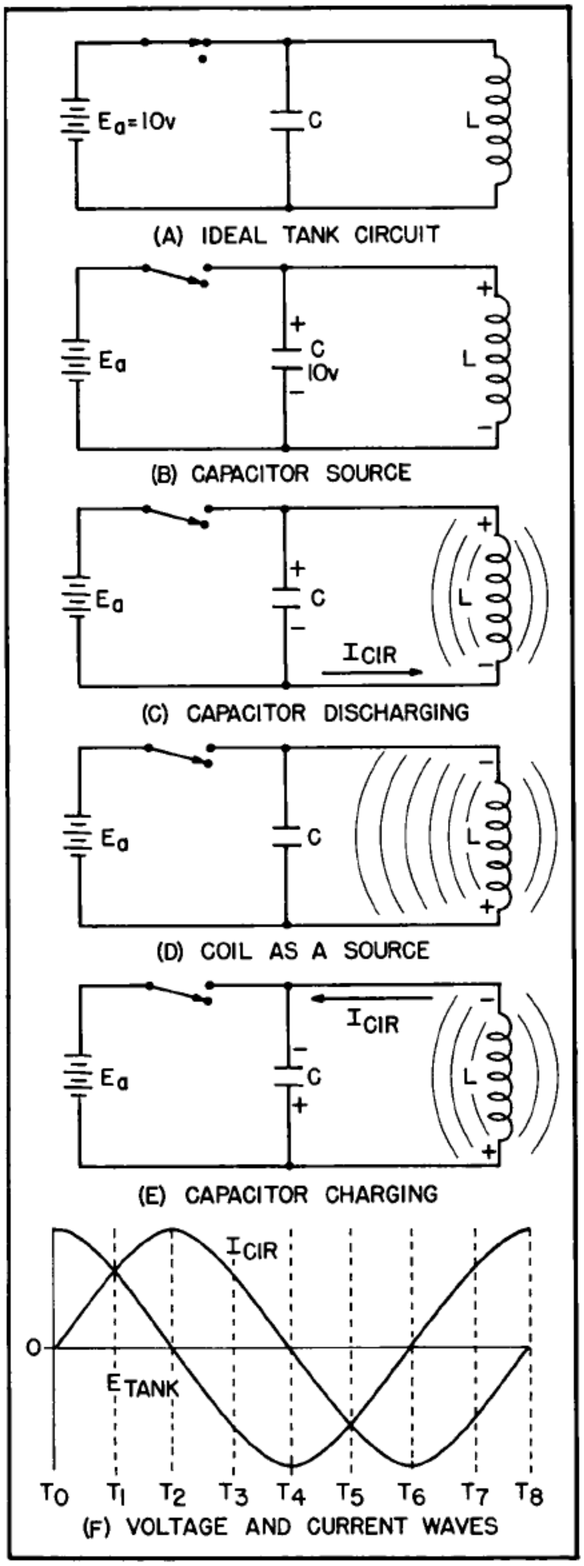

Figure 12-25 - Tank circuit action.

12-25D. It can be seen from the current waveform in Figure 12-25F, that the rate of change of current at t_2 is zero. However, an instant after t_2 the rate of change of current will begin increasing along with induced voltage (E_{tank}). As the inductor field continues to collapse keeping I_{cir} flowing, the capacitor is assuming a charge with a polarity opposite to its original polarity. This is illustrated in Figure 12-25E and the waveform of Figure 12-25F at t_3 where it is seen that the current is still flowing in the positive direction (even though decreasing) while the voltage has changed polarity and is increasing in a negative direction. The above action will continue until t_4 when the inductor field has completely collapsed and the capacitor is fully charged. At this time the circuit has completed a half cycle of operation, the voltage is again maximum (negative) and the current is minimum. The conditions of the circuit are similar to those of Figure 12-25B with the exception that all polarities are reversed.

An instant after t_4 the capacitor will begin to discharge again, only this time, in the direction opposite to the original discharge (the arrow in Figure 12-25C would be reversed). The action of the circuit between t_4 and t_6 of Figure 12-25F will be similar to the action described between t_0 and t_2. In other words, the capacitor will discharge completely and the increasing current will establish a field around the coil. At t_6 the circuit will again appear as in Figure 12-25D (with polarity of coil reversed). The inductive field will again collapse in order to maintain current flow in the same direction (clockwise). At time seven (t_7) the action of the circuit is similar to Figure 12-25E where the inductor field is partially collapsed and the capacitor is partially charged (polarities opposite to those shown). This action continues until time eight (t_8) where the circuit is again equal to Figure 12-25B with the capacitor fully charged to the original potential and the inductive field fully collapsed.

The capacitor has assumed its original potential because this circuit, being ideal, is assumed to have no losses. At time eight the conditions of the circuit are identical to those of time zero. In other words, the circulating current is zero and the tank voltage is maximum.

The tank circuit has gone through a complete cycle of operation. The waveforms of Figure 12-25F are seen to be in the form of a sine wave. Figure 12-26 will be used to briefly summarize the action of the tank circuit.

At t_0 for an instant the switch is closed and the capacitor assumes a charge equal to the source potential. It is from this point (with the capacitor charged, the source disconnected

A17. Increase impedance, due to an increased Q.

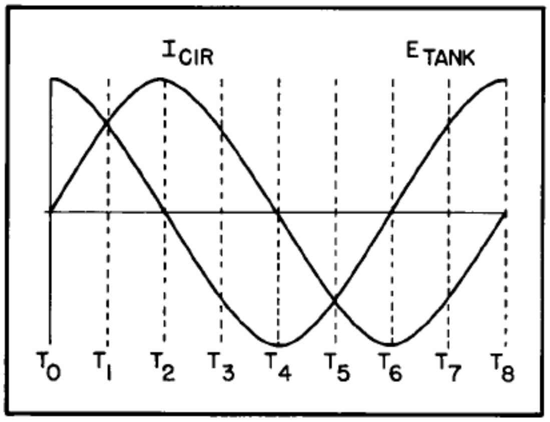

Figure 12-26 - Voltage and current waves.

and no current flowing) that the action of the circuit is assumed to start. For the first quarter of the sine wave (t_0 to t_2) the capacitor acts as the source and the current, due to capacitor discharge establishes a field around the inductor. For the second quarter of the sine wave (t_2 to t_4) the inductor acts as the source and maintains current flow, charging the capacitor in the opposite direction. During the third quarter cycle of the sine wave (t_4 to t_6) the capacitor again becomes the source and the current, due to capacitor discharge, establishes a field around the coil. The direction of circulating current during this second half cycle is oposite to the direction of I_{cir} during the first half cycle. During the final quarter cycle the coil again becomes the source and maintains current flow which results in the capacitor being charged with the original polarity and potential. With no losses being assumed in the circuit the exchange of energy just described would continue indefinitely. The phenomenon of energy continuing to be transferred from one component to the other after the source is removed is called the FLY WHEEL EFFECT. The time it takes the tank circuit to complete a cycle (t_0-t_8) is determined by the value of the capacitor and inductor. The number of complete cycles per second is the natural resonant frequency (f_0) of the tank circuit and is independent of the source frequency.

Due to circuit losses the action of a practical tank circuit is modified somewhat from the ideal. A practical tank circuit containing a lumped resistance is shown in Figure 12-27A.

In the ideal circuit it was assumed that all the energy stored in the capacitor was transferred to the inductor. In the practical circuit energy, in the form of heat, will be dissipated by the resistance of the circuit. The only difference in the action of the ideal tank circuit and the practical tank circuit (Figure 12-27) is that the energy transferred from one reactive component to the other during each cycle of operation will be less by the amount lost to the resistance. Since the amount of energy transferred during each cycle continues to decrease, the charge on the capacitor will continue to decrease. Figure 12-27B graphically shows how the current and voltage of the tank become increasingly less during each cycle until eventually they decrease to zero and the circuit ceases to function. This action is due to DAMPING by the resistance, and the waveforms are called DAMPED WAVEFORMS. Damping will take

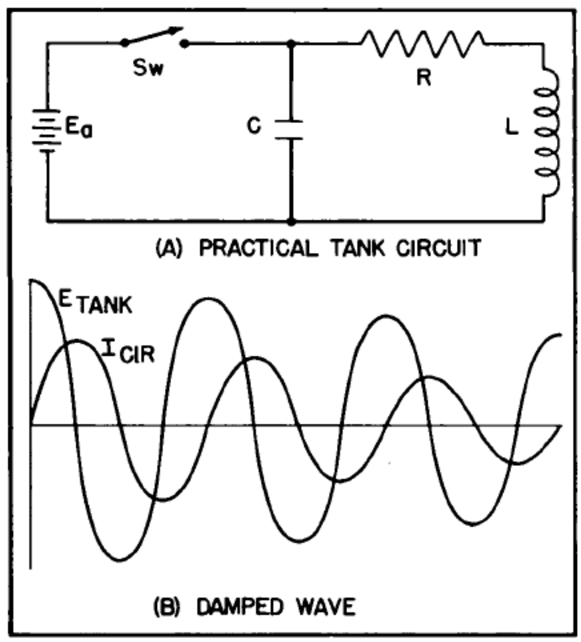

Figure 12-27 - Practical tank circuit and waveforms.

place much more rapidly in a low Q circuit (due to the high resistance) than in a high Q circuit. The damped waveforms result if the switch in Figure 12-27A were only closed once to charge the capacitor and then left open. If an amount of energy equal to the energy dissipated by the resistance of the circuit could be replaced periodically, then the practical tank circuit would be able to operate without damping

out. The dotted portions of the waveforms in Figure 12-28 indicate the damped waveform that would occur if the lost energy were not replaced. Closing the switch for a brief interval at t_1 and t_2 will charge the capacitor to the full source potential at the end of each cycle. Therefore, the circuit will start each new cycle with full source potential. Of course, the capacitor will not require full charging current. The only current drawn from the source will be what is necessary to raise the capacitor potential from the dotted portion (Figure 12-28) of the waveform to the full potential. Since the line current of the circuit consists of this small current drawn from the source, for a brief interval during each cycle, it can be seen that line current will be very small compared to the circulating current. It was stated previously that the waveforms, or OSCILLATIONS will be damped more rapidly in a low Q circuit. This means that the low Q circuit, with its higher inherent resistance, will dissipate more energy during each cycle. If more energy were dissipated each cycle, then more line current would have to be drawn to replace the lost energy. The relationship existing between the line current and the circulating current will, therefore, be dependent on the value of effective resistance present. If the resistance becomes too large (low Q) the fre-

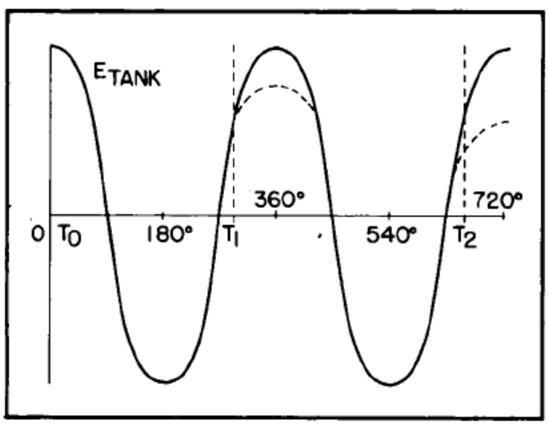

Figure 12-28 - Replacing energy in tank circuit.

quency of the tank may be affected. Since the effective resistance and Q are interdependent, then Q will have an effect on the value of the line current. The relationship is expressed by the equation:

$$I_{line} = \frac{I_{tank}}{Q} \qquad (12\text{-}13)$$

where: I_{line} = the current drawn from the source by the tank circuit.

I_{tank} = the circulating current (I_{cir}) within the tank.

Q = as previously defined.

Example. If a 60 volt source is connected across a tank circuit containing an X_C of 60 ohms and an X_L of 60 ohms, where the effective resistance of the coil is one ohm, what is the line current:

Given: X_L = 60 ohms, X_C = 60 ohms, E_a = 60 volts, R = 1 ohm

Solution: Determine coil Q:

$$Q = \frac{X_L}{R} = \frac{60}{1} = 60$$

Determine tank current:

For all practical purposes at resonance:

$$I_C = I_L = I_{tank}$$

and:

$$I_L = \frac{E_a}{X_L}$$

then: $I_{tank} = \frac{E_a}{X_L} = \frac{60}{60} = 1$ amp

Determine I_{line} current:

$$I_{line} = \frac{I_{tank}}{Q} \qquad (12\text{-}13)$$

$$I_{line} = \frac{1}{60} = 16.66 \text{ ma}$$

If a coil with the same X_L of 60 ohms but an effective resistance of 6 ohms is substituted in the tank circuit, the tank current

$$I_{tank} = \frac{E_a}{X_L}$$

will remain the same, but since the circuit now contains a lower Q coil as determined by

$$Q = \frac{X_L}{R} = \frac{60}{6} = 10$$

The line current will increase as follows:

$$I_{line} = \frac{I_{tank}}{Q} = \frac{1}{10} = 100 \text{ ma}$$

Therefore, it can be seen that the tank circuit draws an increased amount of line current when a lower Q coil is used.

Q18. The frequency of the flywheel effect of a high Q parallel resonant circuit is dependent upon what factors?

Q19. What is the relationship that exists between the tank current above and below the resonant frequency, and the tank current at the resonant frequency?

The terms bandwidth, bandpass, and selectivity may also be applied to the practical two branch parallel circuit because the shape of the resonant curve that describes the impedance variations with respect to frequency has the same general shape as the curve used for series resonant curves. Curves of a high Q and a low Q coil are shown in Figure 12-29. Notice that in the high Q circuit the selectivity is good. The low Q curve shows a wider bandwidth, hence poor selectivity. A comparison pertaining to the merit of each curve cannot be made because each curve has its advantages and disadvantages when applied to a particular circuit. Bandwidth may still be calculated by the equation:

$$BW = \frac{f_o}{Q} \qquad (11\text{-}20)$$

At, above, and below the resonant frequency the practical two branch parallel LC circuit will display various characteristics. At the resonant frequency, the capacitive reactance and the inductive reactance are equal resulting in the cancellation of their respective currents.

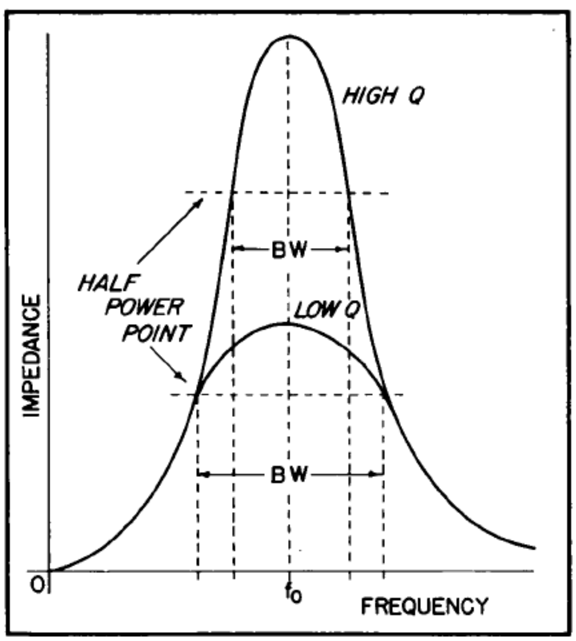

Figure 12-29 - Resonant characteristic.

As was shown, the flow of line current replaced the energy dissipated in the effective resistance. The line current (with the reactive components cancelled) was equal in magnitude to the resistive component of current. Since the source "sees" only a resistive component drawing current, it is said that, at resonance, the tank circuit offers a resistive opposition.

The series resonant circuit also offered a resistive opposition to the source at the resonant frequency. Because the reactance and the associated voltage drops were equal but 180 degrees out of phase, the applied voltage was felt across the circuit resistance. It may then be said that at the resonant frequency, both the series and parallel resonant circuits offer a purely resistive opposition to the source.

When the generator is supplying an increasing frequency above the natural resonance frequency of a parallel LC circuit, the inductive reactance increases and the capacitive reactance decreases. The affected reactances will cause a change in the magnitude of the current flowing through each branch of the parallel network. A greater amount of current will flow in the capacitive branch of the network. The resulting current, after addition of the unequal reactive currents, will lead the applied voltage and the circuit is said to be acting capacitively.

An increase in frequency applied to a series resonant circuit will also cause an increase in the inductive reactance and a decrease in the capacitive reactance. In a series circuit a larger voltage drop is produced across the component which offers the larger impedance. A series resonant circuit operated above the resonant frequency will offer an inductive opposition to the source. Therefore, above the resonant frequency, the parallel circuit will act capacitively and the series circuit will act inductively.

If the frequency applied to a parallel LC circuit decreases, capacitive reactance increases and inductive reactance decreases. In a parallel circuit below the resonant frequency, greater current will flow through the inductive branch and the circuit will appear inductive to the source. In a series circuit below the resonant frequency, greater voltage will be dropped across the capacitor. Thus, when the frequency applied to the series circuit is below the resonant frequency the circuit will offer a capacitive opposition to the source. Therefore, below the resonant frequency, the parallel circuit will act inductive and the series circuit will act capacitive.

12-18. Three Branch Parallel Resonant Circuit

The three-branch parallel resonant circuit

differs from the two branch in the number of possible current paths. The three branch circuit configuration is shown in Figure 12-30.

The combination of L and C in Figure 12-30

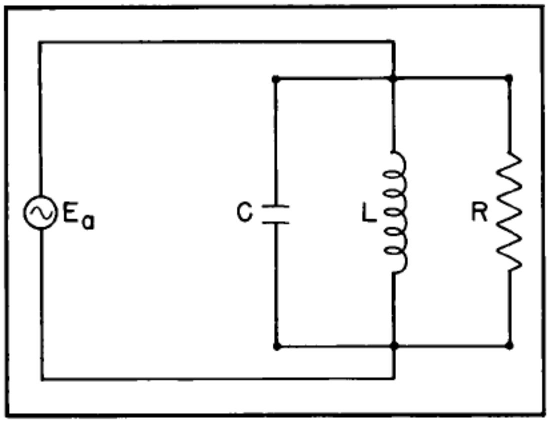

Figure 12-30 - Three branch resonant circuit.

form a parallel resonant circuit. Notice that there is no resistance shown in series with the inductive branch. At resonance,. the vector diagram of the branch currents will be as shown in Figure 12-31.

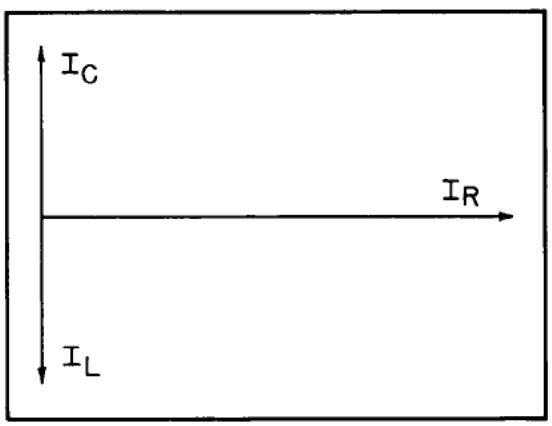

Figure 12-31 - Vector diagram of currents for the three branch parallel RCL circuit.

If the length of the vectors representing the reactive currents are equal, the circuit will appear to be a purely resistive circuit having a resistance value that is equal to R. The circuit shown in Figure 12-32 may be solved for its currents, impedance, circuit Q and other characteristics in the following manner.

Example. E_a = 25 volts, f = 1.2 kc
X_C = 51 ohms, R = 100 ohms
X_L = 51 ohms

Find: I_C = ? I_{tank} = ?
I_L = ? I_{line} = ?
Q = ?

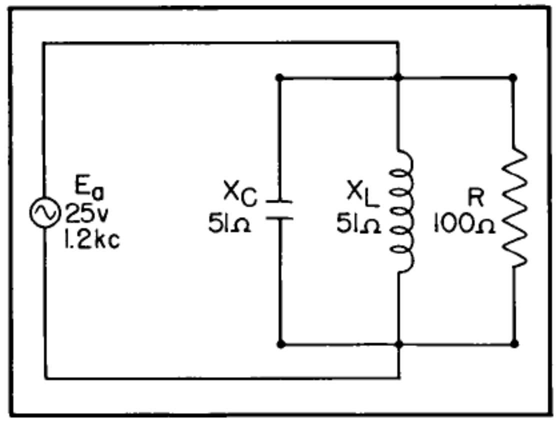

Figure 12-32 - Three branch parallel RCL circuit.

Solution: Determine I_C

$$I_C = \frac{E_a}{X_C}$$

$$I_C = \frac{25}{51}$$

$$I_C = 490 \text{ ma or } 0 + j\,490 \text{ ma}$$

Determine I_L:

$$I_L = \frac{E_a}{X_L}$$

$$I_L = \frac{25}{51}$$

$$I_L = 490 \text{ ma or } 0 - j\,490 \text{ ma}$$

Determine I_R:

$$I_R = \frac{E_a}{R}$$

$$I_R = \frac{25}{100}$$

$$I_R = 250 + j\,0 \text{ ma}$$

To determine the current drawn from the source (line current) insure all currents are in rectangular form and add:

A18. The values of capacity and inductance.

A19. Tank current above and below the resonant frequency will always be less than the current at the resonant frequency.

$$\begin{aligned} I_C &= 0 + j490 \\ I_L &= 0 - j490 \\ I_R &= 250 + j\ \ 0 \\ \hline I_{line} &= 250 + j\ \ 0 \end{aligned}$$

Since the reactive currents cancel, the line current is equal to the current drawn by the resistive component. At resonance, in a three branch circuit:

$$I_{line} = I_R$$

Since the circulating current of the tank is the same for either reactive component, then tank current may be found by determining current flow through the capacitor or inductor.

$$I_{tank} = \frac{E_a}{X_C}$$

$$I_{tank} = \frac{25}{51}$$

$$I_{tank} = 490 \text{ ma}$$

or at resonance:

$$I_{tank} = I_C = I_L$$

The LC combination forms an ideal parallel resonant network. The impedance of the network under these conditions may be considered to be infinite. The equivalent resistance offered to the source by the parallel circuit composed of an infinite impedance in parallel with a known resistance is equal to the value of the known resistance. The circuit impedance can be determined in the following manner.

$$Z = \frac{E_a}{I_{line}}$$

$$Z = \frac{25}{0.25}$$

$$Z = 100 \text{ ohms}$$

Determining the value of Q for a three branch circuit:

$$I_{line} = \frac{I_{tank}}{Q} \qquad (12\text{-}13)$$

transposing:

$$Q = \frac{I_{tank}}{I_{line}}$$

$$Q = \frac{490}{250}$$

$$Q = 1.96$$

Q may be determined by:

$$Q = \frac{I_{tank}}{I_{line}}$$

but:

$$I_{tank} = \frac{E_a}{X_L}$$

and:

$$I_{line} = \frac{E_a}{R}$$

Therefore, by substitution:

$$Q = \frac{\frac{E_a}{X_L}}{\frac{E_a}{R}}$$

To divide, invert denominator and multiply:

$$Q = \frac{E_a}{X_L} \times \frac{R}{E_a}$$

Cancelling E_a:

$$Q = \frac{R}{X_L}$$

$$Q = \frac{100}{51}$$

$$Q = 1.96$$

Notice that this formula for "Q" is just the opposite of the formula used in the series circuit and the two branch practical, LC circuit.

The three branch RCL circuit may have the terms bandwidth, bandpass and selectivity applied to it. The formula for bandwidth undergoes slight modification, i.e., it will not be the same as for the two branch circuit if expressed as a function of RL and C. The shape of the current versus frequency curve is the same as the series and two branch parallel resonant circuits.

Since: $$BW = \frac{f_o}{Q} \qquad (11\text{-}20)$$

and: $Q = \frac{R}{X_L}$ in the three branch circuit

therefore: $$BW = \frac{f_o \times X_L}{R} \qquad (12\text{-}15)$$

If the parallel resistance is increased, the line current goes down. As the shunting resistance approaches infinity, the line current approaches zero. As resistance is increased, the bandwidth becomes narrower, and the selectivity increases. Therefore, it can be seen how regulation of the bandwidth may be accomplished by variation of the shunt, or "swamping" resistor. The inverse relationship between resistance and bandwidth may be seen by examination of equation (12-15) and the curves in Figure 12-33.

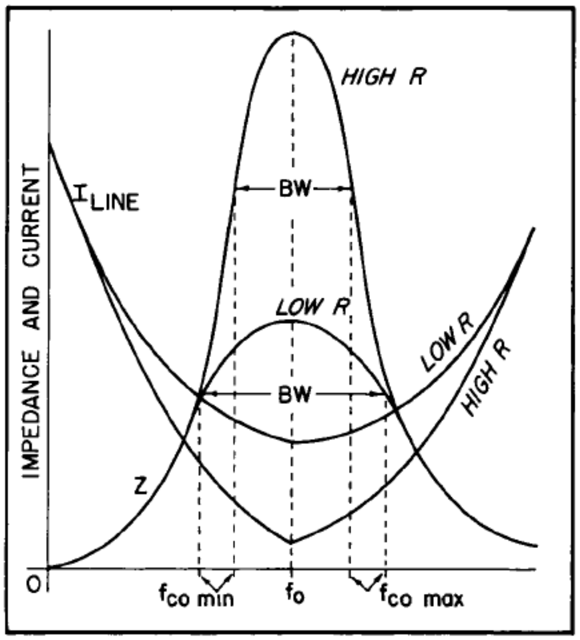

Figure 12-33 - Effect of shunt resistance on BW, I_{line}, and Z.

12-19. Bandpass and Band-Elimination Filters

A parallel resonant circuit may be used as either a bandpass filter or a band-elimination filter depending on where the output terminals are placed. If it is desired to pass only a certain band of frequencies, the output is taken across the tank circuit as in Figure 12-34A. The tank circuit impedance and the resistor R_2 form a voltage divider. Maximum voltage is developed across the tank at resonance and falls off, above and below the resonant frequency. As indicated in the curve of Figure 12-34A, this circuit would only pass a band of frequencies determined by the bandwidth.

If it were desired to pass all frequencies EXCEPT those within the bandwidth of the circuit the output would be taken across the resistor R_2, as shown in Figure 12-34B. Since the line current OFF resonance would be high, the voltage drop across R_2 would be high at all frequencies except the bandwidth around the resonance frequency. At resonance the high impedance of the tank circuit would make I_{line} very low. Therefore, the output waveform shows a considerable decrease around the resonant frequency. This circuit would then eliminate

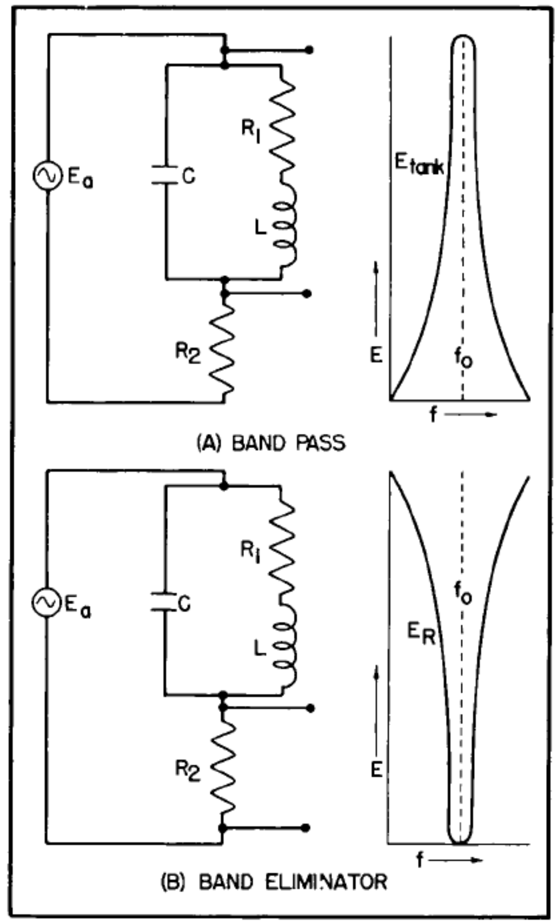

Figure 12-34 - Band filter circuits.

the frequencies within the bandwidth.

It can be seen, from the above, that a band elimination filter might be inserted in the front end of a receiver to "trap" an interferring frequency without affecting all the other desired frequencies. When used in this manner the circuit is called a WAVE TRAP.

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EXERCISE 12

1. A parallel RL circuit consists of a 6.4 mh inductor and a six hundred ohm resistor. The applied voltage is 125V and the frequency is 28 kc. Find the following: I_L, I_R, θ, P.F., and P_t.
2. How is the circuit in problem one acting?
3. If an inductance of 3.6 mh is connected in parallel with the circuit in problem one, find the following: X_{Lt}, I_t, Z_t, and P_t.
4. A parallel RL circuit has an inductance of 3 henrys connected in parallel with a resistor of 12k. The frequency applied is 1.4 kc and the applied voltage is two hundred and fifty volts. Find the following: I_L, I_R, I_t, θ, Z_t, and P_t.
5. Find the frequency at which $X_L = R$ in problem four.
6. Convert $Z = 146 \angle -51^\circ$ ohms to rectangular form.
7. What type of parallel circuit is expressed by the equation in problem six?
8. If the frequency applied is 3.7 kc, find the values of L and R for the circuit in problem six.
9. If the following impedances are connected in parallel: $Z_1 = 28 \angle 0^\circ$ ohms, $Z_2 = 36 \angle +66^\circ$ ohms, find the following: Z_t, coil Q.
10. If there is 60 volts applied to the circuit in problem nine, find the following: I_L, I_R, I_t, θ, P.F. and P_t.
11. A capacitor of 6.2 microfarads is connected in parallel with a resistance of 92 ohms. The frequency applied is 920 cycles and the applied voltage is fifteen volts. Find the following: I_C, I_R, I_t, Z, θ, P_a, P_t, and P.F.
12. Convert $Z = 60 + j47$ to polar form.
13. If the frequency applied is one kilocycle determine the values of R and C used in the circuit in problem twelve.
14. In the circuit shown in Figure 12-35, if the circuit values are: R = 50 ohms, L = 11 mh, C = 3.7 uf, f = 1.3 kc, $E_a = 25$V. Find I_C, I_R, I_L, Z, θ and P_t.
15. In the circuit shown in Figure 12-35, if R = 75 ohms, L = 6.4 mh, C = 10.2 uf, f = 800 cps, $E_a = 30$V: find I_C, I_R, I_L, Z, θ, P.F. and P_t.

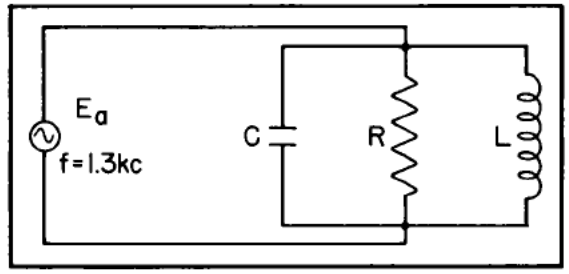


Figure 12-35 - Parallel RCL circuit.

16. An inductor of 3.6 mh and a capacitor of 9.4 uf has what resonant frequency?
17. What is the total impedance of a parallel network composed of the following series impedances: $Z_1 = 0 - j40$, $Z_2 = 20 + j70$? Give answer in polar form.
18. How is the circuit for problem seventeen acting? What is the power factor?
19. An inductance of 90 uh with a Q of 40 is connected in parallel with 275 picofarad capacitor. What is the resonant frequency, the impedance, and the inherent resistance of the inductance?
20. In problem nineteen, find the voltage drop across the coil resistance.
21. A five-hundred picofarad capacitor is connected in parallel with an inductor. The resonant frequency is 360 kc and the impedance is determined to be 65 k ohms. What is the value of the inductance? What is the value of Q?
22. A capacitor of 300 picofarad is connected in parallel across an inductance. Five-hundred volts at the resonant frequency of 1.5 mc is applied. The resulting line current is 9.5 ma. What is the "Q" of the inductance?
23. In problem twenty-two find the bandwidth and bandpass of the circuit.
24. A resistance of 6.5 megohms is connected in parallel with a capacitor of 75 microfarad and an inductor of 60 mh. Find the resonant frequency and the "Q" of the circuit.
25. What is the bandwidth, bandpass, impedance, and total current in the circuit for problem twenty-four?

CHAPTER 13

AC NETWORK ANALYSIS

Many of the circuits employed in electronics equipment are neither simple series networks nor simple parallel networks. These circuits contain a combination of series and parallel components. If the technician is to troubleshoot these complex circuits he must be able to logically analyze what is taking place within the circuit.

The purpose of this chapter is to acquaint the student with some of the common methods used to solve series-parallel circuits. Several practical applications of series-parallel circuits are included to emphasize the importance of this topic.

COMPUTING IMPEDANCE

Whenever a combination circuit is to be analyzed there is usually a possibility of several alternate methods of solution. Undoubtedly, one of these methods is superior to the others from the standpoint of speed and ease of computation. Since the solution of circuit problems is intended to be a teaching aid and not just busywork, the simplest and easiest method of solution should be selected. The first step in any solution should consist of a careful consideration of methods, and the selection of the one most appropriate for the particular problem involved.

In many network problems it is necessary to solve for the impedance of either the circuit as a whole, or for the impedance of a section of the complete network. In some cases it is helpful to group certain portions of a network and treat that portion as a single impedance.

13-1. Parallel Impedances

Although the circuit illustrated in Figure 13-1 contains three components, it can be considered to consist of two parallel impedances. In the diagram the two branches which are to be considered as separate impedances are labeled Z_1 and Z_2. Once a value has been determined for each branch impedance, the total impedance can be computed by one of several appropriate methods (reciprocal method, product over the sum method, assumed source, etc.). The solution of the circuit in Figure 13-1 is accomplished as follows:

The impedance of Z_1 in rectangular form is:

$$Z_1 = R_1 + jX$$

$$Z_1 = 50 + j0 \text{ ohms}$$

The impedance of Z_2 in rectangular form is:

$$Z_2 = R_2 - jX_c$$

$$Z_2 = 16 - j75 \text{ ohms}$$

The phase angle of Z_2 is:

$$\theta = \arctan \frac{-jX_c}{R2}$$

$$\theta = \arctan -\frac{75}{16}$$

$$\theta = -77.9^o$$

The magnitude of Z_2 in polar form is:

$$Z_2 = \frac{X_c}{\sin\theta}$$

$$Z_2 = \frac{75}{0.978}$$

$$Z_2 = 76.6\angle{-77.9^o} \text{ ohms}$$

The total impedance Z_t is now computed by the product over the sum method.

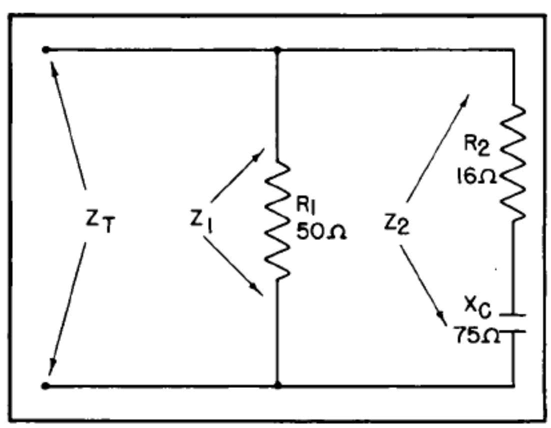


Figure 13-1 - Parallel RC circuit.

$$Z_t = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Expressing the numerator in polar form and the denominator in rectangular form:

$$Z_t = \frac{(50\angle 0^\circ)(76.6\angle -77.9^\circ)}{(50 + j\,0)+(16 - j75)}$$

$$Z_t = \frac{3.83 \times 10^3 \angle -77.9^\circ}{66 - j75}$$

Convert the denominator to polar form.

The angle θ_d of the denominator is:

$$\theta_d = \arctan \frac{-75}{66}$$

$$\theta_d = -48.6^\circ$$

The magnitude M of the denominator is:

$$M = \frac{75}{\sin \theta_d}$$

$$M = \frac{75}{0.75}$$

$$M = 100\angle -48.6^\circ$$

Therefore:

$$Z_t = \frac{3.83 \times 10^3 \angle -77.9^\circ}{100\angle -48.6^\circ}$$

$$Z_t = 38.3\angle -29.3^\circ \text{ ohms}$$

This problem could have been solved by assuming a convenient value of applied voltage. Using an assumed voltage the branch currents are computed and then added to obtain the total current. The impedance is obtained by dividing the assumed voltage by the total current. This method will be illustrated in a later example.

The following example is chosen to illustrate the solution of a parallel circuit in which both branches contain reactive components.

Example. Find the impedance and total current for the circuit shown in Figure 13-2.

As in the example just completed, the two branches of the network will be considered as separate impedances Z_1 and Z_2. The solution will be approached in the same manner as in the last problem.

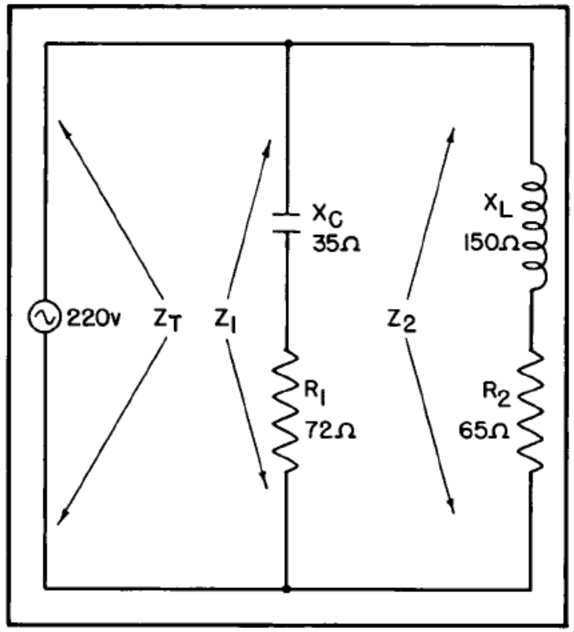


Figure 13-2 - Example parallel circuit.

The impedance of Z_1 in rectangular form is:

$$Z_1 = R_1 - jX_c$$

$$Z_1 = 72 - j35 \text{ ohms}$$

The phase angle θ_1 of Z_1 is:

$$\theta_1 = \arctan \frac{-jX_c}{R_1}$$

$$\theta_1 = \arctan \frac{-35}{72}$$

$$\theta_1 = -25.9^\circ$$

The magnitude of Z_1 in polar form is:

$$Z_1 = \frac{X_c}{\sin \theta_1}$$

$$Z_1 = \frac{35}{0.437}$$

$$Z_1 = 80.2\angle -25.9^\circ \text{ ohms}$$

The impedance of Z_2 in rectangular form is:

$$Z_2 = R_2 + jX_L$$

$$Z_2 = 65 + j\,150 \text{ ohms}$$

The phase angle θ_2 of Z_2 is:

$$\theta_2 = \arctan \frac{jX_L}{R_2}$$

$$\theta_2 = \arctan \frac{150}{65}$$

$$\theta_2 = 66.6^o$$

The magnitude of Z_2 in polar form is:

$$Z_2 = \frac{X_L}{\sin \theta_2}$$

$$Z_2 = \frac{150}{0.918}$$

$$Z_2 = 163.5\underline{/66.6^o} \text{ ohms}$$

Using the values determined for Z_1 and Z_2, the total impedance Z_t can be computed by the product over the sum method.

$$Z_t = \frac{(80.2\underline{/-25.9^o})(163.5\underline{/66.6^o})}{(72 - j\,35) + (65 + j\,150)}$$

$$Z_t = \frac{13.1 \times 10^3 \underline{/40.7^o}}{137 + j\,115}$$

Convert the denominator to polar form:

The angle θ_d of the denominator is:

$$\theta_d = \arctan \frac{115}{137}$$

$$\theta_d = 40^o$$

The magnitude M of the denominator is:

$$M = \frac{115}{\sin \theta_d}$$

$$M = 178.8\underline{/40^o}$$

Therefore:

$$Z_t = \frac{13.1 \times 10^3 \underline{/40.7^o}}{178.8 \underline{/40^o}}$$

$$Z_t = 73.3\underline{/0.7^o} \text{ ohms}$$

To find the total current the applied voltage is divided by the circuit impedance.

$$I_t = \frac{E_a}{Z_t}$$

$$I_t = \frac{220\underline{/0^o}}{73.3\underline{/0.7^o}}$$

$$I_t = 3\underline{/-0.7^o} \text{ amps}$$

Q1. Could the circuit in Figure 13-2 be considered to be operating near its resonant frequency?

Q2. How does this circuit appear to the source (resistive, capacitive, or inductive)?

13-2. Series Impedances

In many instances it may be convenient to treat a complex network as though it consisted of two or more series impedances. A circuit in which the series grouping of components can simplify the solution is illustrated in Figure 13-3.

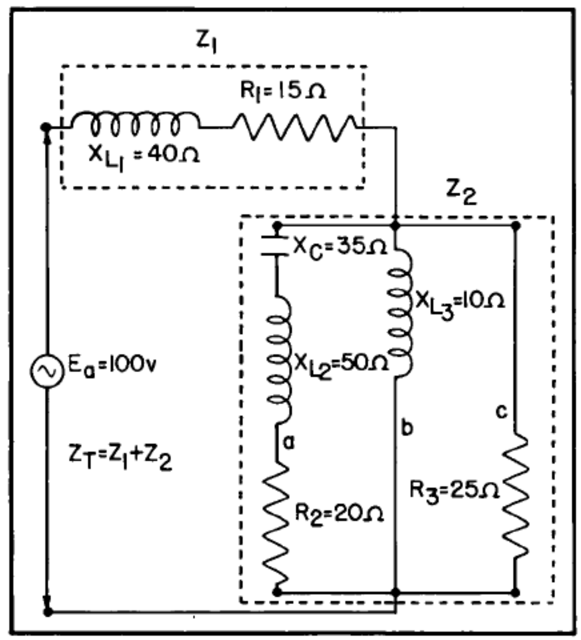


Figure 13-3 - Series impedances.

If R_3 was some type of load device and it was necessary to know the voltage across it, the network could be grouped into two series impedances Z_1 and Z_2 as shown in the diagram. The voltage across R_3 is the same as the voltage across Z_2 and can be readily computed using the voltage divider formula once values have been obtained for Z_1 and Z_2.

A1. Yes, since the circuit phase angle is almost zero degrees.

A2. Almost purely resistive.

Since Z_1 must be in rectangular form for addition to Z_2 there is no need to convert Z_1 to polar form. Therefore Z_1 is:

$$Z_1 = R_1 + jX_{L1}$$

$$Z_1 = 15 + j\,40 \text{ ohms}$$

A simple method for solving a complex parallel network is to assume a voltage across the network and solve for total current. The impedance can then be found by Ohm's law using the assumed voltage and the computed current. This method will be applied to determine the magnitude of Z_2. If the voltage across Z_2 is assumed to be 25 volts the current I_c through branch (c) would be:

$$I_c = \frac{25\underline{/0^\circ}}{25\underline{/0^\circ}}$$

$$I_c = 1\underline{/0^\circ} \text{ amp}$$

In rectangular form:

$$I_c = 1 + j\,0 \text{ amp}$$

The current I_b through branch (b) would be:

$$I_b = \frac{25\underline{/0^\circ}}{10\underline{/90^\circ}}$$

$$I_b = 2.5\underline{/-90^\circ} \text{ amps}$$

In rectangular form:

$$I_b = 0 - j\,2.5 \text{ amps}$$

The impedance of branch (a) must be computed before the current in that branch can be found. The impedance Z_a of branch (a) is:

$$Z_a = R_2 + jX_{L2} - jX_c$$

$$Z_a = 20 + j\,50 - j\,35$$

$$Z_a = 20 + j\,15$$

The phase angle θ_a of Z_a is:

$$\theta_a = \arctan \frac{15}{20}$$

$$\theta_a = 36.9^\circ$$

The magnitude of Z_a in polar form is:

$$Z_a = \frac{15}{\sin 36.9^\circ}$$

$$Z_a = 25\underline{/36.9^\circ} \text{ ohms}$$

The current I_a in branch (a) can now be found and is:

$$I_a = \frac{25\underline{/0^\circ}}{25\underline{/36.9^\circ}}$$

$$I_a = 1\underline{/-36.9^\circ} \text{ amp}$$

Convert I_a to rectangular form:

$$I_a = 1(\cos 36.9^\circ + j \sin - 36.9^\circ)$$

$$I_a = 1(0.800 + j(-0.600))$$

$$I_a = 0.8 - j\,0.6 \text{ amp}$$

The total current is now found by adding the branch currents.

$$\begin{aligned} I_c &= 1 \quad + j\,0 \\ I_b &= 0 \quad - j\,2.5 \\ \underline{I_a} &\underline{= 0.8 - j\,0.6} \\ I_t &= 1.8 - j\,3.1 \text{ amps} \end{aligned}$$

Converting I_t to polar form:

$$\theta = \arctan \frac{-3.1}{1.8}$$

$$\theta = -59.9^\circ$$

$$I_t = \frac{3.1}{\sin 59.9^\circ}$$

$$I_t = 3.58\underline{/-59.9^\circ} \text{ amps}$$

The impedance Z_2 of this part of the circuit is now computed using the assumed voltage and the value of total current I_t obtained above.

$$Z_2 = \frac{25\underline{/0^\circ}}{3.58\underline{/-59.9^\circ}}$$

$$Z_2 = 7\underline{/59.9^\circ} \text{ ohms}$$

Z_2 in rectangular form is:

$$Z_2 = 7(\cos 59.9^\circ + j \sin 59.9^\circ)$$

$$Z_2 = 3.5 + j\,6.0 \text{ ohms}$$

The total impedance is computed by vector addition of Z_1 and Z_2.

$$Z_t = Z_1 + Z_2$$

$$Z_t = (15+j\,40)+(3.5+j\,6.0)$$

$$Z_t = 18.5 + j\,46 \text{ ohms}$$

Z_t in polar form is:

$$\theta = \arctan \frac{46}{18.5}$$

$$\theta = 68.1^\circ$$

$$Z_t = \frac{46}{\sin 68.1^\circ}$$

$$Z_t = 49.5\underline{/68.1^\circ} \text{ ohms}$$

To compute the voltage across R_3 (this is the same voltage that appears across Z_2) the voltage divider formula is applied as follows:

$$E_{R3} = \frac{E_a Z_2}{Z_1+Z_2}$$

Writing the numerator in polar form and the denominator in rectangular form:

$$E_{R3} = \frac{(100\underline{/0^\circ})(7\underline{/59.9^\circ})}{(15 + j\,40)+(3.5 + j\,6.0)}$$

$$E_{R3} = \frac{700\underline{/59.9^\circ}}{18.5 + j\,46}$$

Writing the denominator in polar form:

$$E_{R3} = \frac{700\underline{/59.9^\circ}}{49.5\underline{/68.1^\circ}}$$

$$E_{R3} = 14.14\underline{/-8.2^\circ} \text{ volts}$$

Q3. Why would the impedance of a parallel circuit as computed by the assumed voltage method be the same regardless of the value of the assumed voltage?

13-3. Application of Thevenin's Theorem

In many instances the solution of an ac network can be simplified by developing a Thevenin's equivalent circuit. The Thevenin's equivalent circuit for an ac network is developed using the same steps outlined in section 7-18 for dc networks. The following solution is included to demonstrate the use of a Thevenin's equivalent circuit for ac applications.

Example. Compute the voltage across and the current through R_L in Figure 13-4. To facilitate the solution, the network impedances can be grouped as shown in Figure 13-5. Notice that the circuit is illustrated with the load resistance removed so that E_{th} and Z_{th} can be evaluated. Also notice that the rectangular form of each group (Z_1, Z_2 and Z_3) is shown in the diagram.

Since some of the steps of the solution require Z_1 and Z_2 to be in polar form, these calculations are completed first.

Convert $Z_1 = 70 - j\,30$ to polar form:

$$\theta = \arctan \frac{-30}{70}$$

$$\theta = -23.2^\circ$$

$$Z_1 = \frac{30}{\sin 23.2^\circ}$$

$$Z_1 = 76.2\underline{/-23.2^\circ} \text{ ohms}$$

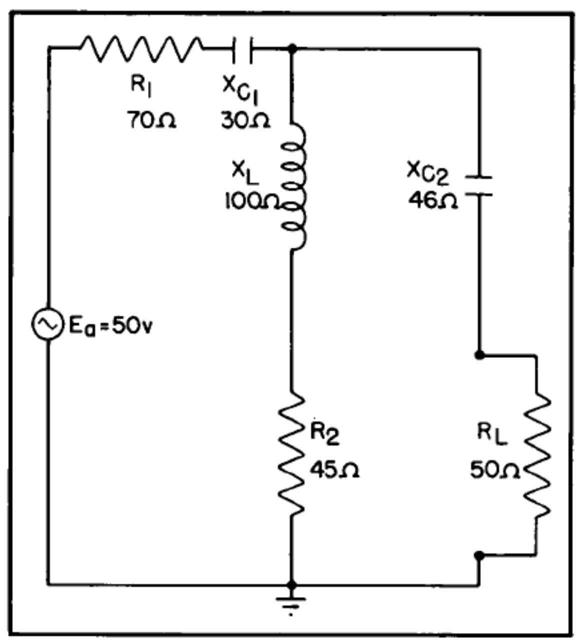


Figure 13-4 - Series-parallel circuit.

A3. An increase of voltage for example would not change the magnitude of capacitance, inductance, or resistance. The impedance of a circuit under fixed frequency conditions is therefore independent of voltage magnitude.

Convert $Z_2 = 45 + j\,100$ to polar form:

$$\theta = \arctan \frac{100}{45}$$

$$\theta = 65.8^\circ$$

$$Z_2 = \frac{100}{\sin 65.8^\circ}$$

$$Z_2 = 110\underline{/65.8^\circ} \text{ ohms}$$

To obtain the value of Thevenin's voltage (E_{th}) the open circuit load voltage must be computed. With the load circuit open, no current

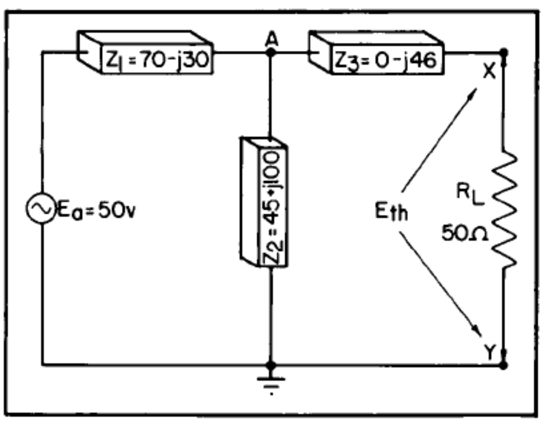


Figure 13-5 - Circuit with impedances grouped

flows through Z_3 and therefore no difference of potential exists across Z_3 (points A to X).

Since points A and X are at the same potential, the voltage between X and Y (E_{th}) is the same as the voltage across Z_2. This voltage can be computed using the voltage divider equation:

$$E_{th} = \frac{E_a Z_2}{Z_1 + Z_2}$$

Substituting: $$E_{th} = \frac{(50\underline{/0^\circ})(110\underline{/65.8^\circ})}{(70 - j\,30) + (45 + j\,100)}$$

$$E_{th} = \frac{5500\underline{/65.8^\circ}}{115 + j\,70}$$

The denominator is now converted to polar form:

$$\theta_d = \arctan \frac{70}{115}$$

$$\theta_d = 31.3^\circ$$

$$Z_d = \frac{70}{\sin 31.3^\circ}$$

$$Z_d = 134.6\underline{/31.3^\circ} \text{ ohms}$$

Thus:

$$E_{th} = \frac{5500\underline{/65.8^\circ}}{134.6\underline{/31.3^\circ}}$$

$$E_{th} = 40.8\underline{/34.5^\circ} \text{ volts}$$

Thevenin's impedance (Z_{th}) is determined by replacing the source with its internal impedance (a short in this case), and calculating the impedance seen looking back into the network from the load terminals X and Y. (See Figure 13-6). Since Z_3 is in series with the parallel combination of Z_1 and Z_2 the equation for determining Thevenin's impedance is:

$$Z_{th} = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Substituting for known quantities:

$$Z_{th} = 0 - j\,46 + \frac{(76.2\underline{/-23.2^\circ})(110\underline{/65.8^\circ})}{(70 - j\,30) + (45 + j\,100)}$$

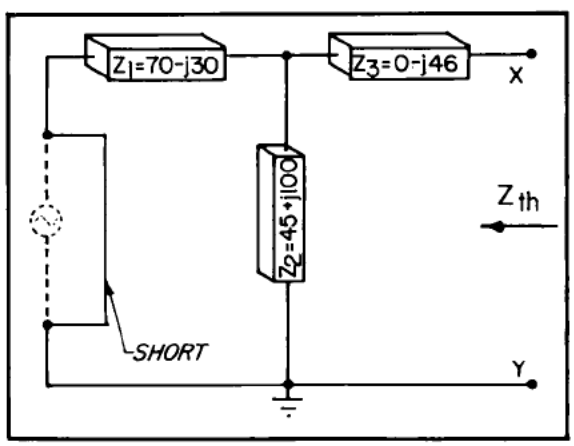


Figure 13-6 - Determination of Thevenin's impedance.

$$Z_{th} = 0 - j\,46 + \frac{8382\underline{/42.6^o}}{115 + j\,70}$$

$$Z_{th} = 0 - j\,46 + \frac{8382\underline{/42.6^o}}{134.6\underline{/31.3^o}}$$

$$Z_{th} = 0 - j\,46 + 62.1\underline{/11.3^o}$$

he last term of the above equation is now con-
erted to rectangular form.

$$62.1\underline{/11.3^o} = 62.1\,(\cos 11.3^o + j \sin 11.3^o)$$

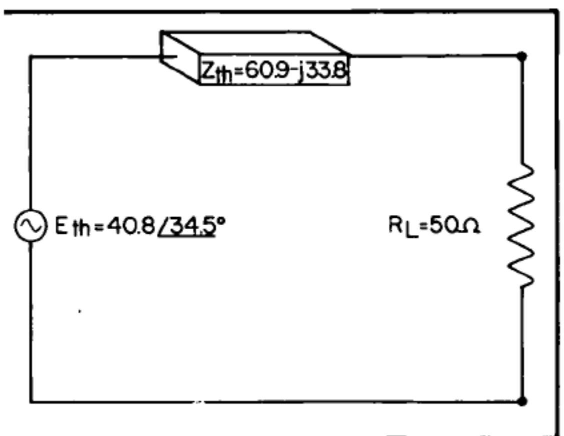
$$62.1\underline{/11.3^o} = 62.1\,(0.981 + j\,0.196)$$

$$62.1\underline{/11.3^o} = 60.9 + j\,12.2$$

herefore: $Z_{th} = 0 - j\,46 + 60.9 + j\,12.2$

$$Z_{th} = 60.9 - j\,33.8 \text{ ohms}$$

ow that values have been obtained for both E_{th} nd Z_{th} the equivalent circuit is set up as shown ı Figure 13-7.



igure 13-7 - Thevenin's equivalent circuit.

he output voltage is computed using the voltage ivider equation:

$$E_L = \frac{E_{th} R_L}{Z_{th} + R_L}$$

$$E_L = \frac{(40.8\underline{/34.5^o})(50\underline{/0^o})}{(60.9 - j\,33.8) + (50 + j\,0)}$$

$$E_L = \frac{2040\underline{/34.5^o}}{110.9 - j\,33.8}$$

Converting the denominator to polar form:

$$\theta_d = \arctan \frac{-33.8}{110.9}$$

$$\theta_d = -17^o$$

$$Z_d = \frac{33.8}{\sin 17^o}$$

$$Z_d = 115.6\underline{/-17^o}$$

Substituting:

$$E_L = \frac{2040\underline{/34.5^o}}{115.6\underline{/-17^o}}$$

$$E_L = 17.6\underline{/51.5^o} \text{ volts}$$

The load current is now found by Ohm's law:

$$I_L = \frac{E_L}{R_L}$$

$$I_L = \frac{17.6\underline{/51.5^o}}{50\underline{/0^o}}$$

$$I_L = 0.352\underline{/51.5^o} \text{ amps}$$

13-4. Solving for Currents and Voltages

To illustrate how the various currents and voltage drops are computed for a combination circuit, the network in Figure 13-8 will be solved. In general, the solution for an ac network is carried out in the same order as the solution of a dc network.

Since values are required for the reactances, these are computed first.

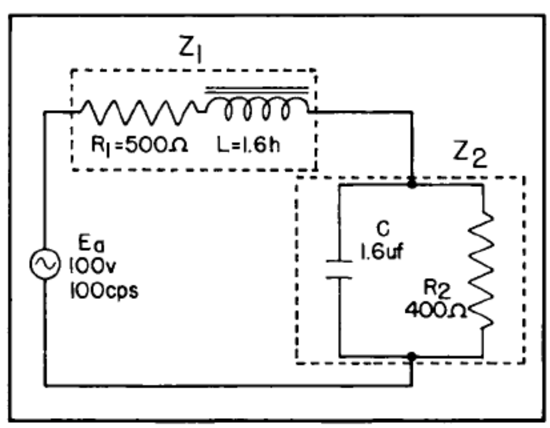


Figure 13-8 - Example circuit.

$$X_L = 2\pi f L$$

$$X_L = 6.28 \times 100 \times 1.6$$

$$X_L = 1000 \text{ ohms}$$

and:

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{(6.28 \times 100)(1.6 \times 10^{-6})}$$

$$X_C = 1000 \text{ ohms}$$

The circuit will be considered to consist of two series impedances Z_1 and Z_2 where:

$$Z_1 = R_1 + jX_L$$

$$Z_1 = 500 + j\,1000$$

and:

$$Z_2 = \frac{R_2(-jX_C)}{R_2 - jX_C}$$

$$Z_2 = \frac{(400\underline{/0^\circ})(1000\underline{/-90^\circ})}{400 - j\,1000}$$

Converting the denominator to polar form:

$$\theta_d = \arctan\frac{-1000}{400}$$

$$\theta_d = -68.2^\circ$$

$$Z_d = \frac{1000}{\sin\theta_d}$$

$$Z_d = 1076.4\underline{/-68.2^\circ}$$

Thus:

$$Z_2 = \frac{(400\underline{/0^\circ})(1000\underline{/-90^\circ})}{1076.4\underline{/-68.2^\circ}}$$

$$Z_2 = 371.6\underline{/-21.8^\circ} \text{ ohms}$$

Convert Z_2 to rectangular form:

$$Z_2 = \mathbb{Z}_2(\cos\theta + j\sin\theta)$$

$$Z_2 = 371.6(0.929 - j\,0.371)$$

$$Z_2 = 345.2 - j\,137.9 \text{ ohms}$$

The total impedance is the sum of Z_1 and Z_2.

$$Z_t = Z_1 + Z_2$$

$$\begin{array}{r} Z_1 = 500 \quad + j\,1000 \\ \underline{Z_2 = 345.2 - j\;\;137.9} \\ 845.2 + j\;\;862.1 \end{array}$$

$$Z_t = 845.2 + j\,862.1 \text{ ohms}$$

Convert Z_t to polar form:

$$\theta = \arctan\frac{862.1}{845.2}$$

$$\theta = 45.6^\circ$$

$$Z_t = \frac{862.1}{\sin\theta}$$

$$Z_t = 1205.7\underline{/45.6^\circ} \text{ ohms}$$

Using the applied voltage and total impedance compute the total current.

$$I_t = \frac{E_a}{Z_t}$$

$$I_t = \frac{100\underline{/0^\circ}}{1205.7\underline{/45.6^\circ}}$$

$$I_t = 82.9\underline{/-45.6^\circ} \text{ ma}$$

The total current I_t flows through resistor R_1 and inductor L. Computing the voltage drops across R_1 and L:

$$E_{R1} = I_t R_1$$

$$E_{R1} = (82.9 \times 10^{-3}\underline{/-45.6^\circ})(500\underline{/0^\circ})$$

$$E_{R1} = 41.5\underline{/-45.6^\circ} \text{ volts}$$

and:

$$E_L = I_t X_L$$

$$E_L = (82.9 \times 10^{-3}\underline{/-45.6^\circ})(1000\underline{/90^\circ})$$

$$E_L = 82.9\underline{/44.4^\circ} \text{ volts}$$

Since the total current through Z_2 and the impedance of Z_2 are known, the voltage E_2 across Z_2 is:

$$E_2 = I_t Z_2$$

$$E_2 = (82.9 \times 10^{-3}\underline{/-45.6^\circ})(371.6\underline{/-21.8^\circ})$$

$$E_2 = 30.8\underline{/-67.4^\circ} \text{ volts}$$

Computing the currents through C and R_2:

$$I_C = \frac{E_2}{X_C}$$

$$I_C = \frac{30.8\angle -67.4^\circ}{1000\angle -90^\circ}$$

$$I_C = 30.8\angle 22.6^\circ \text{ ma}$$

and:

$$I_{R2} = \frac{E_2}{R_2}$$

$$I_{R2} = \frac{30.8\angle -67.4^\circ}{400\angle 0^\circ}$$

$$I_{R2} = 77\angle -67.4^\circ \text{ ma}$$

The computed data is now summarized:

$X_L = 1000\angle 90^\circ$ ohms	$X_C = 1000\angle -90^\circ$ ohms
$E_L = 82.9\angle 44.4^\circ$ volts	$E_C = 30.8\angle -67.4^\circ$ volts
$I_L = 82.9\angle -45.6^\circ$ ma	$I_C = 30.8\angle 22.6^\circ$ ma
$E_{R1} = 41.5\angle -45.6^\circ$ volts	$E_{R2} = 30.8\angle -67.4^\circ$ volts
$I_{R1} = 82.9\angle -45.6^\circ$ ma	$I_{R2} = 77.0\angle -67.4^\circ$ ma
$Z_t = 1205.7\angle 45.6^\circ$ ohms	$I_t = 82.9\angle -45.6^\circ$ ma

The following questions pertain to the circuit in Figure 13-8 and the summarized data above.

Q4. How many degrees of phase difference are there between I_C and I_{R2}? Is this the same number of degrees that exist between I_C and I_R in a simple two branch parallel RC circuit?

Q5. Does a parallel RC circuit act the same when placed in series with other resistances and reactances as it does when used alone?

Q6. What would happen to E_{R2} if the applied frequency were increased?

PHASE SHIFT NETWORKS

An important aspect of a reactive network is its ability to SHIFT THE PHASE of a signal voltage or current. Whether or not this phase shift is desirable depends on the function of the circuit. Some phase shift networks are simple devices while others are complex. However, regardless of the function and construction of the network, the basic principle of operation is simple and easy to understand.

13-5. Simple Phase Shift Network

A practical phase shift network consists of resistance and reactance, so arranged as to produce a required amount of phase shift. A signal is applied to the input terminals of the network and a phase shifted signal appears at the output terminals, usually with somewhat reduced amplitude. The result of phase shift action is illustrated in Figure 13-9. As shown a sine wave having a peak amplitude of 20 volts is applied to the input terminals of the network. The voltage at the output of the network is also a sine wave, however, the output sine wave lags the input sine wave by 30°. Due to losses with-

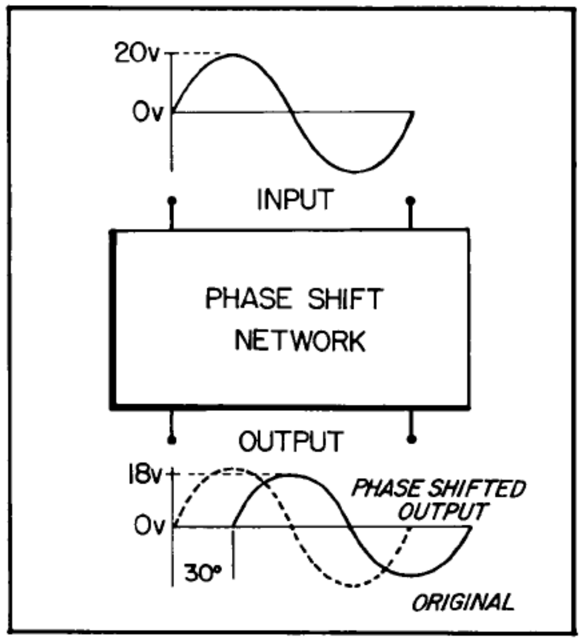


Figure 13-9 - Phase shift network.

in the network, the phase shifted output signal has a peak amplitude of only 18 volts.

The operating principle of a phase shift network can be analyzed using the circuit shown in Figure 13-10. Since the circuit contains capacitive reactance and resistance, the circuit current leads the applied voltage by some number of degrees between zero and ninety. The output voltage across the resistor is in phase with this leading current and therefore leads the input voltage. The number of degrees by which the output voltage leads the input voltage is

A4. The phase difference is 90° since 22.6° minus $(-67.4^\circ) = 90^\circ$.
Yes.

A5. Yes. The basic characteristics of a parallel RC circuit remain the same regardless of how the circuit is used.

A6. E_{R2} would decrease. The reactance of C would decrease causing less voltage to be dropped across the parallel combination of C and R_2.

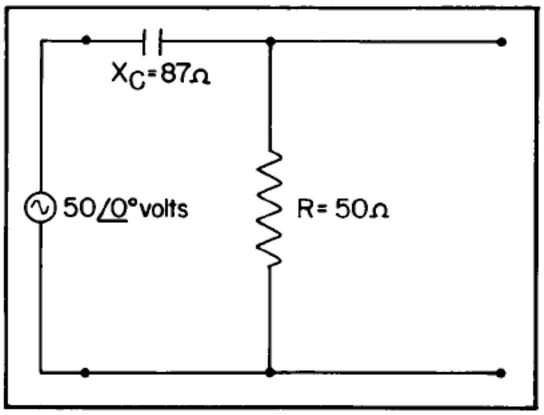


Figure 13-10 - RC phase shift network.

determined by the relative magnitudes of resisance and capacitance and the frequency of the applied voltage.

If the values of reactance, resistance, and applied voltage are known, the amount of phase shift and the magnitude of the output voltage can be readily computed using the voltage divider formula. Using the values from Figure 13-10 the solution is as follows:

$$E_R = \frac{E_a R}{R - jX_C}$$

$$E_R = \frac{(50\angle 0^\circ)(50\angle 0^\circ)}{50 - j\,87}$$

$$E_R = \frac{2500\angle 0^\circ}{50 - j\,87}$$

Convert the denominator to polar form:

$$\theta_d = \arctan \frac{-87}{50}$$

$$\theta_d = -60.1^\circ$$

$$Z_d = \frac{87}{\sin \theta_d}$$

$$Z_d = 100\angle -60.1^\circ$$

thus:

$$E_R = \frac{2500\angle 0^\circ}{100\angle -60.1^\circ}$$

$$E_R = 25\angle 60.1^\circ \text{ volts}$$

Notice that the output voltage leads the input voltage by 60.1° and that it has been reduced in amplitude to 25 volts.

If the resistance in Figure 13-10 is reduced the phase shift will be greater and the output voltage will be smaller than in the example. With a two element network the maximum phase shift that can be obtained is approximately 90°. However, as the phase shift approaches 90°, the output voltage approaches zero.

Q7. Why is the output voltage of a phase shift network, like the one shown in Figure 13-10, always less than the input voltage?

Q8. Describe the results that would be obtained if the output voltage were taken across the capacitor instead of the resistor in Figure 13-10.

Q9. What would happen to the phase shift and output voltage in Figure 13-10 if the frequency of the input voltage were increased?

13-6. Multi-section Phase Shift Network

When a phase shift of 90° or more is required, additional sections can be added to the network. A phase shift network consisting of two sections is shown in Figure 13-11. Each of the two sections is identical to the network of Figure 13-10.

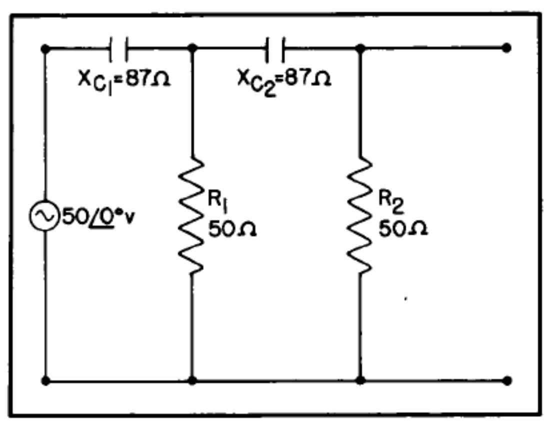


Figure 13-11 - Two section phase shift network.

The phase shift and output voltage will be calculated and then compared to the single section network to show the effects of adding additional sections. The output voltage will be obtained through the use of a Thevenin's equivalent circuit.

Since the output voltage of the network is the voltage across R_2, this resistor will be considered as the load resistor. Figure 13-12 shows the network with the load resistor open so that Thevenin's voltage (E_{th}) can be computed. With the load resistor open no current flows through X_{C2}, therefore no voltage drop occurs across X_{C2}. E_{th} is thus equal to the voltage drop across R_1. This voltage is the same as the output voltage of the single section phase shift network in Figure 13-10, hence:

$$E_{th} = 25\underline{/60.1^\circ} \text{ volts}$$

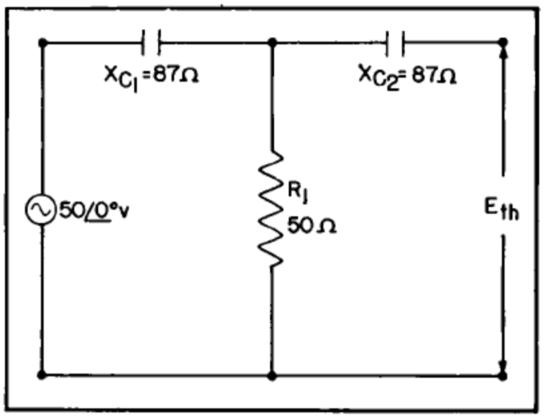


Figure 13-12 - Finding Z_{th}.

The source is now replaced with its internal impedance (a short) and the impedance (Z_{th}) looking back into the network from the load terminals is computed. The circuit now appears as shown in Figure 13-13. Z_{th} is composed of X_{C2} in series with the equivalent parallel impedance of X_{C1} and R_1. Z_{th} is therefore:

$$Z_{th} = -jX_{C2} + \frac{(R_1)(-jX_{C1})}{(R_1) + (-jX_{C1})}$$

$$Z_{th} = -j\,87 + \frac{(50\underline{/0^\circ})(87\underline{/-90^\circ})}{(50) + (-j87)}$$

The denominator is converted to polar form. (This computation was done in section 13-5.) Therefore:

$$Z_{th} = -j\,87 + \frac{(50\underline{/0^\circ})(87\underline{/-90^\circ})}{100\,\underline{/-60.1^\circ}}$$

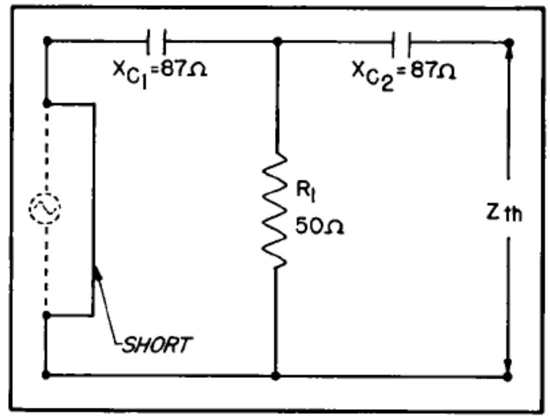


Figure 13-13 - Finding Z_{th}

$$Z_{th} = -j\,87 + 43.5\underline{/-29.9^\circ}$$

Converting the second term to rectangular form:

$$43.5\underline{/-29.9^\circ} = 43.5(\cos 29.9^\circ + j \sin -29.9^\circ)$$

$$43.5\underline{/-29.9^\circ} = 43.5(0.867 - j\,0.499)$$

$$43.5\underline{/-29.9^\circ} = 37.7 - j\,21.7$$

Therefore:

$$Z_{th} = -j87 + 37.7 - j\,21.7$$

$$Z_{th} = 37.7 - j\,108.7 \text{ ohms}$$

The Thevenin's equivalent circuit including values can now be constructed. The equivalent circuit is shown in Figure 13-14.

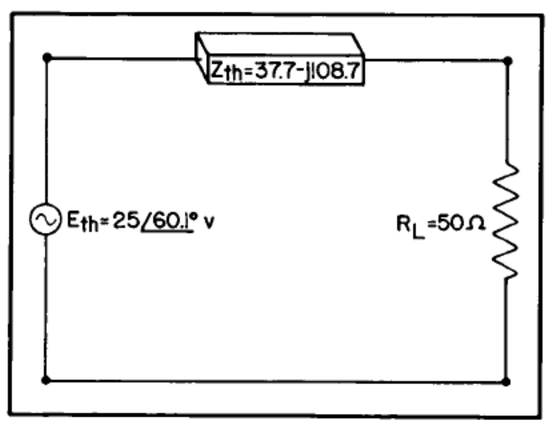


Figure 13-14 - Thevenin's equivalent.

A7. Some of the input voltage is dropped across the capacitor thus reducing the output voltage.

A8. The output voltage would lag the applied voltage by 29.9^{o} and would have a magnitude of 43.35 volts.

A9. The phase shift would decrease and the output voltage would increase.

Next, the output voltage is found using the voltage divider equation.

$$E_{RL} = \frac{E_{th}R_L}{Z_{th}+R_L}$$

$$E_{RL} = \frac{(25\angle 60.1^{o})(50\angle 0^{o})}{37.7 - j\,108.7 + 50}$$

$$E_{RL} = \frac{1250\angle 60.1^{o}}{87.7 - j\,108.7}$$

Convert the denominator to polar form:

$$\theta_d = \arctan \frac{-108.7}{87.7}$$

$$\theta_d = -51.1^{o}$$

$$Z_d = \frac{108.7}{\sin 51.1^{o}}$$

$$Z_d = 139.7\angle -51.1^{o}$$

Thus:

$$E_{RL} = \frac{1250\angle 60.1^{o}}{139.7\angle -51.1^{o}}$$

$$E_{RL} = 8.9\angle 111.2^{o} \text{ volts}$$

The total phase shift of the two sections is thus seen to be 111.2^{o} and the output voltage is approximately 9 volts. If additional phase shift is desired more sections can be added to the network. However, it must be remembered that each section added causes a further reduction in output voltage.

The following example illustrates a special case in which certain assumptions can be made. The circuit shown in Figure 13-15 is designed to produce a phase shift of 90^{o}. The value of the two capacitors and the input frequency have been chosen so that the reactance of the capacitors are equal to each other and also equal to the value of the resistors.

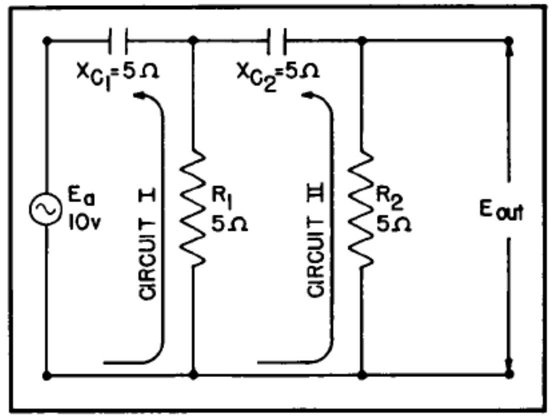


Figure 13-15 - Network for 90^{o} phase shift.

To simplify the analysis of this circuit, R_1 and C_1 are considered to comprise a series circuit, and R_2 and C_2 comprise a second series circuit. To find the phase shift caused by each circuit it is necessary to find the impedance of each circuit.

Since $R_1 = R_2$ and $X_{C1} = X_{C2}$, the impedance of the two circuits is the same.

$$Z_1 = Z_2 = 5 - j\,5 \text{ ohms}$$

$$\theta = \arctan \frac{-5}{5}$$

$$\theta = -45^{o}$$

$$Z_1 = \frac{5}{\sin 45^{o}}$$

and:

$$Z_1 = 7.07\angle -45^{o} \text{ ohms}$$

$$Z_2 = 7.07\angle -45^{o} \text{ ohms}$$

Using the voltage divider equation the voltage across R_1 is:

$$E_{R1} = \frac{E_a R_1}{R_1 - (jX_{C_1})}$$

$$E_{R1} = \frac{(10\angle 0^{o})(5\angle 0^{o})}{7.07\angle -45^{o}}$$

$$E_{R1} = \frac{50\angle 0^{o}}{7.07\angle -45^{o}}$$

$$E_{R1} = 7.07\angle 45^{o} \text{ volts}$$

Assuming the voltage across R_1, as computed above, to be the input voltage to the second section the voltage across R_2 is:

$$E_{R2} = \frac{E_{R1}R_2}{R_2 - jX_{C2}}$$

$$E_{R2} = \frac{(7.07\angle 45^{\circ})(5\angle 0^{\circ})}{7.07\angle -45^{\circ}}$$

$$E_{R2} = 5\angle 90^{\circ} \text{ volts}$$

This answer shows that the output voltage is leading the input voltage by ninety degrees. The answer obtained for the output voltage is correct insofar as the phase shift is concerned, but is only an approximation of the magnitude of output voltage (actual voltage is 3.3V). The discrepancy arises from the shunting effect of C_2 and R_2 on R_1, which lowers the input voltage to the second section of the network. This simplified method of finding the total phase shift can only be used when the impedance of all the components in the network are the same.

RC phase shift circuits will be encountered again in later chapters where they will be used in conjunction with vacuum tubes and transistors.

Q10. Would it be possible to construct a phase shift network using inductors and resistors?

A10. Yes. Either capacitive reactance or inductive reactance can be used to produce a phase shift.

EXERCISE 13

1. Compute the magnitude and angle of the current in a circuit having an applied voltage of $60\angle -30^\circ$ volts and an impedance of 30+j 40 ohms.

2. Find the applied voltage for the circuit in Figure 13-16.

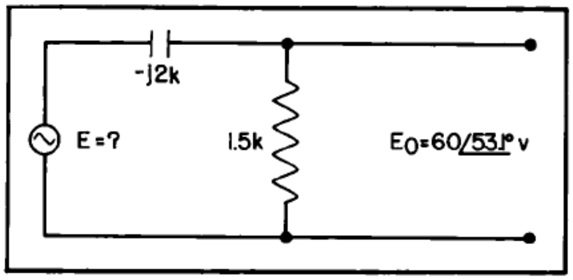


Figure 13-16

3. Solve for the total current, total impedance, branch current, and individual voltage drops in the circuit shown in Figure 13-17.

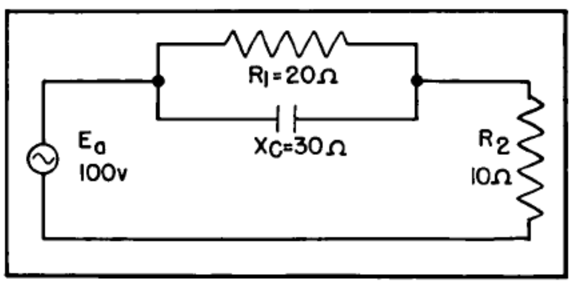


Figure 13-17

4. Draw the Thevenin's equivalent circuit for the network shown in Figure 13-18, including values for E_{th} and Z_{th}.

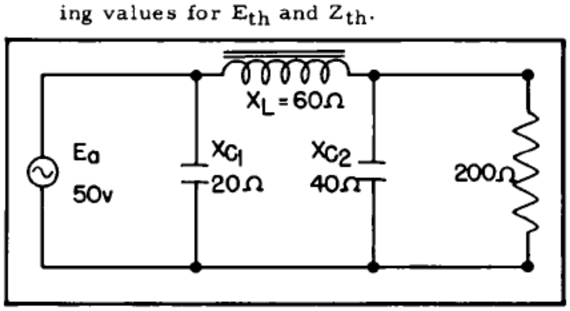


Figure 13-18

5. Compute the output voltage E_o for the network shown in Figure 13-19.

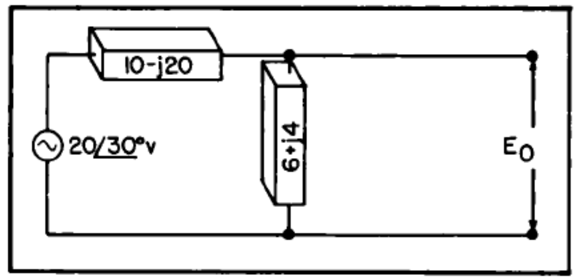


Figure 13-19

6. Assuming no loading effect, what value of voltage will the meter in Figure 13-20 indicate?

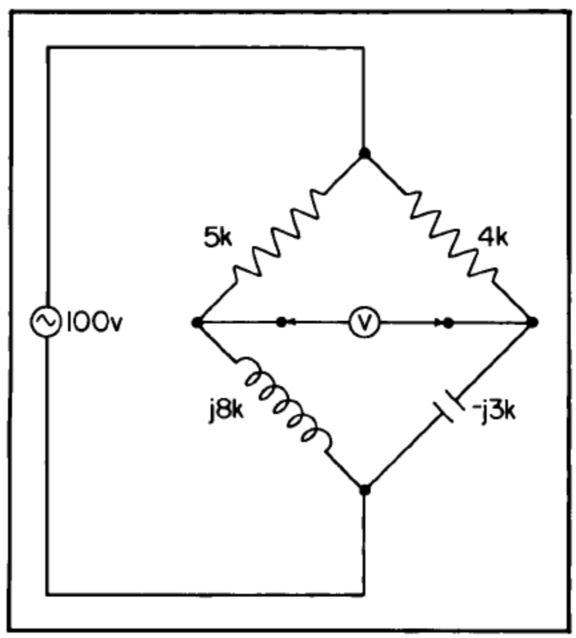


Figure 13-20

7. Solve for the total current, total impedance, individual currents, and individual voltage drops for the circuit in Figure 13-21.

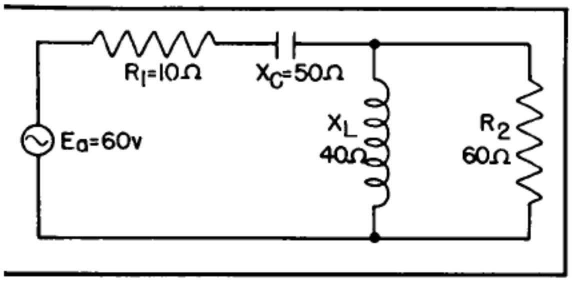


Figure 13-21

8. Construct a simple series circuit that would have an impedance of 20-j 40 ohms.

9. Construct a simple parallel circuit that would have an impedance of 20-j 40 ohms.

10. Describe what would happen to the current in a series circuit having an impedance of 75+j 25 ohms if the applied frequency were increased?

11. What value of voltage would a meter having an internal impedance of 25+j 15 ohms indicate, when connected to the output terminals of the circuit in Figure 13-22?

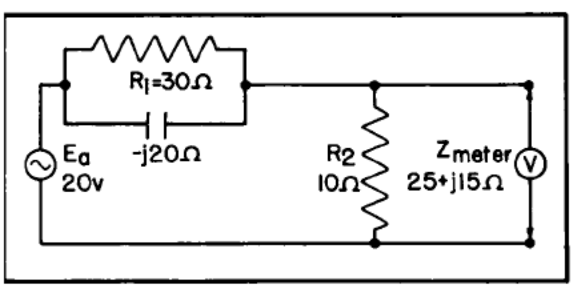


Figure 13-22

12. Two generators each having an internal impedance of $8.6\underline{/54.5^{\circ}}$ ohms and a generated EMF of 20 volts connected series aiding and placed across a 20 ohm resistor. How much current is drawn from the generators? What is the terminal voltage of the two generators under load?

13. Find the output voltage for the circuit in Figure 13-23. Assume L_1 and L_2 to be connected series opposing with a co-efficient of coupling of 0.5.

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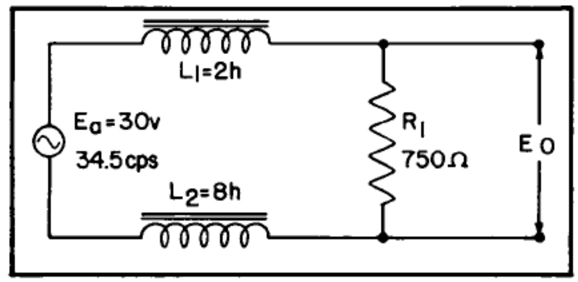

Figure 13-23

14. Find the resonant frequency for the circuit in Figure 13-24, then assuming the circuit to be operating at its resonant frequency find the Q of the circuit, the impedance of the tank circuit, the total impedance, and the line current.

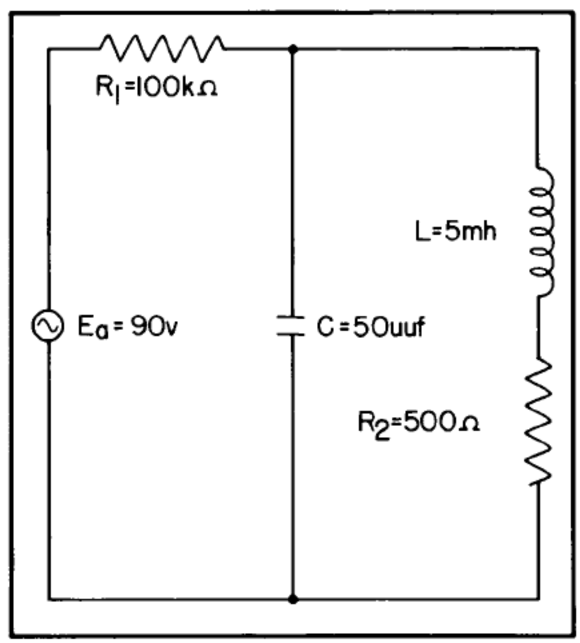

Figure 13-24

15. Describe the indication on the RF ammeter in Figure 13-25 as C is varied from its maximum value to its minimum value.

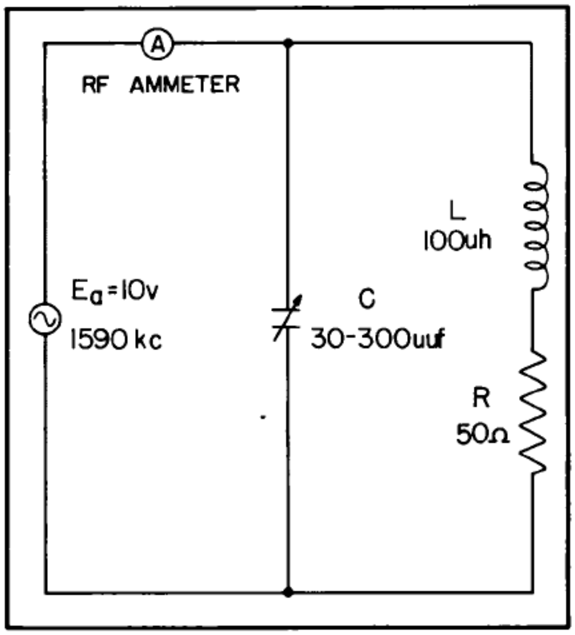

Figure 13-25

16. Compute the phase shift produced by the circuit in Figure 13-26.

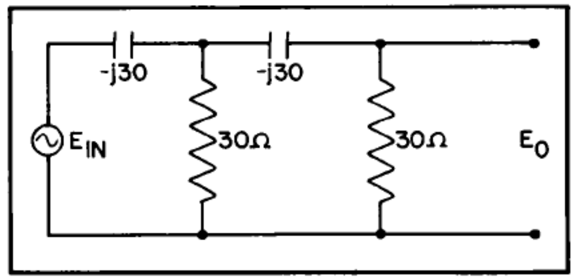

Figure 13-26

CHAPTER 14

MEASURING DEVICES

In the field of electricity, as in all the other physical sciences, accurate quantitative measurements are essential. This involves two important items—numbers and units. Simple arithmetic is used in most cases, and the units are well-defined and easily understood. The standard units of current, voltage, and resistance as well as other units are defined by the National Bureau of Standards. At the factory various instruments are calibrated by comparing them with established standards.

The technician commonly works with ammeters, voltmeters, and ohmmeters; but he may also have many occasions to use wattmeters, oscilloscopes, and various other types of measuring devices.

Electrical equipments are designed to operate at certain efficiency levels. To aid the technician in maintaining the equipment, technical instruction books and sheets containing optimum performance data, such as voltages and resistances, are prepared for each Navy equipment.

To the technician, a good understanding of the functional design and operation of electrical instruments is important. In electrical service work, one or more of the following methods are commonly used to determine if the circuits of an equipment are operating properly.

1. Use an ammeter to measure the amount of current flowing in a circuit.

2. Use a voltmeter to determine the voltage existing between two points in a circuit.

3. Use an ohmmeter or megger (megohmmeter) to measure circuit continuity and total or partial circuit resistance.

The technician may also find it necessary to employ a wattmeter to determine the total POWER being consumed by certain equipments. If he wishes to measure the ENERGY consumed by certain equipments or certain circuits, a watt-hour or kilowatt-hour meter is used.

For measuring other quantities the technician must employ the appropriate instrument. In order to employ the proper instrument the technician must be aware of the capabilities of various instruments. In each case the instrument indicates the value of the quantity measured, and the technician interprets the information in a manner that will help him understand the way the circuit is operating.

A thorough understanding of the construction, operation, and limitations of the basic types of electrical measuring instruments, coupled with the theory of circuit operation, is most essential in selecting the proper instrument and in servicing and maintaining electrical-equipment.

All measuring instruments must have some form of indicating device in order to be of any use to the technician. The most basic indicating device, used in instruments measuring current and voltage, is called a GALVANOMETER. This device (and many other indicating devices) operate by virtue of the magnetic field associated with current flow. Therefore, the properties of magnetism and electromagnetism will be discussed prior to the explanation of basic meters.

14-1. Magnetism

The high points of magnetism pertaining to the information in this chapter will be set forth here. The student desiring to refresh his memory is directed to review Chapter Three.

The magnetic force surrounding a magnet is not uniform. The magnetic force is concentrated at the ends of the magnet. The two ends are called the poles of the magnet. A magnet will always have two magnetic poles and both poles will have equal magnetic strength. The two poles of the magnet are designated the north pole and the south pole. A force of attraction exists between two unlike poles and a force of repulsion exists between two like poles. The intensity of attraction or repulsion between magnetic poles is described by the equation:

$$F = \frac{m_1 m_2}{ud^2} \qquad (3\text{-}1)$$

The space surrounding a magnet, where magnetic forces act, is known as the magnetic field. Exploration of this field by experimental means yields the fact that the magnetic field is very strong at the poles and weakens as the distance from the poles increases. It is also apparent,

from experimental results obtained, that the field extends from one pole of the magnet to the other constituting a loop about the magnet.

To further describe and work with magnetic phenomena, lines are used to represent the force existing in the area surrounding a magnet (refer to Figure 14-1). These lines, called magnetic lines of force, do not actually exist but are imaginary lines used to illustrate and describe the pattern of the magnetic field. The magnetic lines of force are assumed to eminate from the north pole of a magnet, pass through the surrounding space, and enter the south pole. The lines of force then travel inside the magnet from the south pole to the north pole thus forming closed loops.

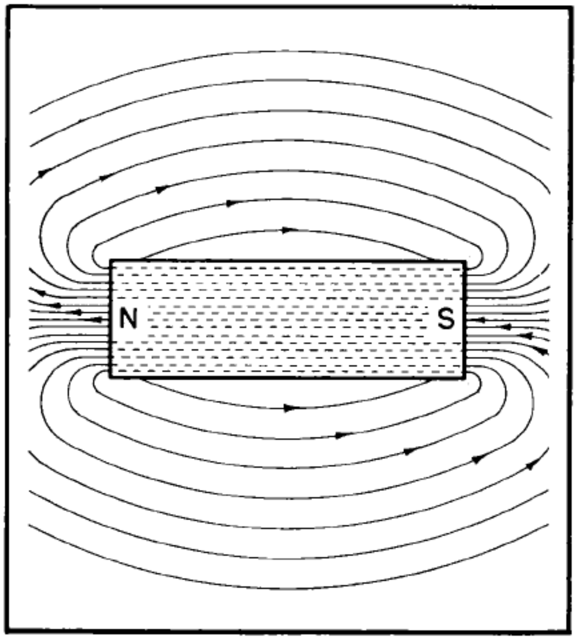

Figure 14-1 - Magnetic lines of force.

Although magnetic lines of force are imaginary, a simplified version of many magnetic phenomena can be explained by assuming the magnetic lines to have certain real properties. The main characteristics of magnetic lines of force are listed below.

1. Magnetic lines of force are continuous and will always form closed loops.

2. Magnetic lines of force will never cross.

3. Magnetic lines of force travelling in the same direction repel one another. Magnetic lines travelling in opposite directions tend to attract each other and combine.

4. Magnetic lines of force tend to shorten themselves (take the shortest route).

5. Magnetic lines of force pass through all materials, both magnetic and non-magnetic.

6. Magnetic lines of force always enter or leave a magnetic material at right angles to the surface.

The total number of magnetic lines of force leaving or entering a surface are called magnetic flux. The number of flux lines per unit area is known as flux density.

The intensity of a magnetic field is directly related to the magnetic force exerted by the field.

There is not a known insulator against magnetic flux. Any material, when placed within a magnetic field, will be penetrated by the passage of magnetic flux. Since sensitive instruments become inaccurate when subjected to the influence of stray magnetic fields, it is necessary to protect them in some manner. Because an instrument's mechanism cannot be insulated from magnetic flux, it is necessary to redirect the passage of the flux lines. It is known that the magnetic lines of force take the path of least opposition. Therefore, if the mechanism is surrounded with a material having a high permeability, the flux lines will take the easy path through the surrounding material. A sensitive instrument is protected by enclosing it in a soft iron case called a magnetic shield (Figure 14-2). It must be emphasized again that there is no insulator for magnetic lines of force, but by placing an instrument inside the iron shield, an effective insulation occurs.

Q1. What is the direction of travel of lines of force within a magnet?

Q2. If the intensity of magnetic field is doubled, what happens to the magnetic force of this field.

14-2. Electromagnetism

Whenever an electron moves, it generates a magnetic field whose lines of force appear as concentric circles about the electron. A conductor (such as a copper wire) contains many free electrons. Applications of an EMF to a conductor causes many of these free electrons to move in the same direction through the conductor. This movement of electrons constitutes a current flow. The individual magnetic fields of all the electrons moving in the same direction in the conductor will be additive with the result that a magnetic field will exist around a conductor when a current is passing through it. Figure 14-3 illustrates the direction of flux lines

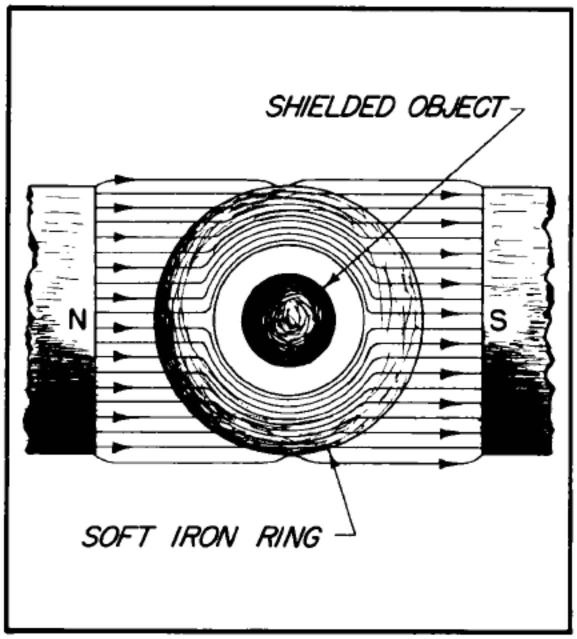

Figure 14-2 - Magnetic shielding.

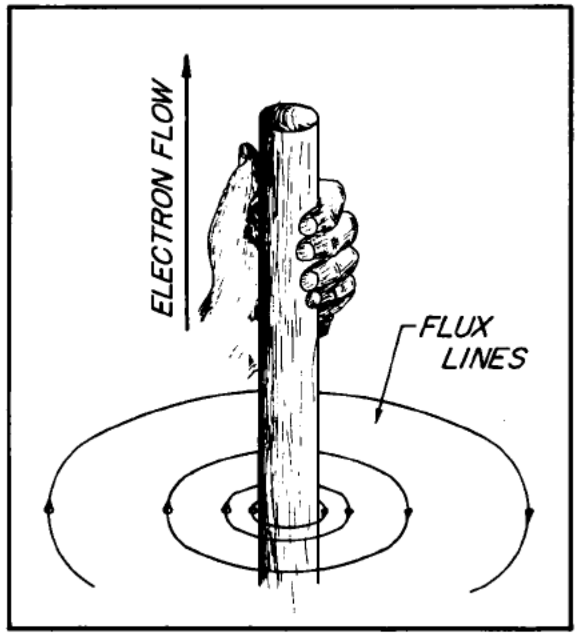

Figure 14-3 - Magnetic field around a conductor.

around a conductor in relation to the electron flow. The relation between the direction of the magnetic lines of force around a conductor and the direction of current flow along the conductor can be determined by means of the LEFT-HAND RULE FOR A CONDUCTOR.

LEFT-HAND RULE FOR CONDUCTORS: If the left hand is placed so that the thumb points in the direction of ELECTRON flow, the curled fingers will point in the direction of the flux lines encircling the conductor.

Figure 14-3 also illustrates the left-hand rule for conductors. If the electron flow (Figure 14-3) were reversed the hand would have to be turned upside down with the thumb pointing toward the bottom of the page. The fingers will now indicate that, with the electron flow reversed, the direction of the flux lines is also reversed.

PARALLEL CONDUCTORS: Arrows generally are used in electric diagrams to denote the direction of current flow along the length of wire. Where cross sections of wire are shown, a special view of the arrow is used. A cross-sectional view of a conductor that is carrying current toward the observer is illustrated in Figure 14-4A. The direction of current is indicated by a dot, which represents the head of the arrow. A conductor that is carrying current away from the observer is illustrated in

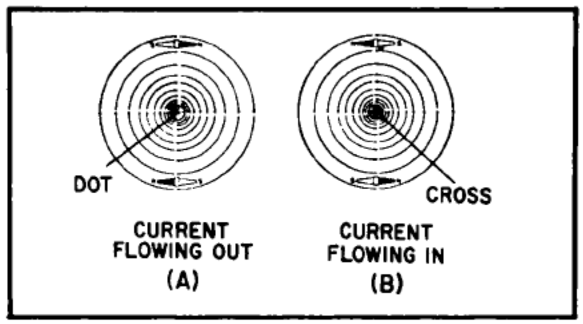

Figure 14-4 - Magnetic field around a current-carrying conductor, detailed view.

Figure 14-4B. The direction of current is indicated by a cross, which represents the tail of the arrow.

When two parallel conductors carry current in the same direction, the magnetic fields tend to encircle both conductors, drawing them together with a force of attraction, as shown in Figure 14-5A. Two parallel conductors carrying currents in opposite directions are shown in Figure 14-5B. The field around one conductor is opposite to the field around the other conductor. The resulting lines of force are crowded together in the space between the wires, and tend to push the wires apart. Therefore, two parallel conductors carrying currents in opposite directions repel each other.

A1. From the south pole to the north pole.

A2. The magnetic force is directly related to the intensity and therefore will also be doubled.

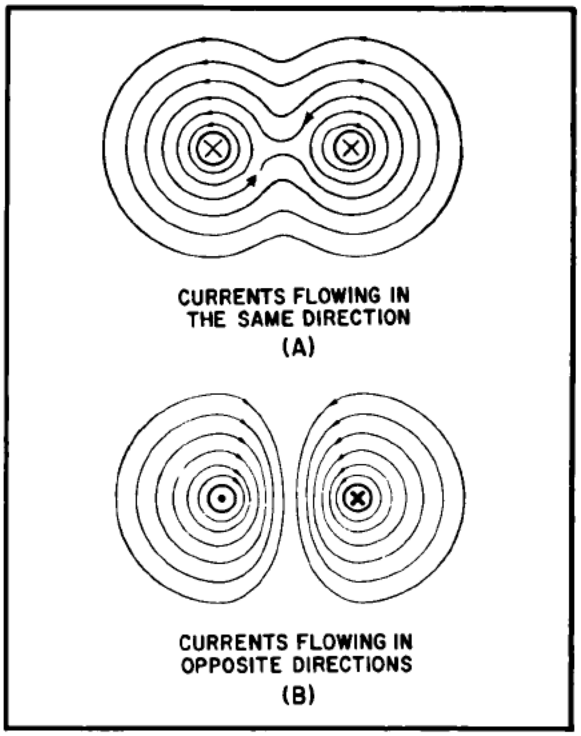

Figure 14-5 - Magnetic field around two parallel conductors.

MAGNETIC FIELD OF A COIL: The magnetic field around a current-carrying wire exists at all points along its length. The field consists of concentric circles in a plane perpendicular to the wire. (See Figure 14-3.) When this straight wire is wound around a core, as shown in Figure 14-6A, it becomes a coil and the magnetic field assumes a different shape. Part (A) is a partial cutaway view which shows the construction of a simple coil. Part (B) is a complete cross-sectional view of the same coil. The two ends of the coil are identified as point a and point b. When current is passed through the coiled conductor, as indicated, the magnetic field of each turn of wire links with the fields of adjacent turns, as explained in connection with Figure 14-5A. The combined influence of all the turns produce a two-pole field similar to that of a simple bar magnet. One end of the coil will be a north pole and the other end will be a south pole.

It was shown that the direction of the mag-

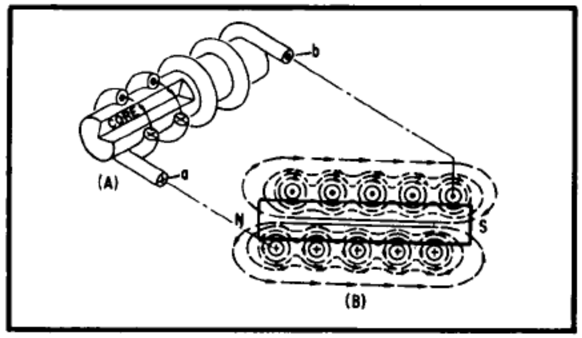

Figure 14-6 - Magnetic field produced by a current-carrying coil.

netic field around a straight conductor depends on the direction of current flow through that conductor. Thus, a reversal of current flow through a conductor causes a reversal in the direction of the magnetic field that is produced. It follows that a reversal of the current flow through a coil also causes a reversal of its two-pole field. This is true because that field is the product of the linkage between the individual turns of wire on the coil. Therefore, if the field of each turn is reversed, it follows that the total field (coils field) is also reversed.

When the direction of electron flow through a coil is known, its polarity may be determined by use of the LEFT-HAND RULE FOR COILS. This rule is illustrated in Figure 14-7, and is stated as follows:

LEFT-HAND RULE FOR COILS: Grasping the coil in the left hand, with the fingers "wrapped around" in the direction of electron flow, the thumb will point toward the north pole.

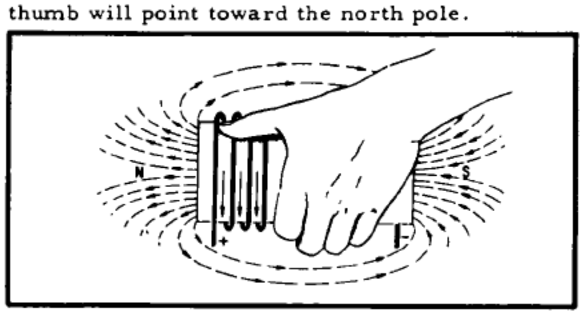

Figure 14-7 - Left-hand rule for coil polarity.

STRENGTH OF AN ELECTROMAGNETIC FIELD: The strength or intensity of a coil's field depends on a number of factors. Some of the factors are listed below. All of these factors are discussed under headings that follow.

1. The number of turns of conductor.
2. The amount of current flow through the coil.
3. The ratio of the coil's length to its width.
4. The type of material in the core.

NUMBER OF TURNS: When current is flowing in the same direction in two parallel conductors the flux lines of the individual conductors will be additive (Figure 14-5A). When a conductor is wound in the form of a coil, the current in adjacent turns is in the same direction. Therefore, the flux from individual turns will be additive (Figure 14-6). Increasing the number of turns will increase the number of flux lines and thereby the intensity of the coil's field.

CURRENT: The strength of the magnetic field around a conductor carrying a current is a function of the current; the larger the current, the stronger the magnetic field. When the conductor is wound into a coil this relationship between current and flux still exists. Increasing the current will increase the flux around the coil turns and thereby increase the intensity of the coil's field.

RATIO OF LENGTH TO WIDTH: In order for the field intensity to be uniform throughout the cross section of the coil, the ratio of length to width should be at least 10 to 1. In other words the length of the coil should be at least 20 times the radius.

CORE MATERIAL: In Chapter 3 it was shown that some materials offered more opposition to magnetic lines of force than did others. If a soft iron core is inserted in a coil, the number of flux lines will be greatly increased as compared to those present when just the air core was used. It must be noted that the additional lines of force are produced by the magnetization of the iron core and not by an increase in the field intensity (field intensity can only be increased by increasing current or number of turns).

Two terms that are used in describing core materials are RELUCTANCE and PERMEABILITY.

The RELUCTANCE, R, similar to resistance in the Ohm's law formula, is the opposition offered by the magnetic circuit to the passage of magnetic flux. The unit of reluctance has not been named officially. However, the REL has been proposed, and the symbol R is commonly used. The unit of reluctance is the reluctance of 1 cubic-centimeter of air. The reluctance of a magnetic substance varies directly as the length of the flux path and inversely as the cross-sectional area and the permeability, u, of the substance.

PERMEABILITY, designated by the Greek letter mu, u, is a measure of the relative lines of force within a material, as compared with air. The permeability of air is taken as 1. Therefore, any core material having a permeability with a value greater than one will produce a greater flux density.

AMPERE TURNS: Since current and the number of turns are the major factors in determining the field strength of a coil the unit used to compare the relative strength of electromagnetic fields is the AMPERE TURN and is equal to the product of the current through the coil and the number of coil turns.

Q3. What is the effect on two magnetic lines of force, traveling in opposite directions when they are brought into close proximity?

Q4. Name two methods of increasing the intensity of the magnetic field of a coil.

14-3. Electromagnetic Forces

It has been established that a conductor carrying a current will have a magnetic field surrounding it. It has also been established (Chapter 3) that if a magnet is shaped in such a manner as to have its poles close together a magnetic field will exist between these poles. The magnetic field between a north and a south pole of a magnet is shown in Figure 14-8A. The lines of force, comprising the field extend from the north pole to the south pole. A cross section of a current-carrying conductor is shown in Figure 14-8B. The plus sign in the wire indicates that the electron flow is away from the observer. The direction of the flux loops around the wire is counterclockwise as shown. This follows from the left-hand rule for conductors.

If the conductor (carrying the electron flow away from the observer) is placed between the poles of the magnet, as in Figure 14-8C, both fields will be distorted. Above the wire the field is weakened, and the conductor tends to move upward. The force exerted upward depends on the strength of the field between the poles and on the strength of the current flowing through the wire.

If the current through the conductor is reversed, as in Figure 14-8D, the direction of the flux around the wire is reversed. The field below the conductor is now weakened, and the conductor tends to move downward.

A convenient method of determining the direction of motion of a current-carrying conductor in a magnetic field is by the use of the right-hand motor rule, illustrated in Figure 14-9.

RIGHT-HAND MOTOR RULE FOR ELECTRON FLOW: To find the direction of motion of a conductor, the thumb, first finger, and second finger of the right hand are extended at right

A3. They will tend to attract each other and combine.

A4. Increase the number of turns, increase the current through the coil, increase the permeability of the core material.

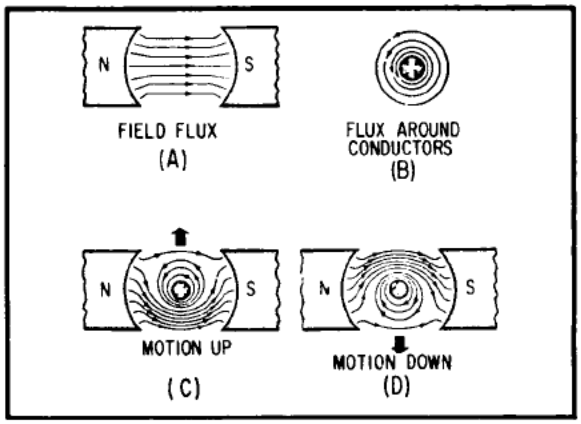

Figure 14-8 - Force acting on a current-carrying conductor in a magnetic field.

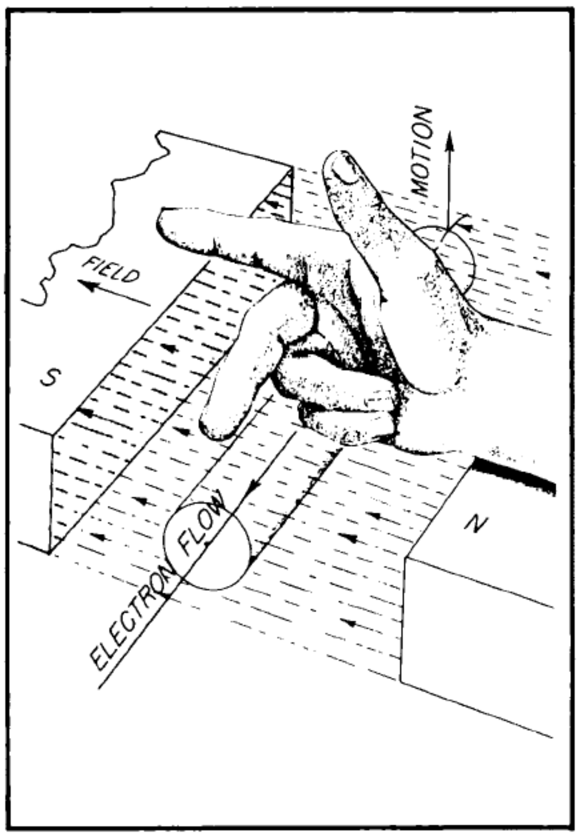

Figure 14-9 - Right-hand motor rule.

angles to each other, as shown. The first finger is pointed in the direction of the flux (toward the south pole) and the second finger is pointed in the direction of electron flow in the conductor. The thumb then points in the direction of motion of the conductor with respect to the field. The conductor, the field, and the force are mutually perpendicular to each other.

The force acting on a current-carrying conductor in a magnetic field is directly proportional to the field strength of the magnet, the active length of the conductor, and the intensity of the electron flow through it.

14-4. Basic Meter Movement

The stationary permanent-magnet moving-coil meter is the basic movement used in most measuring instruments for servicing electrical equipment. The basic movement consists of a stationary permanent magnet and a movable coil. When current flows through the coil the resulting magnetic field reacts with the magnetic field of the permanent magnet and causes the coil to rotate. The greater the amount of current flow through the coil the stronger the magnetic field produced and the stronger this field the greater the rotation of the coil.

A simplified diagram of one type of stationary permanent-magnet moving-coil instrument is shown in Figure 14-10. Such an instrument is commonly called a GALVANOMETER. The galvanometer indicates very small amounts (or the relative amounts) of current or voltage, and is distinguished from other instruments used for the same purpose in that the movable coil is suspended by means of metal ribbons instead of by means of a shaft and jewel bearings.

The movable coil of the galvanometer in Figure 14-10 is suspended between the poles of the magnet by means of thin flat ribbons of phospher bronze. These ribbons provide the conducting path for the current between the circuit under test and the movable coil. They also provide the restoring force for the coil. The restoring force, exerted by the twist in the ribbons, is the force against which the driving force of the coil's magnetic field (to be described later) is balanced in order to obtain a measurement of the current strength. The ribbons thus tend to oppose the motion of the coil, and will twist through an angle that is proportional to the force applied to the coil by the action of the coil's magnetic field against the permanent field. The ribbons thus restrain or provide a counter force, for the magnetic force acting on the coil. When the driving force of the coil current is removed, the restoring force returns the coil to its zero position. In order to determine the amount of current flow a means must be provided to indicate the amount of coil

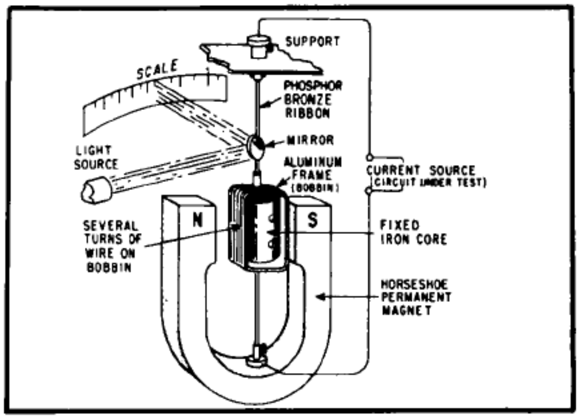

Figure 14-10 - Simplified diagram of a galvanometer.

rotation. Either of two methods may be used—(1) the pointer arrangement, and (2) the light and mirror arrangement. In the pointer arrangement, the end of the pointer is fastened to the rotating coil and as the coil turns the pointer also turns. The other end of the pointer moves across a graduated scale and indicates the amount of current flow. An advantage of this arrangement is that it permits overall simplicity. A disadvantage of the pointer arrangement is that it introduces the problem of coil balance, especially if the pointer is long. The use of a mirror and a beam of light simplifies the problem of coil balance. When this arrangement is used to measure the turning of the coil, a small mirror is mounted on the supporting ribbon (Figure 14-10) and turns with the coil. An internal light source is directed to the mirror, and then reflected to the scale of the meter. As the moving coil turns, so does the mirror, causing the light reflection to move over the scale of the meter. The movement of the reflection is proportional to the movement of the coil, thus the amount of current being measured by the meter is indicated.

If a beam of light and mirrors are used, the beam of light is swept to the right or left across a central-zero translucent screen (scale) having uniform divisions. If a pointer is used, the pointer is moved in a horizontal plane to the right or left across a central-zero scale having uniform divisions. The direction in which the beam of light or the pointer moves depends on the direction of current through the coil.

This instrument is used to measure minute current as, for example, in bridge circuits. In modified form, the galvanometer has the highest sensitivity of any of the various types of meters in use today.

Q5. When the current under test is removed from a galvanometer, why does the indicating device return to zero?

14-5. D'Arsonval Meter Movement

In the D'Arsonval-type meter, the length of the conductor is fixed and the strength of the field between the poles of the magnet is fixed. Therefore, any change in I causes a proportionate change in the force acting on the coil.

The principle of the D'Arsonval movement may be more clearly shown by the use of the simplified diagram (Figure 14-11) of the D'Arsonval movement commonly used in dc instruments. In the diagram, only one turn of wire is shown; however, in an actual meter movement many turns of fine wire would be used, each turn adding more effective length to the coil. The coil is wound on an aluminum frame or bobbin, to which the pointer is attached. Oppositely wound hairsprings (one of which is shown in Figure 14-11) are also attached to the bobbin, one at either end. The circuit to the coil is completed through the hairsprings. In addition to serving as conductors, the hairsprings serve as the restoring force that returns the pointer to the zero position when no current flows.

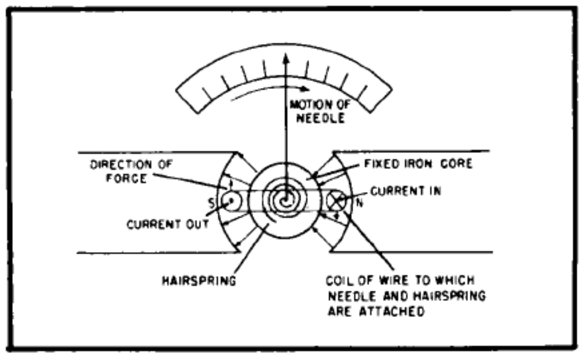

Figure 14-11 - D'Arsonval Movement.

As has been stated, the deflecting force is proportional to the current flowing in the coil. The deflecting force tends to rotate the coil against the restraining force of the hairspring. The angle of rotation is proportional to the deflecting force. When the deflecting force and the restraining force are equal, the coil and the pointer cease to move. Since the deflecting force is proportional to the current in the coil and the angle of rotation is proportional to the deflecting force; then the angle of rotation is proportional to the current through the coil. When current ceases to flow in the coil, the driving force ceases, and the restoring force of the springs returns the pointer to the zero position.

A5. Due to the restoring force of the phosphor bronze ribbons.

If the current through the single turn of wire is in the direction indicated (away from the observer on the right-hand side and toward the observer on the left-hand side), the direction of force, by the application of the right-hand motor rule, is upward on the left-hand side and downward on the right-hand side. The direction of motion of the coil and pointer is clockwise. If the current is reversed in the wire, the direction of motion of the coil and pointer is reversed.

A detailed view of the basic D'Arsonval movement, as commonly employed in ammeters and voltmeters, is shown in Figure 14-12. The principle of operation is the same as that of the simplified versions discussed previously. The iron core is rigidly supported between the pole pieces and serves to concentrate the flux in the narrow space between the iron core and the pole piece—in other words, in the space through which the coil and the bobbin moves. Current flows into one hairspring, th.·ough the coil, and out of the other hairspring. The restoring force of the spiral springs returns the pointer to the normal, or zero, position when the current through the coil is interrupted. Conductors connect the hairsprings with the outside terminals of the meter. If the instrument is not DAMPED, that is if some type of loss is not introduced to absorb the energy of the moving element, the pointer will oscillate for a long time about its final position before coming to rest. This action makes it nearly impossible to obtain a reading and some form of damping is necessary to make the meter practicable. Damping is accomplished in many D'Arsonval movements by means of the motion of the aluminum bobbin upon which the coil is wound. As the bobbin oscillates in the magnetic field, an EMF is induced in it because it cuts through the lines of force. Therefore, induced currents flow in the bobbin in such a direction as to oppose the motion, and the bobbin quickly comes to rest in the final position after going beyond it only once.

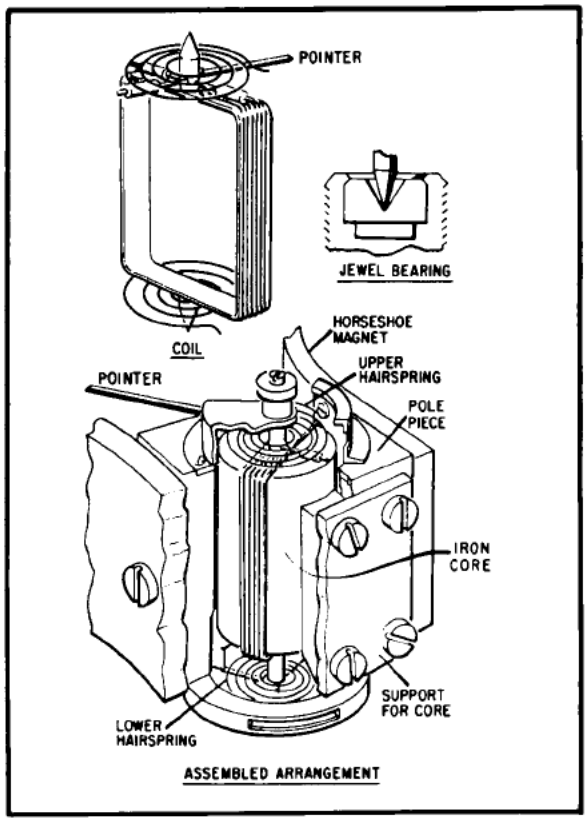

Figure 14-12 - Detailed view of basic D'Arsonval movement.

In addition to factors such as increasing the flux density in the air gap, the overall sensitivity of the meter can be increased by the use of a lightweight rotating assembly (bobbin, coil, and pointer) and by the use of jewel bearings as shown.

It is noted that the pole pieces (Figures 14-11 and 14-12) have curved faces. The advantage of this type of construction can be seen if it is remembered that lines of force enter and leave a magnetic surface at right angles. The curved surfaces produce a uniform magnetic field in the air gap at right angles to the coil regardless of the coil's angular position. This type of construction makes possible a more linear scale than if the pole faces were flat.

Most dc instruments use meters based on some form of the D'Arsonval movement.

Q6. What is the effect on a D'Arsonval movement if the deflecting force exceeds the restraining force by a large amount?

Q7. What is "damping" and why is it necessary in a meter movement?

Q8. If the pointer of a meter movement is deflected in the wrong direction what can be done to correct this?

Q9. Why is iron used as a core in a meter movement?

BASIC DC METER CIRCUITS

14-6. DC Ammeter

The small size of the wire with which an ammeter's movable coil is wound places limits on the current that may be passed through the coil. Consequently, the basic D'Arsonval movement discussed thus far may be used to indicate or measure only very small currents.

To measure a larger current, a shunt must be used with the meter. A shunt is a heavy low-resistance conductor connected across the meter terminals to carry most of the load current. This shunt has the correct amount of resistance to cause only a small part of the total circuit current to flow through the meter coil. The meter current is proportional to the load current. If the shunt is of such a value that the meter is calibrated in milliamperes, the instrument is called a MILLIAMMETER. If the shunt is of such a value that the meter is calibrated in amperes, it is called an AMMETER.

A single type of standard meter movement is generally used in all ammeters, no matter what the range of a particular meter. For example, meters with working ranges of zero to 10 amperes, zero to 5 amperes, or zero to one ampere all use the same movement. The designer of the ammeter simply calculates the correct shunt resistance required to extend the range of the meter movement to measure any desired amount of current. This shunt is then connected across the meter terminals. Shunts may be located inside the meter case (internal shunt) or somewhere away from the meter (external shunt), with leads going to the meter. An external shunt arrangement is shown in Figure 14-13A. Some typical external shunts are shown in part (C). Part (B) of Figure 14-13

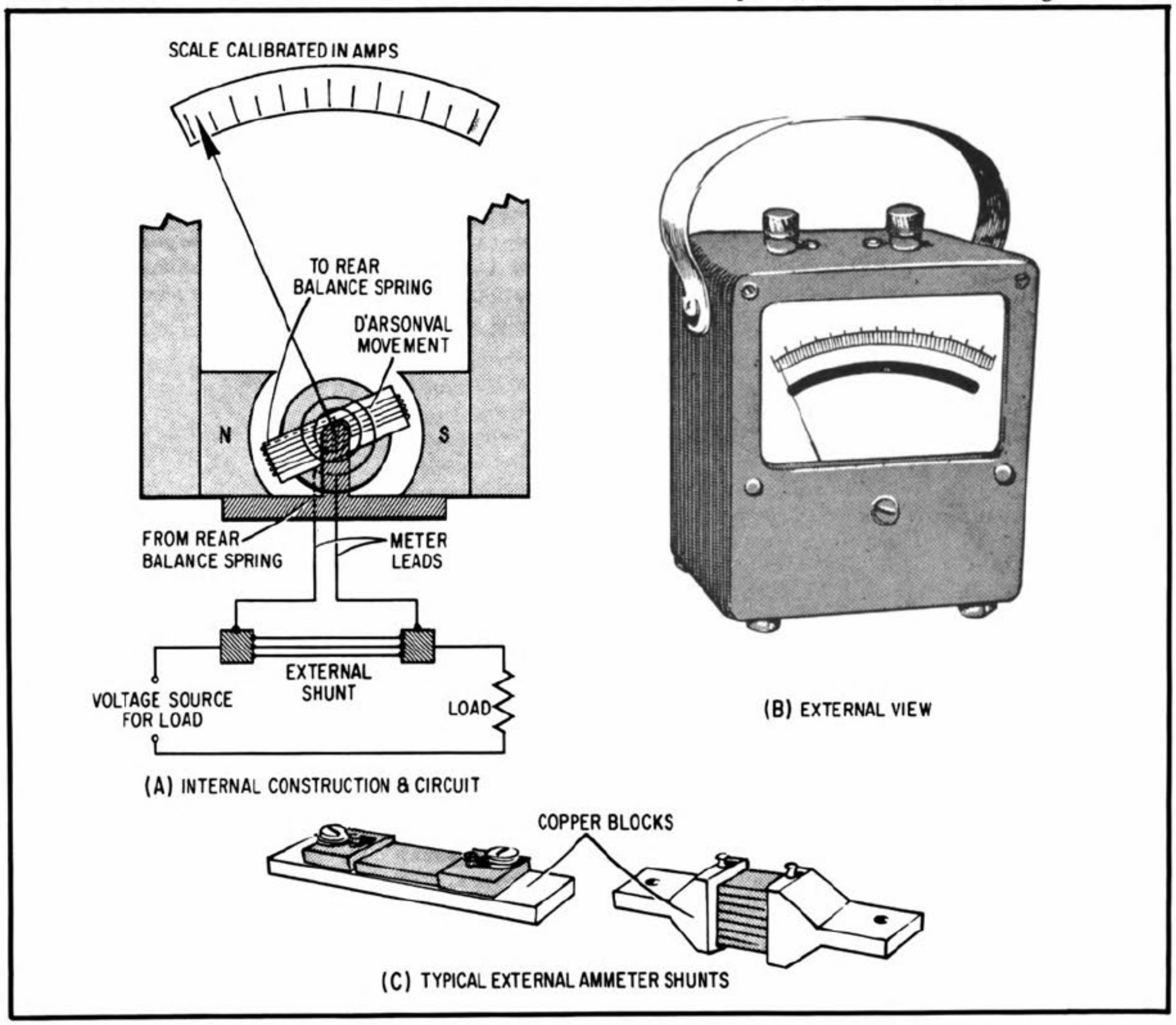

Figure 14-13 - Weston ammeter employing D'Arsonval principle in its movement.

A6. The pointer will be deflected far off scale and probable damage will result.

A7. Damping is a form of loss introduced to absorb the energy of the meter movement and prevent oscillation when the pointer has reached its desired value (position).

A8. Reverse the direction of the current through the coil by reversing the leads.

A9. Iron has a low reluctance and flux lines pass through it easily making the meter more efficient.

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shows a meter movement mounted in a case which provides protection against breakage, magnetic shielding in some cases, portability, etc.

The shunt strips are usually made of manganin; an alloy having an almost zero temperature coefficient of resistance. The ends of the shunt strips are embedded in heavy copper blocks to which are attached the meter coil leads and the line terminals. To insure accurate readings, the meter leads for a particular ammeter should not be used interchangeably with those for a meter of a different range. Slight changes in lead length and size may vary the resistance of the meter circuit and thus its current, and may cause an incorrect meter reading. External shunts are generally used where currents greater than 50 amperes must be measured.

It is important to select a suitable shunt when using an external shunt ammeter so that the scale indication is easily read. For example, if the scale has 150 divisions and the load current to be measured is known to be between 50 and 100 amperes, a 150-ampere shunt is suitable. Under these conditions each division of the scale will represent one ampere. In other words a full scale deflection of the pointer would indicate that 150 amperes of load current is flowing. At half scale deflection the pointer would rest on the 75th division mark indicating that 75 amperes of load current is flowing.

A shunt having exactly the same current rating as the estimated normal load current should never be selected because any abnormally high load would drive the pointer off scale and might damage the movement. A good choice of shunts will bring the needle somewhere near the midscale indication, when the load is normal.

14-7. Extending Ammeter Range With Internal Shunts

For current ranges (below 50 amperes), internal shunts are most often employed. In this manner the range of the meter may be easily changed by means of a switching arrangement which will select the correct internal shunt having the necessary current rating and resistance. Before the required resistance of the shunt for each range can be calculated, the resistance of the meter movement must be known.

For example, suppose it is desired to use a 100 microampere D'Arsonval meter having a meter coil resistance of 100 ohms to measure line currents up to 1 ampere. The meter deflects full scale when the current through the 100 ohm meter coil is 100 microamperes. Therefore, the voltage drop across the meter coil is:

$$E = IR$$

$$E = 0.0001 \times 100$$

$$E = 0.01 \text{ volt}$$

Because the shunt and coil are in parallel, the shunt must also have a voltage drop of 0.01 volt. The current that flows through the shunt is the difference between the full-scale meter current and the line current. In this case, the meter current is 100×10^{-6}, or 0.0001 ampere. This current is negligible compared with the line (shunt) current, so the shunt current is approximately 1 ampere. The resistance, R_s, of the shunt is therefore:

$$R_s = \frac{E}{I_s}$$

$$R_s = \frac{0.01}{1}$$

$$R_s = 0.01 \text{ ohm (approx)}$$

The range of the 100 microampere meter has been increased to 1 ampere (full scale deflection) by paralleling it with the 0.01 ohm shunt.

The 100 microampere instrument may also be converted to a 10 ampere meter by the use of a proper shunt. For full-scale deflection of the meter the voltage drop, E, across the shunt (and across the meter) is still 0.01 volt. The meter current is again considered negligible, and the shunt current is now approximately 10 amperes. The resistance, R_s, of the shunt is therefore:

$$R_s = \frac{E}{I_s}$$

$$R_s = \frac{0.01}{10}$$

$$R_s = 0.001 \text{ ohm (approx)}$$

The same instrument may likewise be converted to a 50 ampere meter by the use of the proper type of shunt. The current, I_s, through the shunt is approximately 50 amperes and the resistance, R_s, of the shunt is:

$$R_s = \frac{E}{I_s}$$

$$R_s = \frac{0.01}{50}$$

$$R_s = 0.0002 \text{ ohm (approx)}$$

The above method of computing the shunt resistance is satisfactory in most cases when the line current is in the ampere range and the current through the meter coil is a very small percentage of the line current. However, when the line current is in the milliampere range and the meter coil current becomes an appreciable percentage of the line current a more accurate calculation must be performed.

Example. It is desired to use a meter movement which has a full scale deflection of one milliampere and a coil resistance of 50 ohms to measure currents up to 10 milliamps.

Using Ohm's law, the voltage across the meter coil (and therefore across the shunt) at full scale deflection is:

$$E = I_m \times R_m$$

$$E = 0.001 \times 50$$

$$E = 50 \text{ mv}$$

The current that flows through the shunt (I_s) is the difference between the line current (I_t) and the meter current (I_m).

$$I_s = I_t - I_m$$

$$I_s = 10 \text{ ma} - 1 \text{ ma}$$

$$I_s = 9 \text{ ma}$$

The resistance of the shunt will be:

$$R_s = \frac{E_s}{I_s}$$

$$R_s = \frac{0.05}{0.009}$$

$$R_s = 5.55 \text{ ohms}$$

A formula for determining the resistance of the shunt is given below:

$$R_s = \frac{I_m}{I_s} \times R_m \qquad (14\text{-}1)$$

where: R_s = the resistance of the shunt in ohms
I_m = meter current at full-scale deflection in amps.
I_s = shunt current at full-scale deflection in amps.
R_m = resistance of the meter coil in ohms

Using equation (14-1):

$$R_s = \frac{I_m}{I_s} \times R_m$$

$$R_s = \frac{0.001}{0.009} \times 50$$

$$R_s = 5.55 \text{ ohms}$$

Various values of shunt resistance may be used, by means of a suitable switching arrangement, to increase the number of current ranges that may be covered by the meter. Two switching arrangements are shown in Figure 14-14. Part (A) is the simpler of the two arrangements from the point of view of calculating the value of the shunt resistors when a number of shunts are used. However, it has two disadvantages

1. When the switch is moved from one shunt resistor to another the shunt is momentarily removed from the meter and the line current then flows through the meter coil. Even a momentary surge of current could easily damage the coil.

2. The contact resistance—that is, the resistance between the blades of the switch when they are in contact—is in series with the shunt but not with the meter coil. In shunts that must pass high currents the contact resistance becomes an appreciable part of the total shunt resistance. Because the contact resistance is of a variable nature, the ammeter indication may not be accurate.

A more generally accepted method of range switching is shown in Figure 14-14B. Although only two ranges are shown, as many ranges as needed can be used. In this type of circuit the range selector switch contact resistance is external to the shunt and meter in each range

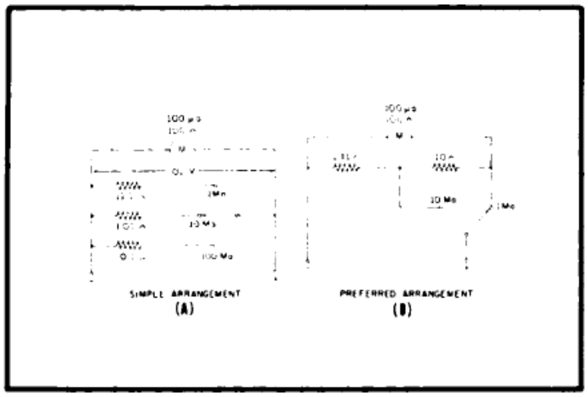

Figure 14-14 - Ways of connecting internal shunts.

position, and therefore has no effect on the accuracy of the current measurement.

CURRENT MEASURING INSTRUMENTS MUST ALWAYS BE CONNECTED IN SERIES WITH A CIRCUIT AND NEVER IN PARALLEL WITH IT. If an ammeter were connected across a constant-potential source of appreciable voltage the shunt would become a short circuit and the meter would burn out.

If the approximate value of current in a circuit is not known, it is best to start with the highest range of the ammeter and switch to progressively lower ranges until a suitable reading is obtained.

Most ammeter needles indicate the magnitude of the current by being deflected from left to right. If the meter is connected with reversed polarity, the needle will be deflected backwards, and this action may damage the movement. Hence the proper polarity should be observed in connecting the meter in the circuit. This is, the meter should always be connected so that the electron flow will be into the negative terminal and out of the positive terminal.

Figure 14-15 shows various circuit arrangements; the ammeter or ammeters are properly connected for measuring current in various portions of the circuits.

Sensitivity is determined by the amount of current required by the meter coil to produce full scale deflection. The less current required the better is the sensitivity of the meter. Thus, a 100 microampere meter movement would have better sensitivity than a 1 milliampere movement.

Q10. A meter has two current ranges in which the calibrated ranges read 5 and 10 amperes respectively. Which range requires the higher resistance shunt?

Q11. Would a milliammeter with a 1 ma full-scale deflection be suitable for use in a circuit where maximum current never goes over one microampere?

Q12. Find R_m if I_s = 90 ma
E_s = 100 mv
I_t = 100 ma

DC VOLTMETER

The 100 microampere D'Arsonval meter used as the basic meter for the ammeter may also be used to measure voltage if a high resistance is placed in series with the moving coil of the meter. For low-range instruments this resistance is mounted inside the case with the D'Arsonval movement and typically consists of resistance wire having a low temperature coefficient and wound either on spools or card frames. For higher voltage ranges, the series resistance may be connected externally. When this is done the unit containing the resistance is commonly called a MULTIPLIER.

A simplified diagram of a voltmeter is shown in Figure 14-16. The resistance coils are treated in such a way that a minimum amount of moisture will be absorbed by the insulation. Moisture reduces the insulation resistance and increases leakage currents, which cause incorrect readings. Leakage currents through the insulation increase with length of resistance wire and become a factor that limits the magnitude of voltage that may be measured.

14-8. Extending the Range

The value of the necessary series resistance is determined by the current required for full-scale deflection of the meter and by the range of voltage to be measured. Because the current through the meter circuits is directly proportional to the applied voltage, the meter scale can be calibrated directly in volts for a fixed series resistance.

For example, assume that the basic meter (microammeter) is to be made into a voltmeter with a full-scale reading of 1 volt. The coil resistance of the basic meter is 100 ohms, and 0.0001 ampere (100 microamperes) causes a full-scale deflection. The total resistance (R_t) of the meter coil and the series resistance is:

$$R_t = \frac{E}{I_m}$$

$$R_t = \frac{1}{0.0001}$$

R_t = 10K ohms (for 1 volt range)

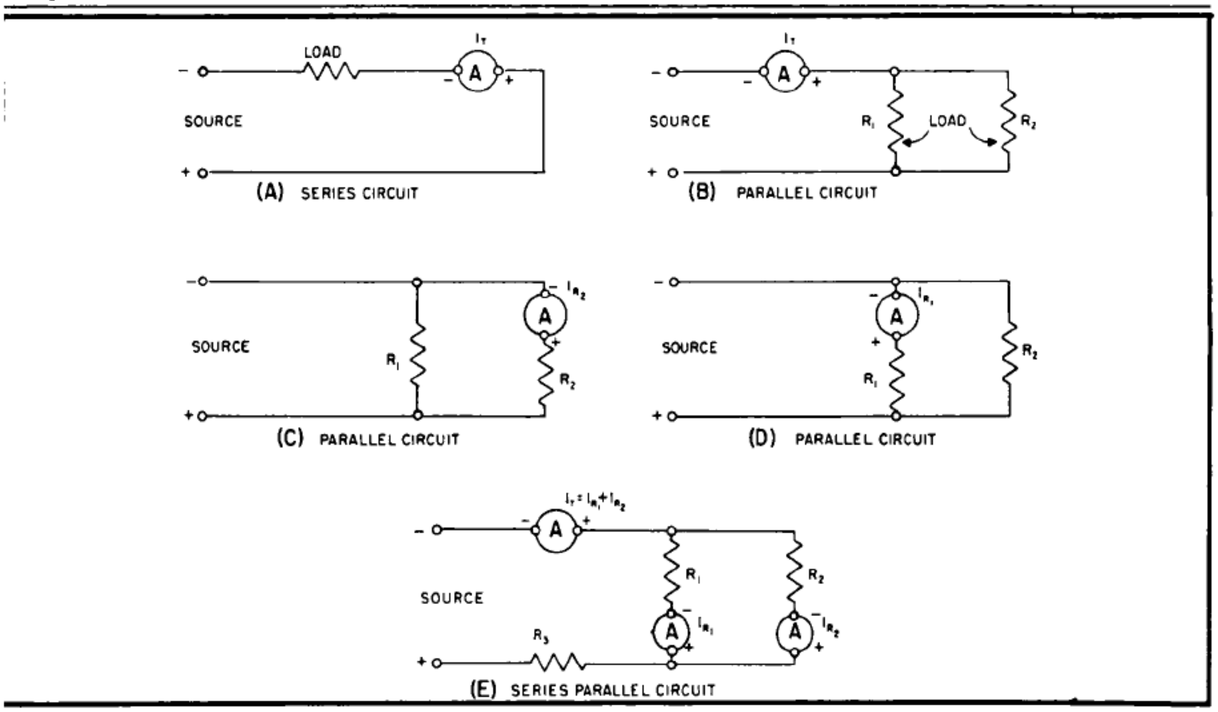

Figure 14-15 - Proper ammeter connections.

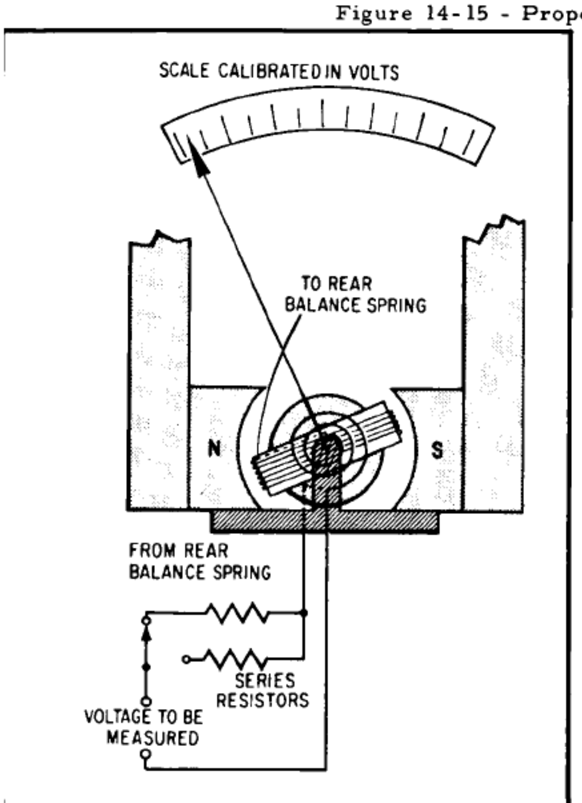

Figure 14-16 - Internal construction and circuit of simplified voltmeter.

The series resistance (R_s) is:

$$R_s = R_t - R_m$$

$$R_s = 10,000 - 100$$

$$R_s = 9.9K \text{ ohms}$$

Multirange voltmeters utilize one meter movement with the required resistances connected in series with the meter by a convenient switching arrangement. A schematic diagram of a multirange voltmeter with three ranges is shown in Figure 14-17. The total circuit resistance for each of the three ranges beginning with the 1 volt range is:

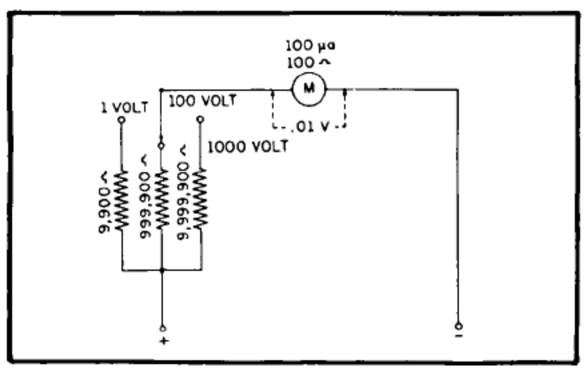

Figure 14-17 - Multirange voltmeter.

A10. The 5 ampere range.

A11. No! There would be no readable deflection of the pointer.

A12. R_m = 10 ohms.

$$R_t = \frac{E}{I} = \frac{1}{0.0001} = 10K \text{ ohm (1V range)}$$

$$R_t = \frac{100}{0.0001} = 1 \text{ meg ohm (100V range)}$$

$$R_t = \frac{1000}{0.0001} = 10 \text{ meg ohm (1000V range)}$$

The multiplying series resistor (R_s) for each of these circuits is 100 ohms less than the total resistance.

VOLTAGE-MEASURING INSTRUMENTS ARE CONNECTED ACROSS (IN PARALLEL WITH) A CIRCUIT. If the approximate value of the voltage to be measured is not known, it is best to start with the highest range of the voltmeter and progressively lower the range until a suitable reading is obtained.

In many cases, the voltmeter is not a central-zero indicating instrument. Thus, it is necessary to observe the proper polarity when connecting the instrument to the circuit, as is the case in connecting the dc ammeter. The voltmeter is connected so that electrons will flow into the negative terminal and out of the positive terminal of the meter.

14-9. Influence in a Circuit

The function of a voltmeter is to indicate the potential difference between two points in a circuit. When the voltmeter is connected across a circuit, it shunts the circuit. If the voltmeter has low resistance it will draw an appreciable amount of current. The effective resistance of the circuit will be lowered and the voltage reading will consequently be lowered.

When voltage measurements are made in high-resistance circuits, it is necessary to use a high-resistance voltmeter to prevent the shunting action of the meter. The effect is less noticeable in low-resistance circuits because the shunting effect is less.

14-10. Sensitivity

The sensitivity of a voltmeter is given in ohms per volt, (Ω/E), and may be determined by dividing the resistance R_m, of the meter plus the series resistance, R_s, by the full-scale reading in volts. Thus,

$$\text{sensitivity} = \frac{R_m + R_s}{E}$$

This is the same as saying that the sensitivity is equal to the reciprocal of the current (in amperes) that is,

$$\text{sensitivity} = \frac{\text{ohms}}{\text{volts}} = \frac{1}{\frac{\text{volts}}{\text{ohms}}} = \frac{1}{\text{amperes}}$$

Thus, the sensitivity of a 100 microampere movement is the reciprocal of 0.0001 ampere, or 10,000 ohms per volt.

14-11. Accuracy

The accuracy of a meter is generally expressed in percent. For example, a meter that has an accuracy of 1 percent will indicate a value that is within 1 percent of the correct value. The statement means that if the correct value is 100 units, the meter indication may be anywhere within the range of 99 to 101 units.

Q13. When a voltmeter having a low internal resistance is used to measure the voltage of a high resistance circuit, will the voltage indicated be higher or lower than the true voltage across the circuit?

Q14. A voltmeter has a sensitivity of 15K ohms per volt, what is the full scale deflection current of the meter movement?

METERS USED FOR MEASURING RESISTANCE

The two instruments most commonly used to check the continuity, or to measure the resistance of a circuit or circuit element, are the OHMMETER and the MEGGER (megohmmeter). The ohmmeter is widely used to measure resistance and check the continuity of electrical circuits and devices. Its range usually extends to only a few megohms. The megger is widely used for measuring insulation resistance, such as between a wire and the outer surface of its insulation, and insulation resistance of cables and insulators. The range of a megger may extend to more than 1,000 megohms.

14-12. Ohmmeter

The ohmmeter consists of a dc milliammeter, which was discussed earlier in this chapter, with a few added features. The added features are:

1. A dc source of potential (usually a three-volt battery).

. One or more resistors (one of which is variable). A simple ohmmeter circuit is shown in Figure 14-18.

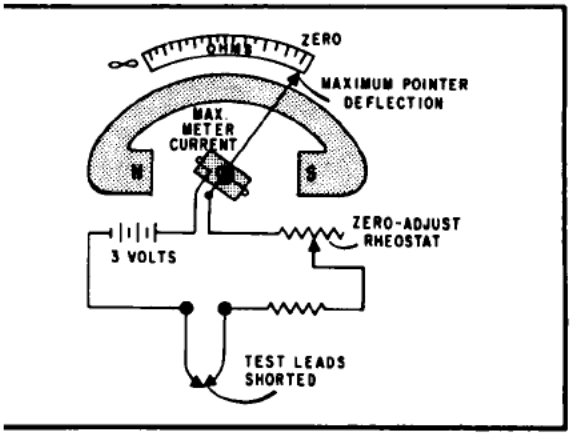

Figure 14-18 - Simple ohmmeter circuit.

The ohmmeter's pointer deflection is con-:olled by the amount of battery current passing ırough the moving coil. Before measuring the esistance of an unknown resistor or electrical ircuit, the ohmmeter must be calibrated. If ıe value of resistance to be measured can be stimated within reasonable limits, a range is elected which will give approximately half-cale deflection when this resistance is inserted etween the probes. If the resistance is un-nown the selector switch is set on the highest ange. Whatever range is selected the meter ıust be calibrated to read zero before the un-nown resistance is measured. Calibration is ccomplished by first shorting the test leads ›gether as shown in Figure 14-18. With the ›st leads shorted, there will be a complete eries circuit consisting of the 3V source, re-istance of the meter coil (R_m), resistance of ıe zero-adjust potentiometer, and the series ıultiplying resistor (R_s). Current will flow nd the meter pointer will be deflected. The ero division of the meter in Figure 14-18 (and ıany commercial ohmmeters) is located at the xtreme right of the scale. With the test leads horted, the zero-adjust potentiometer is set ɔ that the pointer rests on the zero division. herefore, full scale deflection indicates zero esistance between the test leads. If the range ; changed the meter must be "zeroed" again › obtain an accurate reading. When the test ›ads of an ohmmeter are separated, the pointer f the meter will return to the left side of the cale, due to the interruption of current and ıe spring tension acting on the movable coil ssembly. This reading indicates infinity. **(∞)**

After the ohmmeter is adjusted for zero eading, it is ready to be connected in a circuit to measure resistance. A typical circuit and ohmmeter arrangement is shown in Figure 14-19.

The power switch of the circuit to be measured should always be in the OFF position. This prevents the circuit's source voltage from being applied across the meter, which could cause damage to the meter movement.

The test leads of the ohmmeter are connected across (in series with) the circuit to be measured. (See Figure 14-19.) This causes the current produced by the meter's three-volt battery to flow through the circuit being tested. Assume that the meter test leads are connected at points a and b of Figure 14-19. The amount of current that flows through the meter coil will depend on the resistance of resistors, R_1 and R_2, plus the resistance of the meter coil and **zero-adjust rheostat**. Since the meter has been preadjusted (zeroed), the amount of coil movement now depends solely on the resistance of R_1 and R_2. The inclusion of R_1 and R_2 raised the total series resistance, decreased the current, and thus decreased the pointer deflection. The pointer will now come to rest at a scale figure indicating the combined resistance of R_1 and R_2. If R_1, R_2, or both, were replaced with a resistor(s) having a larger ohmic value, the current flow in the moving coil of the meter would be decreased still more. The deflection would also be further decreased and the scale indication would read a still higher circuit resistance. Movement of the moving coil is proportional to the amount of current flow. The scale reading of the meter, in ohms, is inversely proportional to current flow in the moving coil.

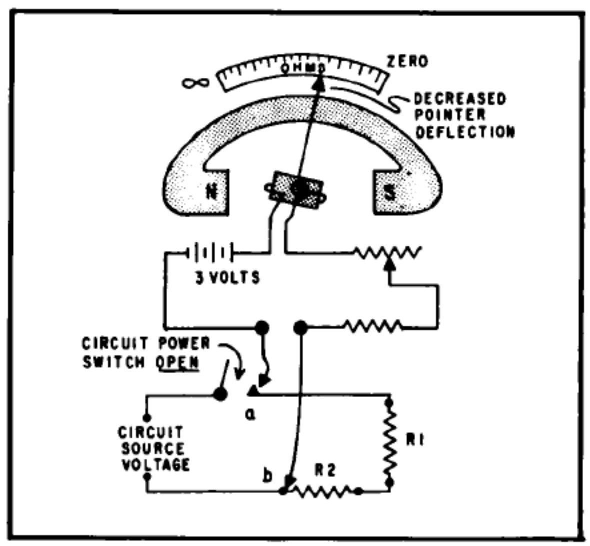

Figure 14-19 - Measuring circuit resistance with an ohmmeter.

A13. Lower because the voltmeter will shunt the circuit.

A14. 66.6 microamperes. (Sensitivity equals the reciprocal of current.)

The amount of circuit resistance to be measured may vary over a wide range. In some cases it may be only a few ohms, and in others it may be as great as 1,000,000 ohms. To enable the meter to indicate any value being measured, with the least error, scale multiplication features are incorporated in most ohmmeters. For example, a typical meter will have four test lead jacks, marked as follows—COMMON, R x 1, R x 10, and R x 100. The jack marked COMMON is connected internally through the battery to one side of the moving coil of the ohmmeter. The jacks marked R x 1, R x 10, and R x 100 are connected to three different size resistors located within the ohmmeter. This is shown in Figure 14-20.

Some ohmmeters are equipped with a selector switch for selecting the multiplication scale desired, so that only two test lead jacks are necessary. The range to be used in measuring any particular unknown resistance (R_x in Figure 14-20) depends on the approximate ohmic value of the unknown resistance. For instance, assume the ohmmeter scale in Figure 14-20 is calibrated in divisions from zero to 1,000. If R_x is greater than 1,000 ohms, and the R x 1 range is being used, the ohmmeter cannot measure it. This occurs because the combined series resistance of resistor R x 1 and R_x is

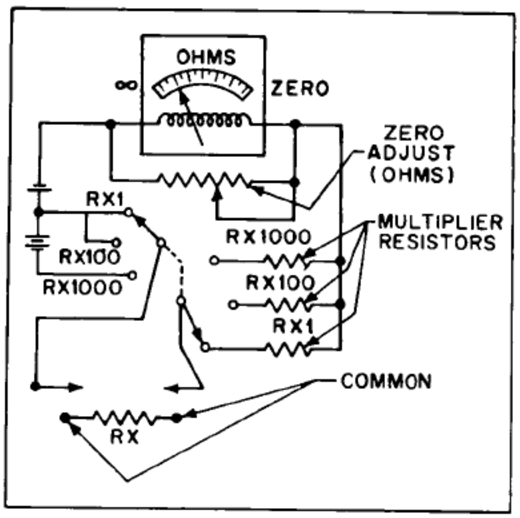

Figure 14-20 - Ohmmeter with multiplication jacks.

too great to allow sufficient battery current to flow to deflect the pointer away from infinity (∞). The test lead would have to be plugged into the next range, R x 10. With this done, assume the pointer deflects to indicate 375 ohms. This would indicate that R_x has 375 x 10 = 3,750 ohms resistance. The change of range caused the deflection because resistor R x 10 has only 1/10 the resistance of resistor R x 1. Thus, selecting the smaller series resistance permitted a battery current of sufficient amount to cause a useful pointer deflection. If the R x 100 range were used to measure the same 3,750 ohm resistor, the pointer would deflect still further, to the 37.5 ohm position. This increased deflection would occur because resistor R x 100 has only 1/10 the resistance of resistor R x 10.

The foregoing circuit arrangement allows the same amount of current to flow through the meter's moving coil whether the meter measures 10,000 ohms on the R x 1 scale, or 100,000 ohms on the R x 10 scale, or 1,000,000 ohms on the R x 100 scale.

It always takes the same amount of current to deflect the pointer to a certain position on the scale (midscale position for example), regardless of the multiplication factor being used. Since the multiplier resistors are of different values, it is necessary to ALWAYS "zero" adjust the meter for each multiplication factor selected. The operator of the ohmmeter should select the multiplication factor that will result in the pointer coming to rest as near as possible to the midpoint of the scale. This enables the operator to read the resistance more accurately, because the scale readings are more easily interpreted at or near midpoint.

14-13. Megger

An ordinary ohmmeter cannot be used for measuring resistance of multimillions of ohms, such as conductor insulation. To adequately test for insulation breakdown, it is necessary to use a much higher potential than is furnished by an ohmmeter's battery. This potential is placed between the conductor and the outside surface of the insulation.

An instrument called a MEGGER (megohmmeter) is used for these tests. The megger (Figure 14-21A) is a portable instrument consisting of two primary elements: (1) a hand-driven dc generator, G, which supplies the necessary voltage for making the measurement, and (2) the instrument portion, which indicates the value of the resistance being measured. The instrument portion is of the opposed-coil type as shown in Figure 14-21A. Coils a and b are mounted on the movable member c with a fixed angular relationship to each other, and

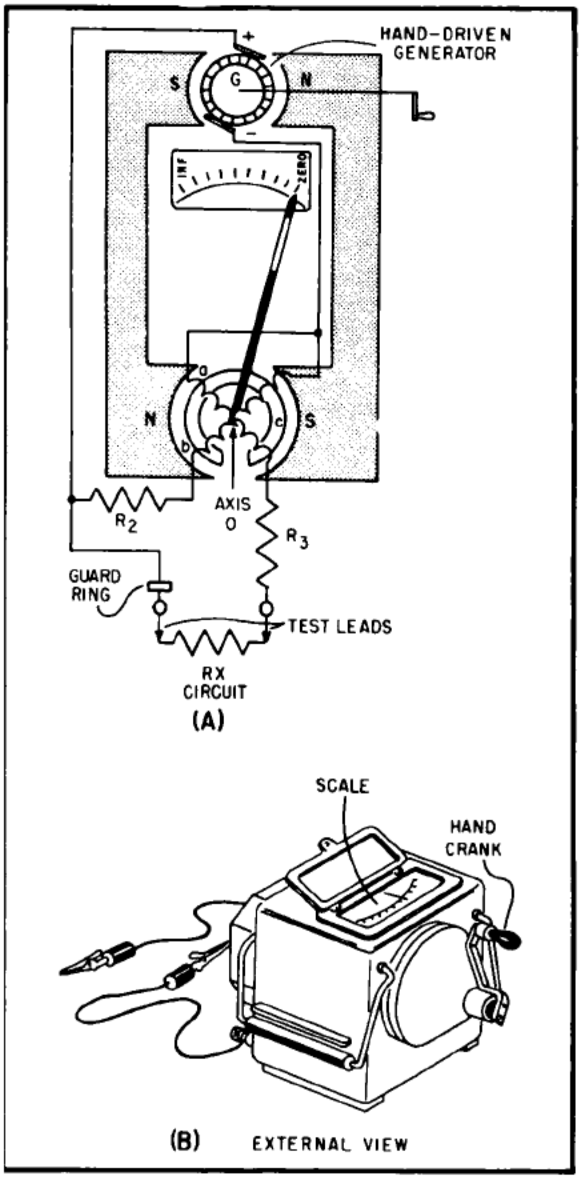

Figure 14-21 - (A) Megger internal circuit; (B) External view of megger.

are free to turn as a unit in a magnetic field. Coil b tends to move the pointer counterclockwise, and coil a clockwise.

Coil a is connected in series with R_3 and the unknown resistance, R_x, to be measured. The combination of coil a, R_3, and R_x form a direct series path between the + and - brushes of the dc generator. Coil b is connected in series with R_2 and this combination is also connected across the generator. There are no restraining springs on the movable member of the instrument portion of the megger. Therefore, when the generator is not operated, the pointer floats freely and may come to rest at any position on the scale.

The guard ring intercepts leakage current. Any leakage currents intercepted are shunted to the negative side of the generator. They do not flow through coil a; therefore, they do not affect the meter reading.

If the test leads are open-circuited, no current flows in coil a. However, current flows internally through coil b, and deflects the pointer to infinity, which indicates a resistance too large to measure. When a resistance such as R_x is connected between the test leads, current also flows in coil a, tending to move the pointer clockwise. At the same time, coil b still tends to move the pointer counterclockwise. Therefore, the moving element, composed of both coils and the pointer, comes to rest at a position at which the two forces are balanced. This position depends upon the value of the external resistance, which controls the relative magnitude of current in coil a. Because changes in voltage affect both coil a and coil b in the same proportion, the position of the moving system is independent of the voltage. If the test leads are short-circuited, the pointer rests at zero because the current in a is relatively large. The instrument is not injured under these circumstances because the current is limited by R_3.

The external view of one type of megger is shown in Figure 14-21B.

Meggers provided aboard ship usually are rated at 500 volts. To avoid excessive test voltages, most meggers are equipped with friction clutches. When the generator is cranked faster than its rated speed, the clutch slips and the generator speed and output voltage are not permitted to exceed their rated values. For extended ranges, a 1,000 volt generator is available. When extremely high resistances such as 10,000 megohms or more, are to be measured, a high voltage is needed to cause sufficient current flow to actuate the meter movement.

Q15. Why should an ohmmeter be zeroed when changing ranges?

Q16. Does an ohmmeter have a linear or nonlinear spacing between scale divisions?

Q17. What will happen to the meter movement in a megger if an extremely high resistance were placed across the leads?

MULTIMETER

The MULTIMETER is a multipurpose instru-

A15. To compensate for variations in multiplying resistor values.

A16. Nonlinear. Because current is an inverse function of voltage with a constant voltage.

A17. Nothing the megger would read infinity.

---

ment combining the feature of an AMMETER, VOLTMETER, and OHMMETER in one instrument. One meter movement is used for all functions with the face of the instrument having separate graduated scales for each of the various functions.

14-14. Description of Multimeter AN/PSM-4A

The PSM-4A is a multimeter that is widely used in the Navy. Figure 14-22 illustrates an external view, showing the switches and jacks, of the PSM-4 multimeter.

There are three controls on the face of the instrument. There is a ten position rotary switch in the lower left hand corner which is used as a function switch. However five of these positions set up ohmmeter connections within the instrument. For these resistance positions, the function switch acts also as a range selector. The eight position switch in the lower right hand corner selects ranges of current and voltage. The zero ohms control is continuously variable and is used to adjust the meter circuit sensitivity to compensate for battery aging in the ohmmeter circuits. It is used to set the pointer at full scale (indicating zero ohms) when the function switch is set to any resistance range and the test probes are shorted together.

This multimeter requires no external source of power. There are two batteries which are self contained and which furnish the required power for resistance measurements. The meter movement has a basic sensitivity of 50 microamperes dc.

Each dc current range uses a separate shunt resistor. The maximum voltage drop across the meter circuit for current measurements is 250 millivolts.

DC voltage measurements are made with ranges which have 20,000 ohms per volt input resistance. The maximum current through the meter for voltage measurements is 50 microamperes.

AC voltage measurements are made with ranges which have 1,000 ohms per volt input resistance. The maximum current through the instrument for ac voltage measurements is 1 milliampere (RMS). The frequency response is flat from 20 cps to over 10,000 cps.

Output voltages are read as ac voltages. They use the 1,000 ohms per volt ac volt circuits and place a blocking capacitor in series with the ac voltage circuits. The purpose of this type of measurement is to determine the strength of the ac portion of voltage when there is a combination of ac and dc voltage present in the circuit which is under test.

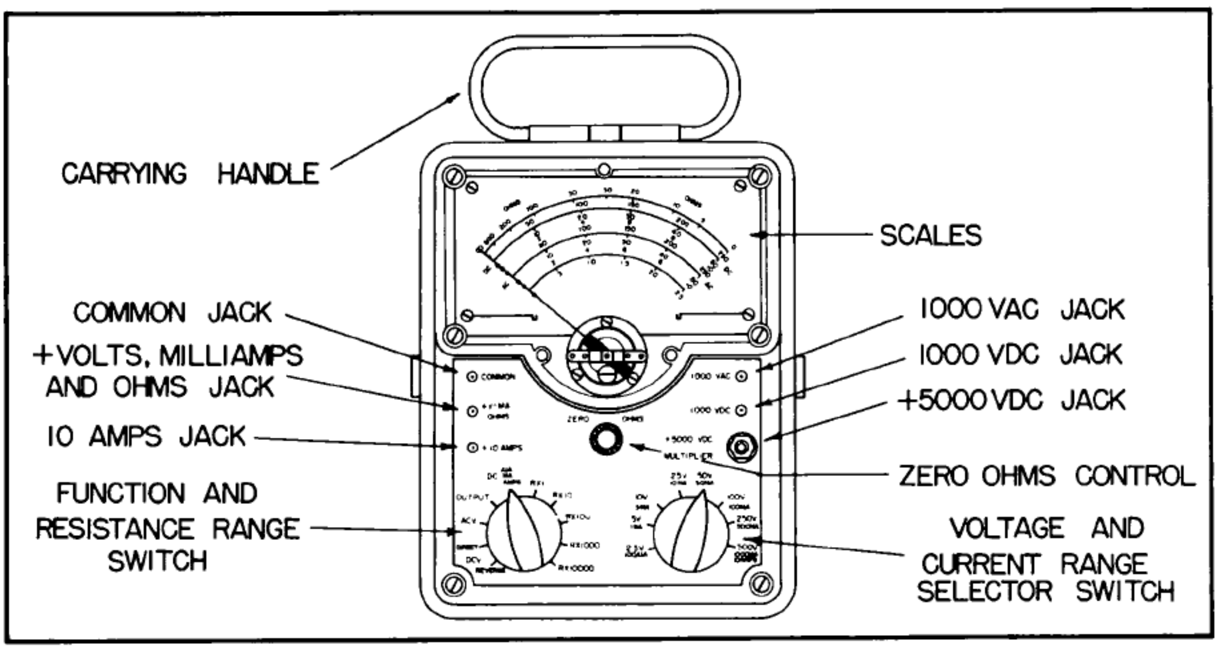

Figure 14-22 - AN/PSM-4A Multimeter

Resistances are measured with a basic circuit which has 30 ohms of internal resistance, and which uses a 1.5 volt battery source. For the R x 10 and R x 100 ranges, circuit resistances are increased to 300 and 3000 ohms respectively, and the same 1.5 volt battery is used. For the R x 1000 and R x 10,000 ranges, the resistances are 30,000 and 300,000 ohms, respectively, and power is furnished from a 22.5 volt battery.

NOTE: The basic ohmmeter circuits discussed in section 14-12 uses series resistance to change the range of the ohmmeter. The AN/PSM-4 uses shunt resistance which accounts for the opposite values of resistance for R x 1, R x 10, etc.

14-15. Operation of Multimeter

DC VOLTMETER CIRCUITS: When measuring dc voltages the function switch is set to either the DIRECT or REVERSE dc volt position. The reverse dc volt position is a feature which allows negative voltages to be measured without reversing the test leads and also protects against shock when measuring high negative voltages by keeping the meter case near equipment ground potential. Zero to 500 VDC may be measured by proper manipulation of the function and range selector switches. The test leads are placed, black in common jack and red in VOM jack. For the 1000 volt range the red lead is inserted in the 1000 VDC jack. To measure from 1000 to 5000 VDC the special test lead (W-103) is inserted in the 5000 volt multiplier jack.

AC AND OUTPUT VOLTAGE CIRCUITS: The circuits which measure AC and OUTPUT voltages are selected by turning the function switch to the ACV or OUTPUT positions. Voltage ranges are selected with the range switch and the test leads are used in the same jacks as for dc.

DC CURRENT CIRCUITS: The circuit which measures dc current is selected by turning the function switch to the position marked DC (uA, MA, amps). For currents up to 1 ampere the range is selected with the range switch. For the 10 ampere range the red test lead is plugged into the 10 amps jack.

OHMMETER CIRCUITS: The ohmmeter circuit and its ranges are selected with the function switch. The position R x 1, R x 10, etc., have been explained previously (section 14-12).

ALTERNATING-CURRENT INSTRUMENTS

In the beginning of this chapter the simpler electrical indicating instruments were discussed. These were used to measure only direct voltage or current. It will be necessary for the technician to become familiar with additional, more advanced ac indicating instruments. Those that will be discussed in this part of the chapter are (1) rectifier-type ac meters; (2) wattmeters and watt-hour meters, (3) electrodynamometer (4) moving vane and (5) thermocouple.

Many of the instruments discussed in this chapter utilize metallic rectifiers. Since these units are common to a number of different instruments, they will be discussed first.

14-16. Metallic Rectifiers

A metallic rectifier is a device that offers a high opposition to current flow through it in one direction but not in the other. It is therefore effectively a unidirectional conductor and is used mostly for converting alternating current into unidirectional current (direct current).

A metallic rectifier element, called a cell, consists of a good conductor and a semiconductor (material of high resistivity) separated by a thin insulating barrier layer. The flow of forward current through a cell consists of a flow electrons from the good conductor, across the barrier layer, and through the semiconductor.

Metallic rectifier cells are usually made in the form of plates, circular or square in shape, with a hole in the center. A number of cells, with the necessary terminals, spacers, and washers, are assembled on an insulated stud passing through their center holes. This assembly is called a rectifier stack. Some assemblies contain fins, which are used to keep the rectifier from overheating; they afford a large surface area for conducting away the heat. Cells and stacks may be connected in series or parallel, with proper polarities, to obtain the required voltage and current ratings and circuit connections for specific applications.

Two types of metallic rectifiers are used in the Navy: (1) a thin film of copper oxide and copper, and (2) selenium and either iron or aluminum. Metallic rectifier units are represented by the symbol shown in Figure 14-23A. The arrowhead in the symbol points against the direction of electron flow.

Figure 14-23B shows a simple ac circuit that utilizes a metallic rectifier. It rectifies the ac voltage and produces a series of dc voltage pulses as its output. Although the copper-oxide rectifier is shown, the selenium rectifier may be used instead.

In the copper-oxide rectifier shown in Fig-

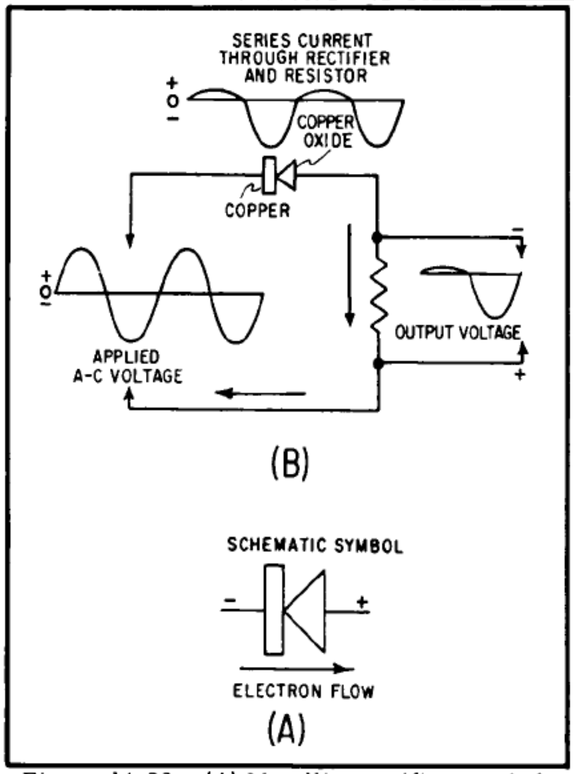

Figure 14-23 - (A) Metallic rectifier symbol; (b) Waveforms in simple ac utilizing a rectifier.

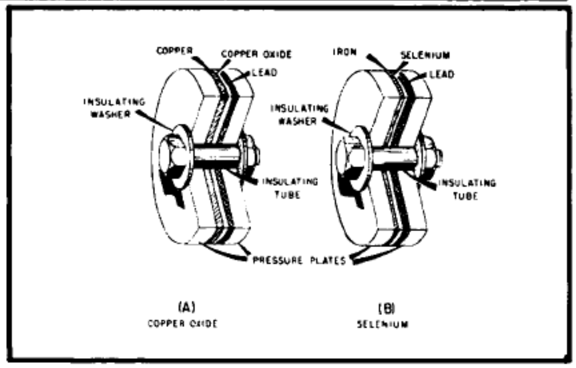

Figure 14-24 - Metallic rectifier construction.

ure 14-24A, the oxide is formed on the copper disk before the rectifier unit is assembled. In this type of rectifier the electrons flow more readily from the copper to the oxide than from the oxide to the copper. External electrical connections may be made by connecting terminal lugs between the left pressure plate and the copper and between the right pressure plate and the lead washer.

For the rectifier to function properly, the oxide coating must be very thin. Thus, each individual unit can stand only a low inverse voltage. Rectifiers designed for moderate and high-power applications consist of many of these individual units mounted in series on a single support. The lead washer enables uniform pressure to be applied to the units so that the internal resistance may be reduced. When the units are connected in series, they normally present a relatively high resistance to the current flow. The resultant heat developed in the resistance must be removed if the rectifier is to operate satisfactorily. Many commercial rectifiers have copper fins between each unit for the purpose of dissipating the excess heat. The useful life of the unit is extended by keeping the temperature low (below 140°F). The efficiency of this type of rectifier is generally between 60 and 70 percent.

Selenium rectifiers function in much the same manner as copper-oxide rectifiers. A selenium rectifier is shown in Figure 14-24B. Such a rectifier is made up of an iron disk that is coated with a thin layer of selenium. In this type of rectifier the electrons flow more easily from the selenium to the iron than from the iron to the selenium.

Commercial selenium rectifier units are designed to pass 50 milliamperes per square centimeter of plate area. This type of rectifier may be operated at a somewhat higher temperature than a copper-oxide rectifier of similar rating. The efficiency is between 65 and 85 percent, depending on the circuit and the loading. As in the case of the copper-oxide rectifier, any practical number of units may be bolted together in series to increase the voltage rating. Larger element disks and the necessary cooling fins may be used for higher current ratings. Also, forced-air cooling may be used.

Metallic rectifiers may be used not only as half-wave rectifiers, as shown in Figure 14-23B but also in full-wave and bridge circuits. In each of these applications the action of the metallic rectifier is similar to that of a diode.

Metallic rectifiers may be used in battery charges, instrument rectifiers, and many other applications including welding and electroplating. Commercial radios also frequently use selenium rectifiers in the high-voltage power supply, as do other electronic equipments.

Q18. Why are cooling fins necessary on most metallic rectifiers?

BASIC AC MEASURING DEVICES

14-17. Moving Iron-Vane Meter

The moving iron-vane method is another

basic type of meter. Unlike the D'Arsonval-type meter, which employs permanent magnets, the moving iron-vane meter depends on induced magnetism for its operation. It employs the principle of repulsion between two concentric iron vanes, one fixed and one movable, placed inside a coil, as shown in Figure 14-25. A pointer is attached to the movable vane.

When current flows through the coil, the two iron vanes become magnetized with north poles at their upper ends and south poles at their lower ends for one direction of current through the coil, as shown in Figure 14-25. Because like poles repel, the unbalanced component of force tangent to the movable element causes it to turn against the force exerted by the springs.

The movable vane is rectangular in shape, and the fixed vane is tapered. This design permits the use of a relatively uniform scale.

When no current flows through the coil, the movable vane is positioned so that it is opposite the larger portion of the tapered fixed vane, and the scale reading is zero. The amount of magnetization of the vanes depends on the strength of the field, which in turn, depends on the amount of current flowing through the coil. The force of repulsion is greater opposite the larger end of the fixed vane than it is nearer the smaller end. Therefore, the movable vane moves toward the smaller end through an angle that is proportional to the magnitude of the coil current. The movement ceases when the force of repulsion is balanced by the restoring force of the spring.

Because the repulsion is always in the same direction (toward the smaller end of the fixed vane) regardless of the direction of current

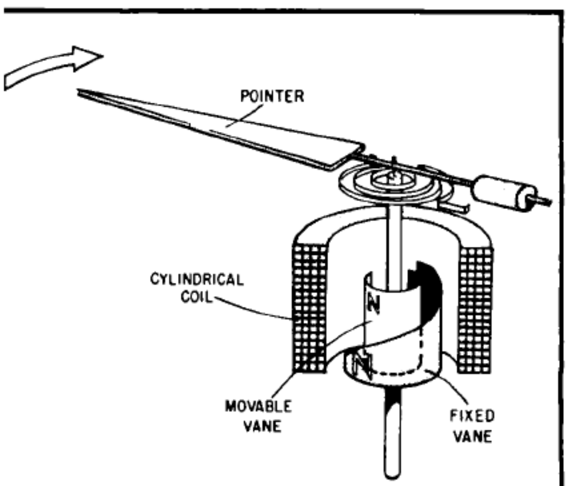

Figure 14-25 - Simplified diagram of a moving iron-vane meter.

flow through the coil, the moving iron-vane instrument operates on either dc or ac circuits.

Mechanical damping in this type of instrument is obtained by the use of an aluminum vane attached to the shaft (not shown in the figure) in such a way that, as the shaft moves, the vane moves in a restricted air space.

When the moving iron-vane meter is designed to be used as an ammeter, the coil is wound with relatively few turns of large wire in order to carry the rated current.

When the moving iron-vane meter is designed to be used as a voltmeter the solenoid is wound with many turns of small wire. Portable voltmeters are made with self-contained series resistance for ranges up to 750 volts. Higher ranges are obtained by the use of additional external multipliers.

The moving iron-vane instrument may be used to measure direct current, but has an error due to residual magnetism in the vanes. The error may be minimized by reversing the meter connections and averaging the readings. When used on ac circuits the instrument has an accuracy of 0.5 percent. Because of its simplicity, its relatively low cost, and the fact that no current is conducted to the moving element, this type of movement is used extensively to measure current and voltage in ac power circuits.

However, because the reluctance of the magnetic circuit is high, the moving iron-vane meter requires much more power to produce full-scale deflection than is required by a D'Arsonval meter of the same range. Therefore, the moving iron-vane meter is seldom used in high-resistance low-power circuits.

Q19. What is the advantage of having a tapered vane on the moving iron-vane meter movement?

14-18. Rectifier-Type Meter

The average value of an alternating voltage or current is zero. Therefore, if an alternating current is applied to a dc meter movement the pointer will indicate zero. If the frequency of the alternating current is low (in the neighborhood of 60 cycles) the pointer will quiver about the zero mark in its attempt to follow the variations of the current through the meter coil. However, if the ac voltage or current is rectified and passed to the meter coil as a series of dc pulses the meter pointer will indicate the average value of these pulses. Therefore, it is possible to connect a D'Arsonval direct-current type instrument and a rectifier so as to measure ac quantities. The rectifier is usually of the dry-plate type (such as copper oxide or selenium rectifiers) and is arranged in a bridge circuit, as shown in Figure 14-26. By the use

A18. To dissipate heat and extend the useful life of the rectifier.

A19. Uniformity of scale division.

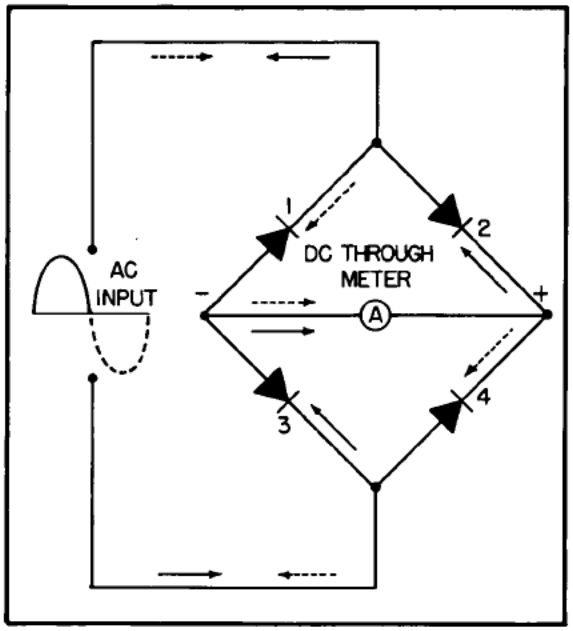

Figure 14-26 - Simple connection of a full-wave, rectifier-type, ac instrument.

of rectifiers in the bridge, it can be seen that current flow through the meter is always in one direction. When the voltage being measured has a waveform as shown in Figure 14-26, the path of current flow will be from the lower input terminal through rectifier No. 3 through the instrument, and then through rectifier No. 2, thus completing its path back to the source's upper terminal. The next half cycle of the input voltage (indicated by dotted sine wave) will cause the current to pass through rectifier No. 1, through the instrument, and through rectifier No. 4, completing its path back to the source. If the instrument is used in the simplified version shown there will be some error introduced due to variations in waveforms and frequency. In a commercial instrument, however, corrective (compensating) networks are added which make it practically free from error up to 100KC. An instrument of this type requires a current from the line of only about one milliampere for full scale deflection. It is widely used for ac voltmeters, especially of the lower ranges.

There is some "aging" of the rectifier with a corresponding change in the calibration of the instrument. Because of this aging such instruments must be calibrated from time to time. A rectifier-type instrument reads the average value of the ac quantity. However, because the EFFECTIVE, or root-mean-square (RMS), values are more useful, ac meters are generally calibrated to read RMS values (1.11 times the average of the instantaneous values).

Example. The average value of an ac voltage applied to the meter is 30 volts. What will be the value indicated by the meter pointer?

The meter is calibrated in RMS volts, thus, the meter will read:

$$E_{RMS} = E_{avg} \times 1.11$$

$$E_{RMS} = 30 \times 1.11$$

$$E_{RMS} = 33.3 \text{ volts}$$

The construction of the practical ac voltmeter and ammeter is the same as the dc meters with the exception of the rectifying element and a capacitor in series with the test lead to remove any dc component of the voltage or current under test.

Q20. Why does a dc voltmeter read zero when placed across an ac voltage source?

Q21. An ac rectifier type meter indicates 100 volts, what is the average value of voltage across the meter movement?

14-19. Electrodynamometer-Type Meter

The electrodynamometer-type meter differs from the galvanometer-type meter in that no permanent magnet is used. Instead, two fixed coils are utilized to produce the magnetic field. Two movable coils are also used in this type meter.

The two fixed coils are connected in series and positioned coaxially, with a space between them. The two movable coils are also positioned coaxially, and are connected in series. The two pairs of coils (fixed pair and movable pair) are further connected in series with each other. The movable-coil unit is pivot-mounted between the fixed coils.

This meter arrangement is illustrated in Figure 14-27.

The central shaft on which the movable coils are mounted is restrained by spiral springs which hold the pointer at zero when no current is flowing through the coil. These springs also serve as conductors for delivering current to the movable coils. Since these conducting

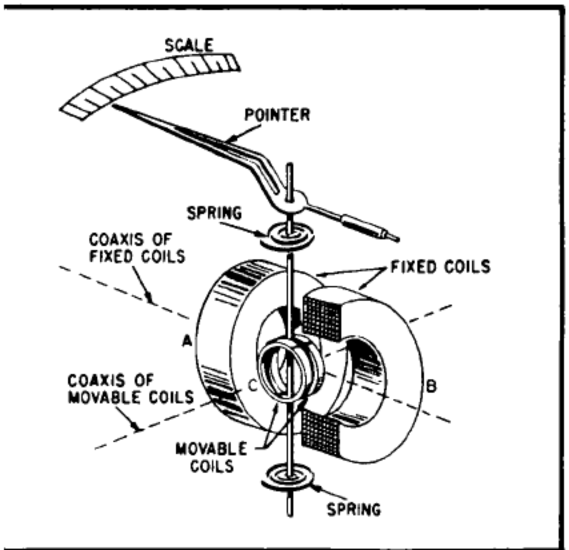

Figure 14-27 - Inside construction of an electrodynamometer.

prings are very small, the meter cannot carry very heavy current.

When used as a voltmeter, no difficulty in onstruction is encountered, because the curent required is not more than 0.1 ampere. This amount of current can be brought in and ut of the moving coil through the springs. When the electrodynamometer is used as a oltmeter, its internal connections and contruction are as shown in Figure 14-28A. The ixed coils a and b are wound with fine wire, ince the current through them will be no more han 0.1 ampere. They are connected directly n series with the movable coil c and the series urrent-limiter resistance. For ammeter aplications, however, a special type construction nust be used, because the large currents that low through the meter cannot be carried through he moving coils.

In the ammeter, the stationary coils a and b f Figure 14-28B, are generally wound of heaver wire, to carry up to five amperes. In parllel with the moving coils is an inductive shunt, which permits only a small part of the total curent to flow through the moving coil. This curent through the moving coil is directly proporional to the total current through the instrunent. The shunt has the same ratio of reactnce to resistance as has the moving coil, thus he instrument will be reasonably correct at all requencies with which it is designed to be used.

The meter is mechanically damped by means f aluminum vanes that move in enclosed air hambers. Although electrodynamometer type neters are very accurate, they do not have the ensitivity of the D'Arsonval type meter. For

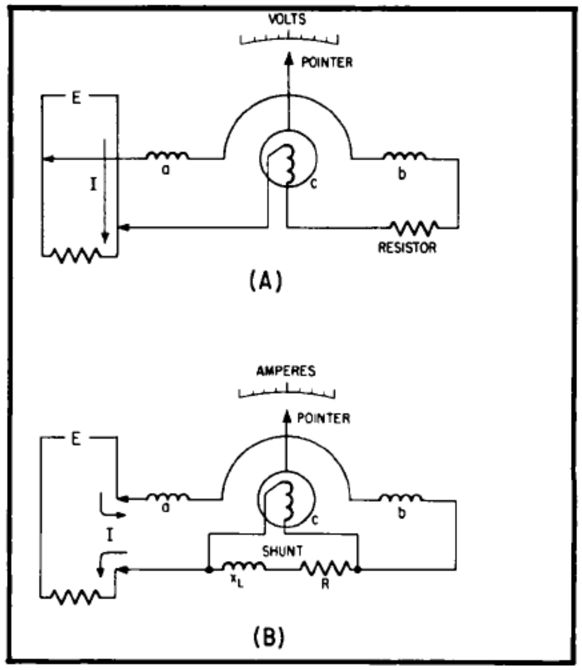

Figure 14-28 - Circuit arrangement of electrodynamometer type meter, (A) voltmeter, (B) ammeter.

this reason they are not widely used outside the laboratory.

Q22. What is the main difference in construction between the electrodynamometer type meter movement and others?

14-20. Thermocouple-Type Meter

If two of the ends of two dissimilar metals are welded together and this junction is heated, a dc voltage is developed across the two open ends. The voltage developed depends on the material of which the wires are made and on the difference in temperature between the heated junction and the open ends.

In one type of instrument, the junction is heated electrically by the flow of current through a heater element. It does not matter whether the current is alternating or direct because the heating effect is independent of current direction. The maximum current that may be measured depends on the current rating of the heater, the heat that the thermocouple can stand without being damaged, and on the current rating of the meter used with the thermocouple. Voltage may also be measured if a suitable resistor is placed in series with the heater.

A simplified schematic diagram of one type of thermocouple is shown in Figure 14-29. The

A20. DC voltmeters indicate average values, and the average value of ac is zero.

A21. 90.09 volts. $E_{avg} = \frac{E_{RMS}}{1.11}$

A22. No permanent magnet is used.

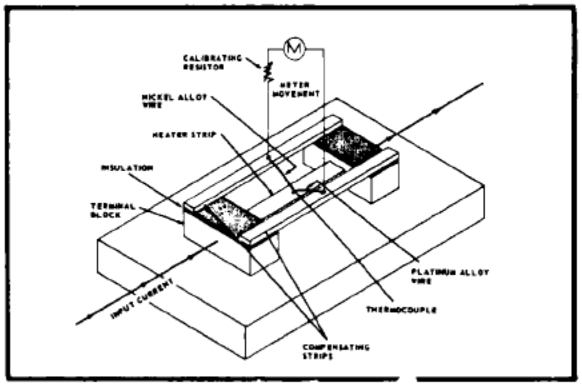

Figure 14-29 - Simplified schematic of one type of thermocouple.

input current flows through the heater strip via the terminal blocks. The function of the heater strip is to heat the thermocouple, which is composed of a junction of two dissimilar wires welded to the heater strip. The open ends of these wires are connected to the center of two copper compensating strips. The function of these strips is to radiate heat so that the open ends of the wires will be much cooler than the junction end of the wires; thus permitting a higher voltage to be developed across the open ends of the thermocouple. The compensating strips are thermally and electrically insulated from the terminal blocks.

The heat produced by the flow of line current through the heater strip is proportional to the square of the heating current ($P=I^2R$). Because the voltage appearing across the two open terminals is proportional to the temperature, the movement of the meter element connected across these terminals is proportional to the square of the current flowing through the heater element. The scale of the meter is crowded near the zero end, and is progressively less crowded near the maximum end of the scale. Because the lower portion of the scale is crowded the reading is necessarily less accurate. For the sake of accuracy in making a given measurement, it is desirable to choose a meter in which the deflection will extend at least to the more open portion of the scale.

The meter used with the thermocouple should have low resistance to match the low resistance of the thermocouple, and it must deflect full scale when rated current flows through the heater. Because the resistance must be low and the sensitivity high, the moving element must be light.

A more nearly uniform meter scale may be obtained if the permanent magnet of the meter is constructed so that as the coil rotates (needle moves up scale); it moves into a magnetic field of less and less density. The torque then increases approximately as the first power of the current instead of as the square of the current, and a more linear scale is achieved.

If the thermocouple is burned out by excessive current through the heater strip, it may be replaced and the meter recalibrated by means of the calibrating variable resistor.

A positive error is introduced into the readings at high frequencies due to skin effect. However, proper design will limit this error to a value less than 1% well into the 20 or 30 megacycle range.

A particular advantage of the thermocouple is that, being very accurate for dc, it can be calibrated with dc and then used to calibrate, low frequency ac instruments.

Q23. What must be done when a thermocouple is replaced?

Q24. What does the voltage developed across a thermocouple depend on?

14-21. Wattmeter

Electric power is measured by means of a wattmeter. This instrument is of the electrodynamometer type. It consists of a pair of fixed coils, known as current coils, and a movable coil, known as the potential coil. (See Figure 14-30.) The fixed coils are made up of a few turns of comparatively large conductor. The potential coil consists of many turns of fine wire; it is mounted on a shaft, carried in jeweled bearings, so that it may turn inside

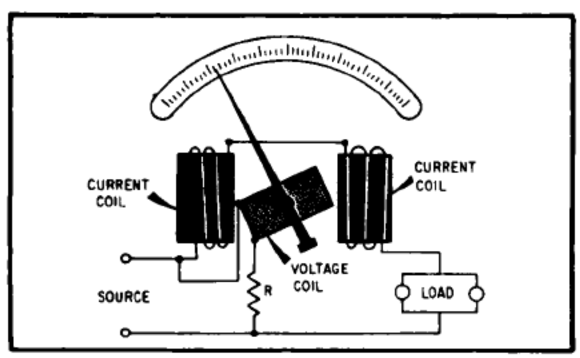

Figure 14-30 - Simplified electrodynamometer wattmeter circuit.

the stationary coils. The movable coil carries a needle which moves over a suitably graduated scale. Flat coil springs hold the needle to a zero position.

The current coil (stationary coil) of the wattmeter is connected in series with the circuit (load), and the potential coil (movable coil) is connected across the line.

When line current flows through the current coil of a wattmeter, a field is set up around the coil. The strength of this field is proportional to the line current and in phase with it. The potential coil of the wattmeter generally has a high resistance resistor connected in series with it. This is for the purpose of making the potential-coil circuit of the meter as purely resistive as possible. As a result, current in the potential circuit is practically in phase with line voltage. Therefore, when voltage is impressed on the potential circuit, current is proportional to and in phase with the line voltage.

The actuating force of a wattmeter is derived from the interaction of the field of its current coil and the field of its potential coil. The force acting on the movable coil at any instant (tending to turn it) is proportional to the product of the instantaneous values of line current and voltage.

The wattmeter consists of two circuits, either of which will be damaged if too much current is passed through them. This fact is to be especially emphasized in the case of wattmeters, because the reading of the instrument does not serve to tell the user that the coils are being overheated. If an ammeter or voltmeter is overloaded, the pointer will be indicating beyond the upper limit of its scale. In the wattmeter, both the current and potential circuits may be carrying such an overload that their insulation is burning, and yet the pointer may be only part way up the scale. This is because the position of the pointer depends upon the power factor of the circuit as well as upon the voltage and current. Thus, a low power factor circuit will give a very low reading on the wattmeter even when the current and potential circuits are loaded to the maximum safe limit. This safe rating is generally given on the face of the instrument.

A wattmeter is always distinctly rated, not in watts but in volts and amperes.

Figure 14-31 shows the proper way to connect a wattmeter into a circuit.

Q25. Whay type of power does the electrodynamometer wattmeter measure?

CATHODE-RAY OSCILLOSCOPE

The electrical measuring instruments dis-

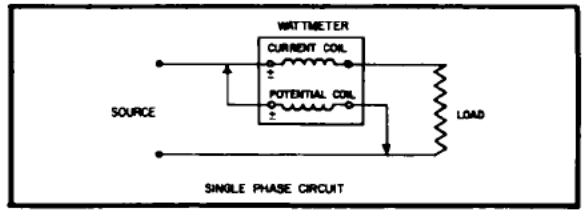

Figure 14-31 - Wattmeter connection.

cussed so far have employed essentially the same system for indicating the value of the property being measured. That is, readings are taken by observing the position of a pointer which is deflected across the face of a scale a distance which is proportionate to an amount of voltage, current, or resistance applied to the input terminals of the instrument. By use of these instruments it is possible to determine only the magnitude of the property connected to the input. For instance, an ac voltmeter is connected across an ac circuit and the pointer indicates a voltage of 120 volts (RMS). This reading merely indicates that the effective value of the ac voltage present is 120 volts, it does not indicate the type of signal (sine wave, or more complex waveforms), frequency, phase, etc. Quite often it is desirable to know these characteristics.

A picture of the amplitude variations of a current with respect to time can be obtained by use of an OSCILLOSCOPE. The CATHODE-RAY OSCILLOSCOPE is an instrument consisting of a cathode-ray tube and associated circuits for use in viewing wave shapes of voltages or currents.

14-22. Cathode-Ray Tube

The cathode-ray tube is a special type of vacuum tube; which finds its most familiar use in the commercial television set.

The CRT contains four major parts:

The CONTAINER, which, in most cases, is an evacuated glass envelope. The size of this envelope will vary according to the type of instrument or equipment the CRT is to be used in.

The ELECTRON GUN. An arrangement of tube elements to introduce electrons into the container, accelerate them, and form them into a narrow beam.

The DEFLECTION SYSTEM. A control system which allows the electron stream to be deflected or moved in a desired manner. There are two types of deflection systems; (1) electrostatic and (2) electromagnetic.

The SCREEN. A flat area of the container, covered on the inside with a fluorescent material. The electron stream is directed at the screen. The fluorescent material on the screen

A23. The meter must be recalibrated by use of the variable resistor.

A24. The difference in temperature between the junction and the open ends of the wires and the material of which the junction is made.

A25. Average power.

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transforms the electrical energy of the electron stream into light.

Figure 14-32 illustrates a very simple form of the CRT, representing a very early step in the development of the more complex tubes of today. The electrons are emitted from the heated cathode (K). The electrons are accelerated by the attraction of the anode (A), and may reach a velocity of 10,000 miles per second or more, depending on this force of attraction. Most of the electrons strike the anode and return to the cathode through the external circuit. Some of the electrons, however, pass through the small opening in the anode and proceed without appreciable loss of velocity straight to the screen. Although the electrons possess equal negative charges, and thus tend to repel each other, the beam is scattered very little, since the electrons are traveling too swiftly for any scattering action to be effective.

In early tubes of the type shown in Figure 14-32 there was little control of either the number or the direction of the electrons emitted from the cathode. This caused high current in the external power supply. Modern tubes have overcome the disadvantages of the early tubes by the addition of control grids, focusing elements, etc.

Figure 14-33 illustrates the elements of a typical CRT using ELECTROSTATIC CONTROL.

Electrostatic control indicates that the deflection and focusing of the CRT are accomplished by virtue of electrostatic fields existing between the various tube elements. In the CRT of Figure 14-33 electrons emitted by the cathode are focused and accelerated by the ac-

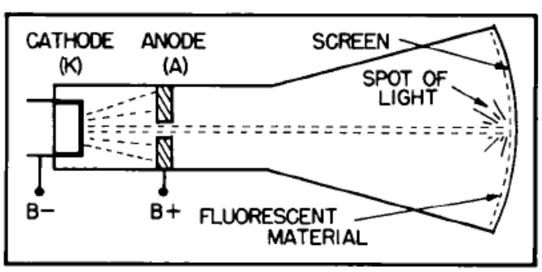


Figure 14-32 - Simple cathode-ray tube.

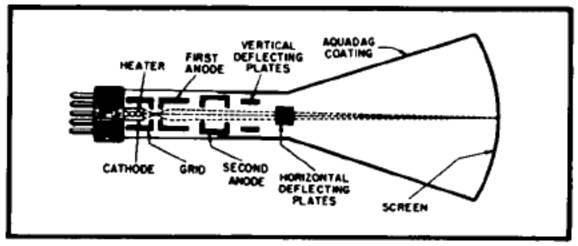


Figure 14-33 - Construction of cathode-ray tube using electrostatic deflection and focusing.

tion of the grid and anodes. By virtue of the apertures in the various tube elements and the form of the electric field around the two anodes, the electron stream is formed into a narrow beam which passes between each of the two sets of deflecting plates before reaching the screen. The beam is caused to move about on the screen by applying various potentials to the deflection plates.

Application of a varying potential between the vertical plates (Figure 14-33) will cause the beam to be deflected vertically (up and down) on the screen. Application of a varying potential between the horizontal plates will cause the beam to be deflected horizontally (back and forth) on the screen.

When no potential is applied to either pair of deflecting plates the electron beam will not be deflected and will produce a spot of light in the center of the screen. Application of an alternating signal between the vertical plates only will cause the beam to be deflected first towards one vertical plate and then towards the other. Since there is no deflection in the horizontal direction the result will be a vertical line in the center of the screen. Removing the signal from the vertical plates and connecting it to the horizontal plates only will result in a horizontal line across the center of the screen. With signals applied to both pairs of deflection plates at the same time the deflection of the beam will be the resultant of the two forces at any instant. In other words, if both the top vertical plate and the right horizontal plate (looking from the front of the screen) acquire the same value of positive potential the spot will be deflected towards the upper right hand corner.

One of the most general uses of the "scope" is the observation of the shape of voltage waveforms with respect to time. In the conventional method of representing a voltage or current waveform the vertical axis represents amplitude and the horizontal axis represents time. Therefore, the oscilloscope must present its information in this form if it is to be of value.

To do so, a voltage must be impressed on the horizontal deflection plates which will move the electron beam from left to right at a constant rate of speed. Since the electron stream strikes the screen at only one point at any instant, it is possible to form a line, only by producing in rapid succession many spots of light which are close together. The human eye retains any image for approximately one sixteenth of a second, so that a motion which is fast enough appears as a blur because successive images overlap. If the horizontal deflection voltage causes the spot to retract its path more than 16 times a second, the image will be a line which appears stationary on the screen. The voltage that causes this horizontal deflection is called a sweep voltage because it sweeps the spot across the screen. The line which the moving spot generates, is called the time base since it is a line whose length represents a definite period of time.

The time-base or sweep generator produces a waveform as shown in Figure 14-34. This is called a sawtooth waveform and is applied between the horizontal deflection plates. At point A of the waveform the spot is at the left hand side of the screen. From point A to point B the spot is swept, in a linear manner, across the screen from left to right. At point B the spot is at the left hand side of the screen. During time B to C of the waveform the spot is returned rapidly to the right side of the screen ready to start the next sweep.

If, during the same time the sawtooth waveform is being applied to the horizontal plates a sine wave is applied to the vertical plates, the sine wave will be reproduced on the screen by the moving spot. Figure 14-35 illustrates how this is accomplished.

At zero degrees the sawtooth, which is applied to the horizontal deflection plates, has moved the beam to the left hand side of the screen. At zero degrees the sine-wave, which is applied to the vertical deflection plates, is at its zero reference line (indicating no difference in potential between the vertical plates). As the sawtooth waveform starts its sweep across the tube, from left to right, the sine-

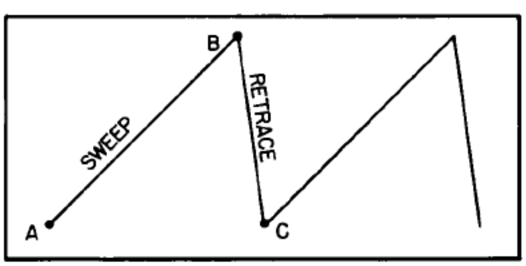


Figure 14-34 - Sawtooth waveform.

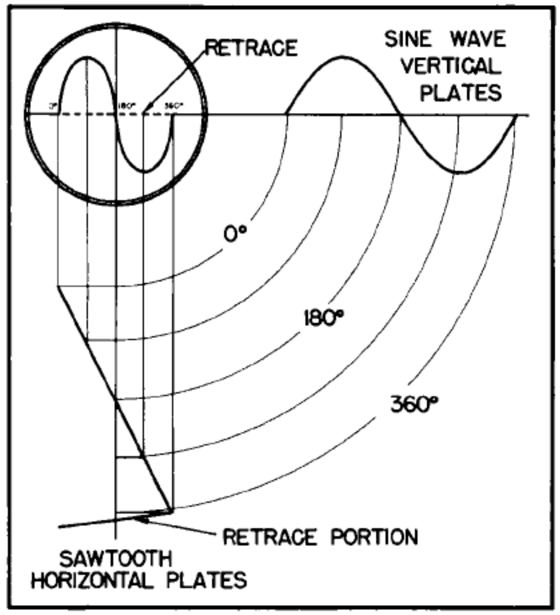


Figure 14-35 - Reproduction of pattern on scope screen.

wave begins to deflect the beam towards the top of the screen. At 90 degrees the sawtooth has completed a fourth of its sweep and the sine-wave has deflected the beam in its maximum direction toward the top of the screen. At 180 degrees both deflection voltages are passing through their respective zero reference lines. Therefore, with equal potentials on all deflecting plates the spot will be in the center of the screen. At a 180 degrees (Figure 14-35) the pattern on the screen is shown passing through the junction of the vertical and horizontal axis. As the sawtooth sweep continues to move the beam to 360 degrees, at the right hand side of the screen, the sine-wave deflects the beam towards the bottom of the screen and then back to the horizontal zero reference line. At 360 degrees the sawtooth sweep retraces the beam rapidly back to the left hand side of the screen ready to begin a new sweep. The retrace is shown on the scope screen (Figure 14-35) as a dotted line because in most cases it will be too swift to produce a bright trace.

The second type of control is ELECTROMAGNETIC. Electromagnetic control indicates that the deflection and focusing of the CRT are accomplished by magnetic fields.

The production of the electron beam in an electromagnetic CRT is essentially the same as in the electrostatic tube. The grid structure is similar, and the use of the grid to control the number of electrons in the beam is identi-

cal. The first anode serves the same purpose of providing acceleration. It is at this point that the similarity ends, for in place of the second anode and the deflecting plates the electromagnetic CRT possesses a focus coil and a pair of deflecting coils. These coils are wound around the neck of the tube.

The advantages of electromagnetic tubes over electrostatic tubes are their structural simplicity (the electromagnetic tube has no deflecting plates or focusing anode that must be carefully aligned); their greater ruggedness, which makes for greater reliability in mobile equipment; and their shorter tube length, which reduces the overall size of the equipment in which these types are used.

Electrostatic tubes, on the other hand, require little or no deflection current or power. The auxiliary circuits are therefore simpler and difficulties from deflecting coil inductances are avoided. Most CRT oscilloscopes use electrostatic tubes.

Q26. What is the difference between electromagnetic and electrostatic control in CRT's?

14-23. Basic Cathode-Ray Oscilloscope

Oscilloscopes are divided into two general types—electrostatic and electromagnetic—depending on the method of deflection used in the CRT. Practically all oscilloscopes used as test instruments are of the electrostatic type. The electromagnetic type is used in television and in radar sets.

Figure 14-36 illustrates the block diagram of a basic oscilloscope. The horizontal deflection amplifier is a high gain amplifier that increases the amplitude of the horizontal input voltage and applies it to the horizontal deflection plates.

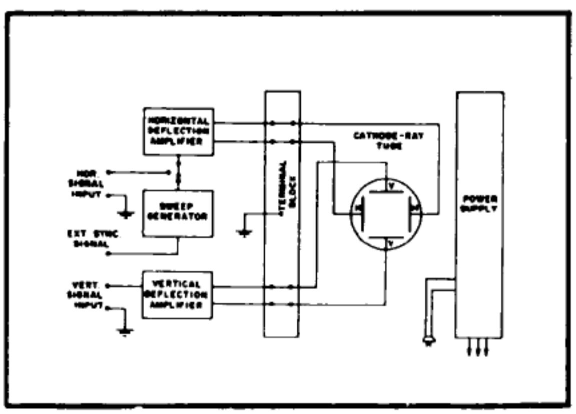


Figure 14-36 - Block diagram of basic oscilloscope.

The sweep generator supplies a saw-tooth voltage to the input of the horizontal amplifier.

The vertical amplifier increases the amplitude of the vertical input voltage before applying it to the vertical deflection plates.

The power supply provides all dc voltages for the tubes, including a high dc accelerating potential for the CRT.

14-24. Oscilloscope Controls

Since Oscilloscope OS-8C/U is operated in a conventional manner, only a basic knowledge of cathode ray oscilloscopes is required for its application and operation. Therefore, this section will be concerned with the specific controls of the equipment and their functions.

The front panel views illustrated in Figure 14-37A and B show the location of all operating controls.

INT.-OFF - Operating the intensity control clockwise turns the power on to the instrument and the pilot light will indicate that the instrument is on. As this control is operated further clockwise, it controls the intensity of the pattern on the cathode ray tube. When moved to full clockwise position, the pattern is at maximum brilliancy.

FOCUS - This control adjusts the focus, or sharpness, of the trace on the screen of the cathode ray tube. If the trace appears fuzzy or blurred the focus control is adjusted until the trace is clear and sharp.

HORIZONTAL POSITION - The purpose of the horizontal positioning control is to adjust the pattern on the screen horizontally (back and forth).

VERTICAL POSITION - The purpose of the vertical positioning control is to adjust the position of the pattern on the screen vertically (up and down).

VERT. ATTEN. - Applications of too large a signal to the vertical input will cause the display to extend beyond the viewing area of the screen. This control attentuates the signal fed in at the vertical input (ac) connector by a factor of 1, 10 or 100. When turned to the "dc" position, it permits the dc voltages fed in between the dc input and GND to be amplified by the vertical amplifier. Positive dc voltages will cause the beam to move up on the screen.

NOTE: Always operate the VERT. ATTEN. switch to the highest attenuator position in which suitable vertical deflection can be obtained. If this is not done, overloading of the cathode follower will generally result. Overloading can be detected by a clipping or squashing of the pattern. A clockwise rotation of the attenuator switch will attenuate the signal to a greater degree.

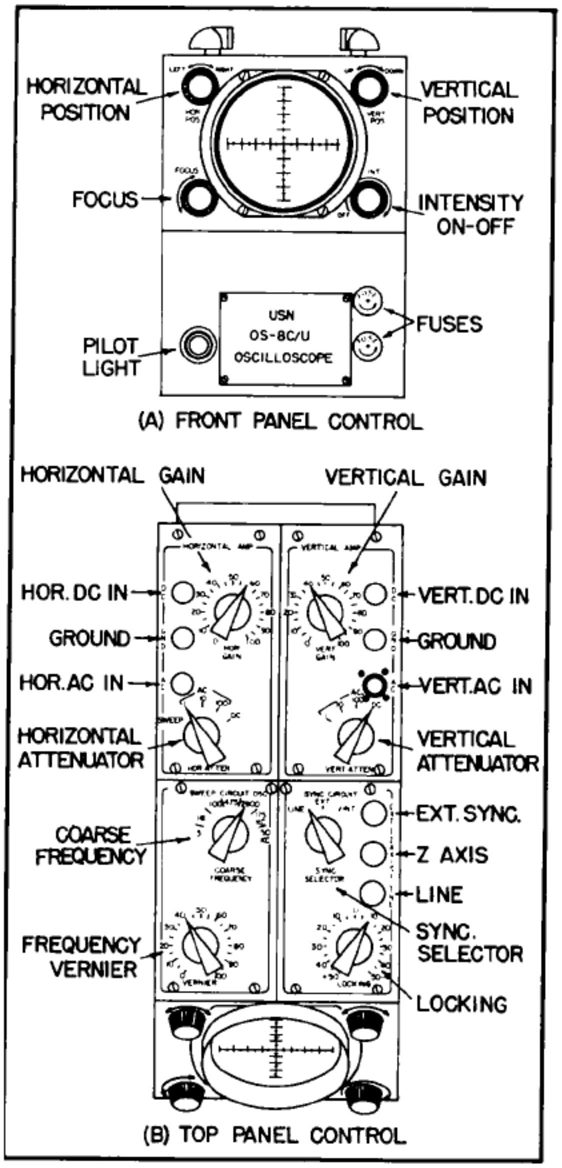


Figure 14-37 - Oscilloscope controls, OS-8C/U.

VERT. GAIN - This control is used as a ernier in connection with the VERT. ATTEN. control the height of the pattern on the screen the case of ac voltages; and in the case of dc oltages, the extent of deflection, either up or own, of the beam. The position of the gain ontrol has no effect on bandwidth when the at-nuator is in the "ac" positions.

HOR. ATTEN. - This control attenuates the ignal fed in at the horizontal input (ac) con-ector by a factor of 1, 10 and 100. When turned to the "dc" position, it permits the dc voltages fed in between the dc input and GND to be amplified by the horizontal amplifier. Positive dc voltages will cause the beam to move to the right on the screen. This control, when turned to the "SWEEP" position, permits the sawtooth from the sweep circuit oscillator to be amplified by the horizontal amplifier, thus providing horizontal deflection.

NOTE: Always operate the HOR. ATTEN. switch to the highest attenuator position in which suitable horizontal deflection can be obtained. If this is not done, overloading of the cathode follower will generally result. Overloading can be detected by a clipping or squashing of the pattern.

HOR. GAIN - This control is used as a vernier in connection with the HOR. ATTEN. to control the width of the pattern on the screen in the case of external ac voltages; and in the case of dc voltages, the extent of deflection, either left or right, of the beam. When the HOR. ATTEN. is in the "SWEEP" position, the HOR. GAIN controls the width of the sweep.

COARSE FREQUENCY - This control selects the range of frequencies in the internal sweep circuit oscillator which operates between the limits of 3 and 50,000 cycles. Although the frequency ranges are marked on the panel for convenience of the operator, these frequencies are only approximate and, in general, the actual frequency range will be much greater so that two consecutive frequency ranges will exhibit a sizeable overlap.

VERNIER-FREQUENCY - This control serves as a vernier on the frequency being generated by the sweep circuit oscillator in any one of the six positions of the COARSE FREQUENCY control.

SYNC. SELECTOR - This control selects synchronizing voltage for applications to the sweep circuit oscillator. These synchronizing voltages may be selected either from an external source, internal source which is the voltages being applied to the vertical amplifiers, or from an internal source of line frequency voltage.

LOCKING - This control permits selection of either positive or negative peaks of synchronizing voltages and, in addition, controls the extent of locking voltage applied to the sweep circuit oscillator.

14-25. Terminals

VERTICAL INPUT (ac) - Input for ac voltages deflecting the beam vertically on the cathode ray tube screen.

A26. Electrostatic control uses electric fields and electromagnetic control uses magnetic fields.

VERTICAL INPUT (dc) - Input for dc voltages applied to the vertical amplifiers.

HORIZONTAL INPUT (ac) - Input for ac voltages deflecting the beam horizontally on the cathode ray tube screen.

HORIZONTAL INPUT (dc) - Input for dc voltages applied to the horizontal amplifiers.

GND (2) - Direct connection to chassis of equipment and to one side of all other externally applied voltage.

EXT. SYNC - Input for external synchronizing voltages to be used in synchronizing the sweep circuit oscillator.

LINE - A source of line supply frequency to be used either in causing deflection for horizontal or vertical inputs, or as a source of line frequency for any other use to which it might be put.

Z AXIS - Connection for an external voltage to be used in intensity modulating the cathode-ray tube beam.

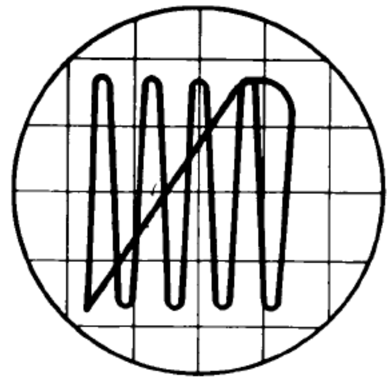
Q27. If a scope is turned on and no trace is visible on the screen which control should be checked first?

Q28. Which controls should be checked next?

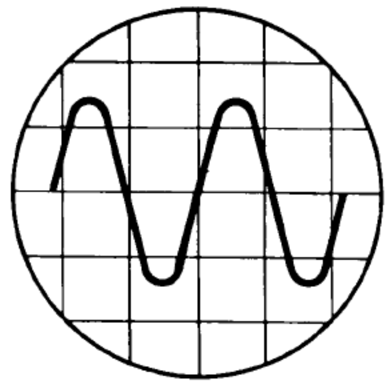
14-26. Oscilloscope Operation

(1) OBSERVING WAVE FORMS USING INTERNAL SWEEP AND SYNC.

Connect the source of alternating voltage to be observed to the vertical input (ac) and GND connections. Set the coarse frequency control to the slowest sweep frequency, position "3-18". The SYNC. SELECTOR should be turned to "INT", while the LOCKING control is turned to the zero position. Adjust VERT. GAIN and VERT. ATTEN. for suitable vertical deflection of pattern on the scope screen. Display should occupy only two thirds of the screen height and should be centered. Stretching the pattern to full height will result in distortion of the wave shape. Adjust HOR. GAIN until the pattern is of the desired width. When the pattern first appears it will usually show many cycles as the picture of the sine wave under observation in Figure 14-38A. Slowly rotate the VERNIER-FREQUENCY until the number of cycles decreases to the desired number. If the number is still greater than convenient, then COARSE FREQUENCY should be rotated to the next clockwise position and fewer cycles will appear as shown in Figure 14-38B. When the desired number of cycles are obtained, the trace can be locked in by rotating the LOCKING control either clockwise or counter-clockwise, depending upon whether it is desired to lock in positive or negative synchronizing pulses.



(A)



(B)

Figure 14-38 - Waveforms.

(2) OBSERVING WAVE FORMS USING INTERNAL SWEEP AND EXT. SYNC.

Follow all steps outlined above with the following exception:

SYN. SELECTOR is turned to "EXT" rather than "INT" and the source of synchronizing voltage is applied between the EXT. binding post and GND.

(3) OBSERVING WAVE FORMS USING INTERNAL SWEEP WITH LINE FREQUENCY SYNCHRONIZING VOLTAGES.

Follow all steps outlined previously with the following exception:

When the sweep circuit is to be locked in

.t line frequency, SYNC. SELECTOR is turned o "LINE".

4) OBSERVING WAVE FORMS USING INTERNAL SINE WAVE LINE FREQUENCY SWEEP.

Connect the source of alternating voltage to be observed between the vertical input (ac) and GND. Set the HOR. ATTEN. to the ac divided by 10 position. Make an electrical connection between the LINE binding post and the horizontal input (ac) binding post. Operate the HOR. GAIN and VERT. GAIN controls to give the desired size of pattern. LOCKING, VERNIER-FREQUENCY and SYNC. SELECTOR controls have no effect upon the operation.

5) OBSERVING PATTERNS WITH SINE WAVE VOLTAGES IN BOTH HORIZONTAL AND VERTICAL INPUTS.

Connect the two voltages for comparison to the oscilloscope, one on the horizontal input (ac) and one on the vertical input (ac). Adjust the HOR. ATTEN., and VERT. ATTEN. to the highest attenuation position that will give suitable deflection in both directions. Adjust the HOR. GAIN and VERT. GAIN controls until the pattern is of the desired size. With the above controls so adjusted, as the two frequencies become exact ratios of one another definite patterns (called LISSAJOUS PATTERNS and explained in the next section) will appear on the screen.

6) VERTICAL DEFLECTION WITH DC INPUT.

Operate the VERT. ATTEN. control to the "dc" position. Apply dc voltage to the (dc) vertical input connection and adjust the VERT. GAIN to give the desired deflection sensitivity.

7) HORIZONTAL DEFLECTION WITH DC INPUT.

Operate the HOR. ATTEN. control to the "dc" position. Apply dc voltages to the (dc) horizontal input connection and adjust the HOR. GAIN control to give the desired deflection sensitivity (explained in next section).

Q29. Why shouldn't the pattern be expanded to the edge of the screen?

OSCILLOSCOPE AS A MEASURING DEVICE

Although the most common use of the cathode-ray oscilloscope is the observation of waveforms there are other measurements to which the scope is well adapted. In many cases when using the scope as a measuring device it is necessary to know the deflection sensitivity of the scope.

14-27. Deflection Sensitivity

The deflection sensitivity of an oscilloscope may be defined as the distance in millimeters that the spot is moved on the screen when 1 volt is applied to the deflection plates, expressed in millimeters per volt. The most accurate way of measuring this quantity is to apply a known dc potential directly to the deflection plates and to measure the distance that the spot is moved by this voltage. The number of millimeters that the spot moves, divided by the voltage applied, is the deflection sensitivity. Most electrostatic cathode-ray tubes have sensitivities which are less than 1 millimeter per volt. This same figure may be expressed in terms of the voltage required to move the spot 1 inch. To convert from millimeters per volt to volts per inch, divide 25.4 by the sensitivity in millimeters per volt.

Example. Application of 1 volt causes the spot to be deflected a distance of 0.528 millimeters. Therefore, the deflection sensitivity is 0.528 millimeters per volt.

Converting into volts per inch:

$$\text{Deflection sensitivity} = \frac{25.4}{0.528} = 48 \text{ V per inch}$$

In other words, if an unknown voltage were applied to the plates of this CRT and the spot were deflected one inch, then the unknown voltage would be 48 volts. If the spot were deflected two inches, then the applied voltage would be 96 volts (2 x 48), etc.

When it is desired to use the amplifier with the oscilloscope, the gain of the amplifier must be calibrated. However, a more direct procedure is to apply a calibration voltage of the same units (dc, peak-peak, etc.) as the voltage to be measured, to the vertical input of the scope. Assume the voltage to be measured is in peak-peak units. A peak-peak signal of 1 volt is applied to the vertical input of the scope (with the VERT. ATTEN. in the times 1 position) the VERT. GAIN control is adjusted so that the pattern occupies 10 vertical squares on the screen. As long as the GAIN CONTROLS ARE NOT TOUCHED 10 squares on the screen equal 1 volt. Upon application of the unknown voltage to the vertical input it is found that the pattern on the screen occupies 15 squares. Therefore, the unknown voltage is equal to 1.5 volts P-P. If the gain control is changed, the overall sensitivity of the scope will also change. It must be remembered that the sensitivity of the cathode-ray tube is NOT affected by changes in the gain control setting. The only factor changed is the amplitude of the voltage applied to the deflecting plates.

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A27. The intensity control.

A28. Deflection controls (positions). Because beam may be centered off screen.

A29. Distortion of the waveshape will result.

14-28. DC Voltmeter

The electrostatic cathode-ray tube is a voltage-operated device. The amount of deflection of the spot is proportional to the magnitude of the voltage applied to the deflection plates. If the deflection sensitivity of the cathode-ray tube is known, the oscilloscope can be used as a voltmeter on either direct or alternating voltages. The oscilloscope has the advantage of extremely high input impedance when the voltage to be measured is applied directly to the deflection plates.

14-29. AC Voltmeter

The cathode-ray oscilloscope is a better device for measuring alternating voltages than most conventional ac voltmeters. The principal difficulty with the oscilloscope is the calibration of its deflection sensitivity. If this factor can be determined accurately, the magnitude of an alternating voltage can be determined very simply. The advantages of the oscilloscope as an ac voltmeter are its very high input impedance, its ability to measure equally well voltages of a very wide frequency range, and its ability to indicate magnitude regardless of waveform. The oscilloscope measures the peak value of the ac voltage applied. The standard ac meters show the RMS value of a sine-wave ac voltage, which may be converted to a peak value, but the results may be very misleading for voltages whose waveforms are other than sinusoidal.

14-30. Frequency Measurement

Determination of an unknown frequency is accomplished by use of a scope and LISSAJOUS FIGURES. A lissajous figure is a pattern created on an oscilloscope screen when sine-wave voltages are applied to both the horizontal and vertical deflection plates. The principal use of the lissajous figure is in the determination of an unknown frequency by comparison with a known frequency. Figure 14-39 is used to show the development of a lissajous figure.

A sine-wave signal is applied to the horizontal input of the scope. After amplification it is applied to the horizontal deflection plates. This signal is shown as one cycle along the vertical axis in Figure 14-39. Another sine-wave signal is applied to the vertical input of the scope.

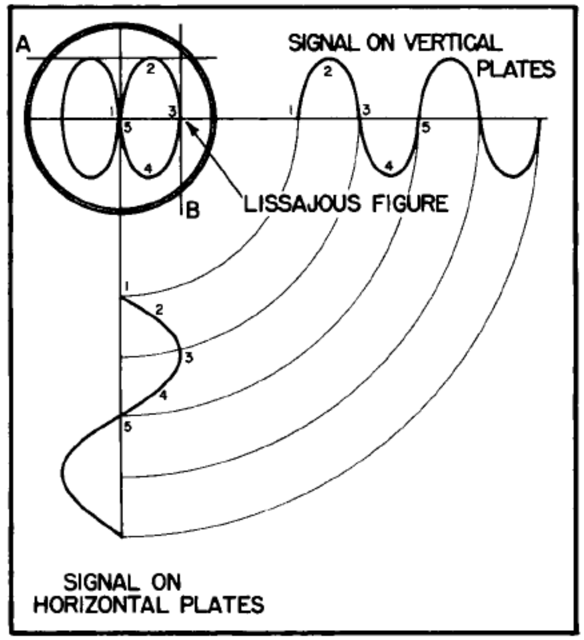

Figure 14-39 - Lissajous figure development.

After amplification it is applied to the vertical deflection plates of the scope. This signal is shown as two cycles along the horizontal axis in the figure. The resultant pattern of these two signals is shown on the scope screen as a figure eight laying on its side. The development of this resultant figure can be explained in the following manner: Point one of both voltages occur on their respective axis at zero. The electron beam will start its trace at the center of the CRT screen (point one on the screen). At point 2 the signal on the horizontal plates, which is acting as the sweep voltage, has moved the beam about half way towards the right hand side of the screen. At the same time the signal on the vertical plates has reached its peak value. The spot on the screen, being the resultant of these two forces, is shown at point 2. At point 3 the sweep signal has reached its peak deflection toward the right side, while at the same time the signal on the vertical plates is going through zero. Therefore, at point 3 the spot on the screen will be at the right edge of the screen and at zero on the X axis. From point 3 to point 5 the sweep signal is tracing its way back to the center of the screen while the signal on the vertical plates goes from zero through a peak point and back to zero. At point 5 both signals are at zero, thus, the spot is again in the center of the screen. The action described above is now repeated with the sweep signal tracing its way to the left hand edge of the screen and back to the

nter, while the signal on the vertical plates es through a complete cycle.

In order to use the lissajous figure in fre-ency measurements it is necessary to super-pose a vertical and a horizontal line (lines and B in Figure 14-39) on the pattern. The mber of times the pattern touches these lines ll form a ratio, which is stated as follows:

equency applied to rt. deflection ıtes	=	Number of times pattern touches horiz. line (B)
equency applied to riz. deflection ıtes		Number of times pattern touches vert. line (A)

$$\frac{f_V}{f_H} = \text{Ratio of vert. freq. to horiz. freq.}$$

For the lissajous pattern in Figure 14-39

$$\frac{f_V}{f_H} = \frac{2}{1}$$

From the above it can be seen that the ver-al frequency is twice the horizontal frequen-.

Using lissajous figures to determine an un-ɔwn frequency is accomplished in the follow-; manner: The known frequency of 1 kc that to be used as the comparison standard is ap-ed to the horizontal input. The unknown fre-ɜncy is applied to the vertical input. A lis-jous pattern (Figure 14-40) is observed on : screen of the scope. Following the proce-re outlined previously:

$$\frac{f_V}{f_H} = \frac{3}{2}$$

therefore: $$\frac{f_V}{1\ \text{kc}} = \frac{3}{2}$$

Transpose and solve for the vertical frequency:

$$f_V = \frac{3}{2} \times 1{,}000$$

$$f_V = 1.5\ \text{kc}$$

The unknown frequency is 1.5 kc

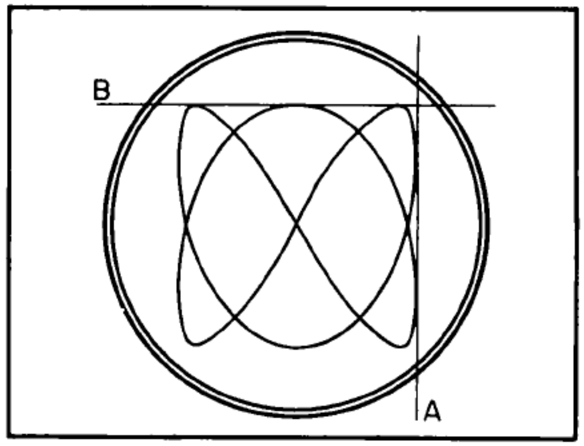

Figure 14-40 - Lissajous pattern vert. to horiz. ratio of 3:2

Q30. Does including the scope amplifiers, when using the scope as a measuring device, effect the sensitivity of the CRT?

Q31. What does a lissajous pattern with a vertical to horizontal ratio of 5:1 indicate?

A30. No. Only the amplitude of the voltage applied to the deflection plates is changed.

A31. The frequency applied to the vertical plates is 5 times that applied to the horizontal plates.

EXERCISE 14

1. On what principle does the D'Arsonval movement operate?
2. Give the uses of the hairsprings in the D'Arsonval meter movement.
3. How is an ammeter hooked in a circuit to measure line current?
4. Explain how a shunt resistor reduces current through a meter movement.
5. Explain how a multiplier resistor reduces current through a meter movement.
6. Why can the connection of a voltmeter to a circuit change the circuit's operation?
7. Explain the purpose of the dry disk rectifiers in an ac voltmeter.
8. What values of voltage is an ac voltmeter calibrated to read, and what values does the meter movement read?
9. Which type of ac voltmeter has the highest frequency response and why?
10. What component in an ohmmeter is replaced when it will no longer zero properly?
11. Does a good voltmeter have a high or low sensitivity? Explain.
12. Explain why an ammeter must never be connected in parallel with a voltage source.
13. Explain why connecting a voltmeter in series with a circuit will give a false indication.
14. How is a wattmeter connected in a circuit?
15. How important is the circuit phase angle when using a wattmeter? Explain.
16. What three basic units does a multimeter read?
17. How many deflection plates are used for horizontal positioning?
18. To which set of deflection plates is the sweep circuit usually connected?
19. Why is a sawtooth used as a sweep voltage?
20. What is the vertical frequency if the horizontal frequency is 400 cps, and the vertical to horizontal ratio of the lissajous pattern is 5:3?

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